Nuclear matter properties at finite temperature from effective interactions

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- Introduction
- Ab initio theory: SCGF, χEMBPT
- Effective theory: ImMDI-GF, Skχm*
- Properties of NM and PNM
- Summary

Based on work [PRC 100, 024618 (2019)] in collaboration with
Arianna Carbone, Jun Xu and Zhen Zhang

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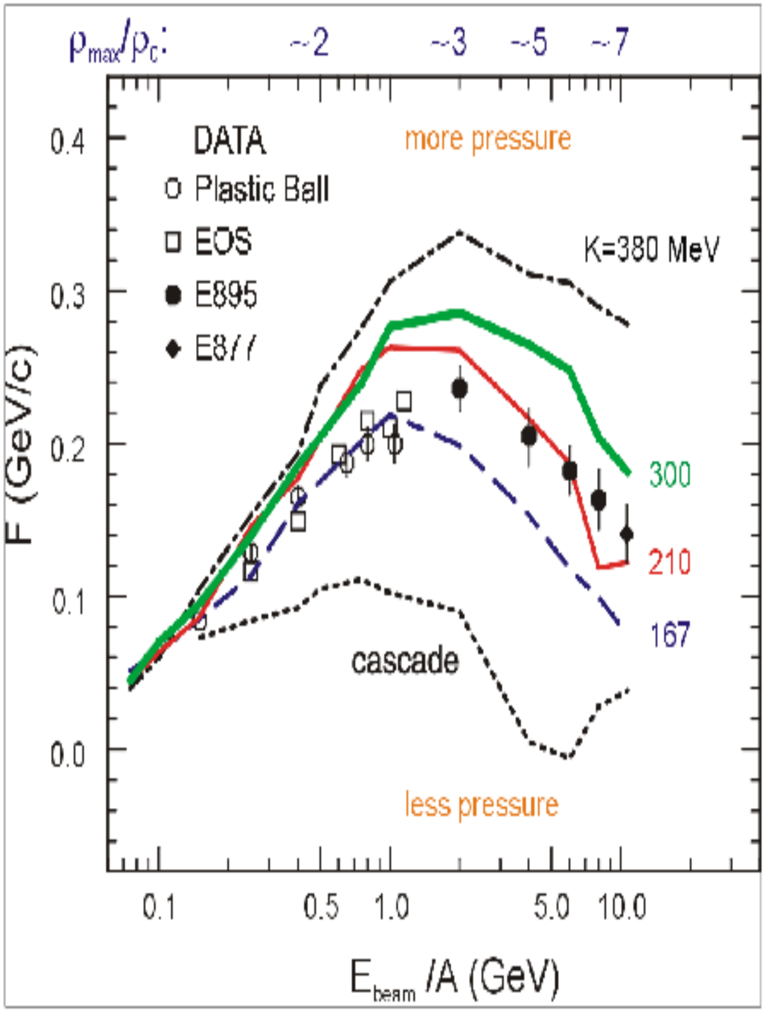
Question raised at NUSYM18

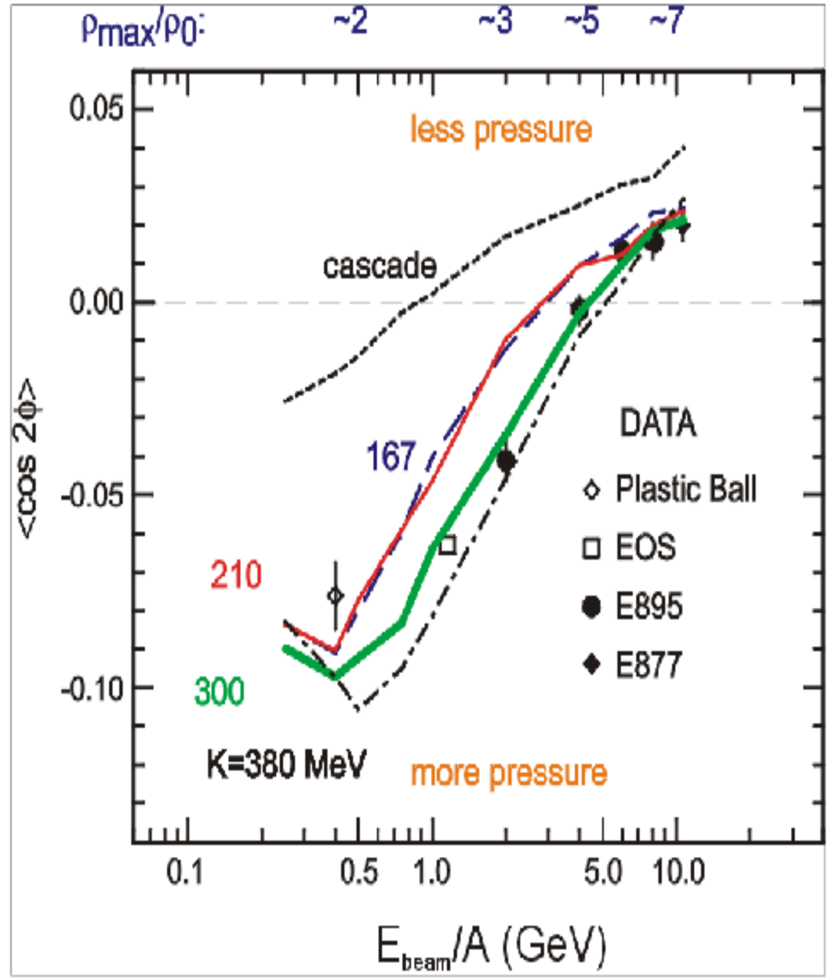
How does the nuclear equation of state determined from heavy ion collisions differ from that for a cold nuclear matter that is needed for studying the properties of neutron stars?

Discussions among Maria Colonna, Pawel Danielewicz, Arianna Carbone, Jeremy Holt, Hermann Walter, Jun Xu, ...

Direct and elliptic flows

Danielewicz, Lacey & Lynch, Science 298, 1592 (2002)





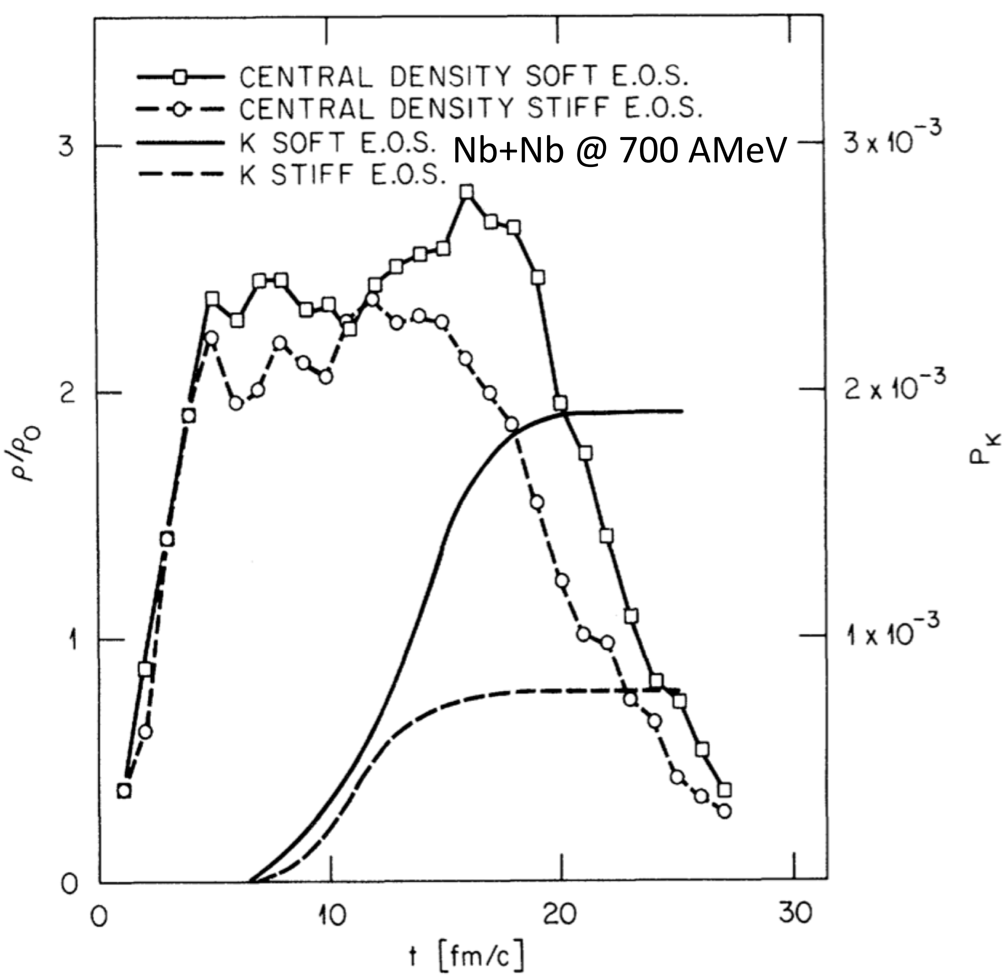
Direct flow $F = \left(\frac{dp_x}{dy}\right)_{y_{cm}}$

Elliptic flow $v_2 = \langle \cos 2\varphi \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$

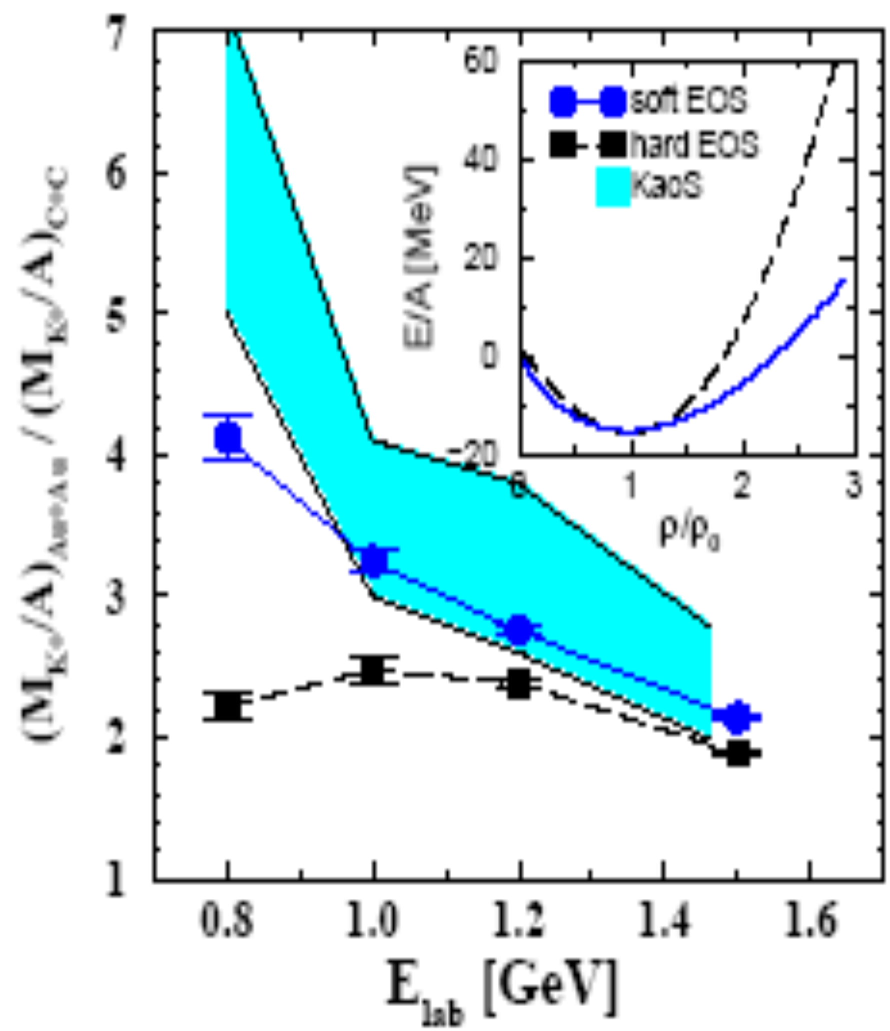
- Data consistent with a nuclear EOS with K ≈ 200 -300 MeV.

Subthreshold kaon production in high-energy HIC

Aichelin & Ko, PRL 55, 2661 (1985)

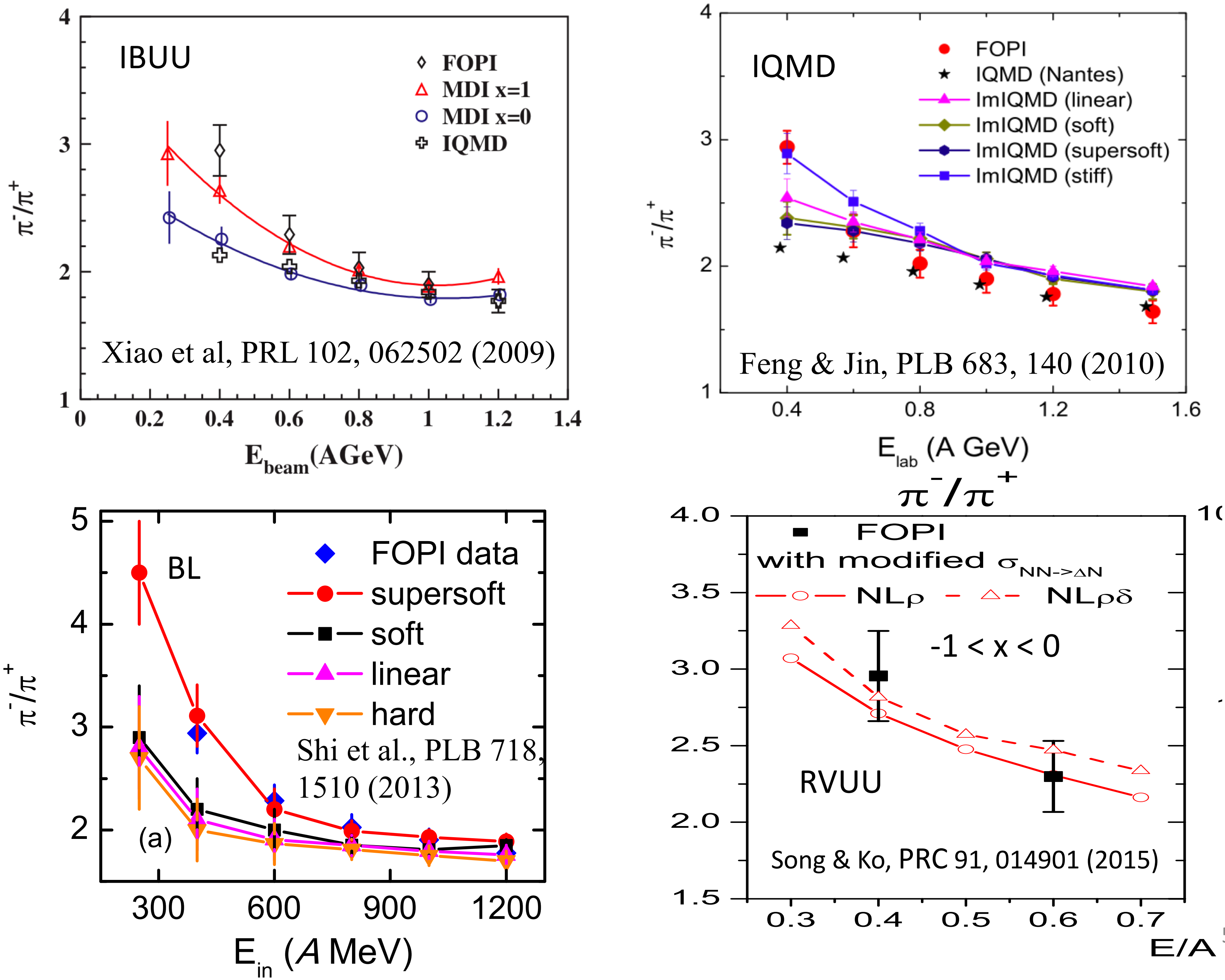


Fuchs, PRL 86, 1974 (2001)



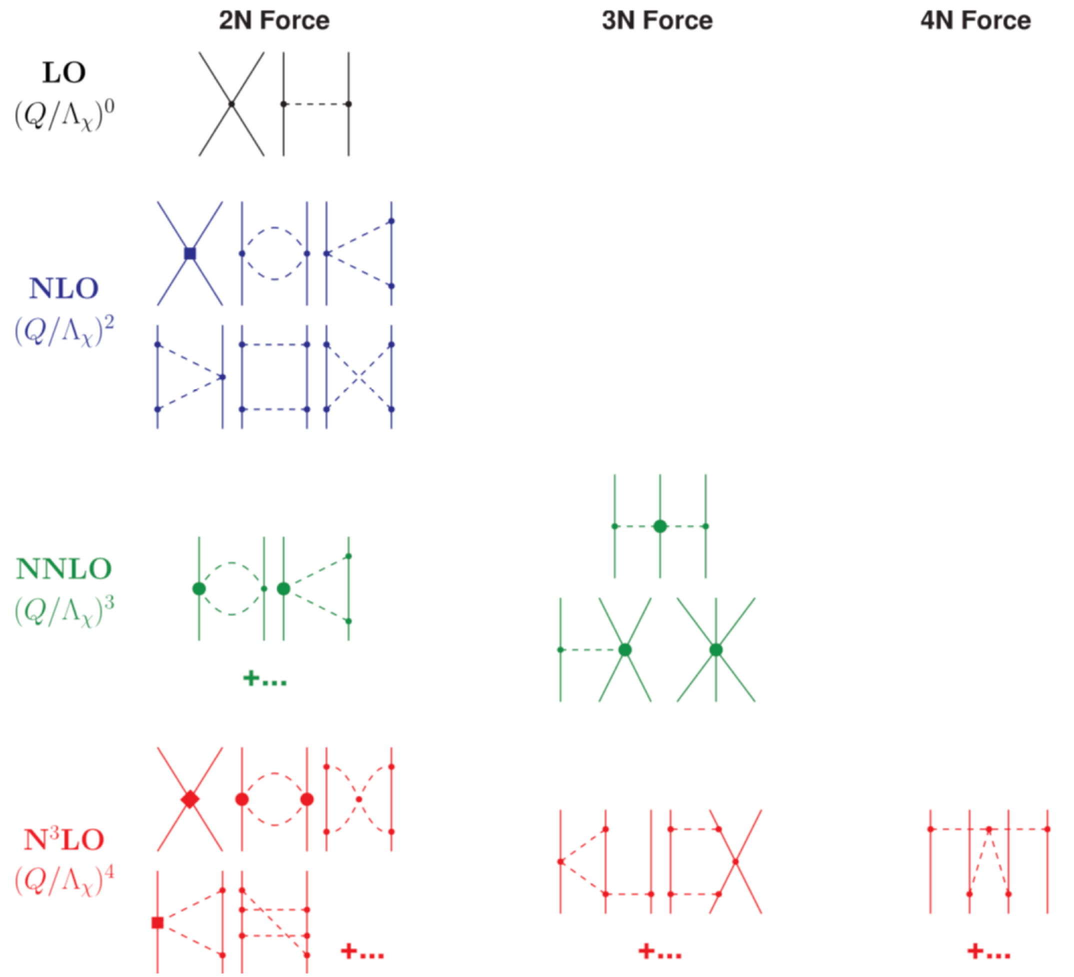
- Kaon production at subthreshold energy in HI collisions is sensitive to nuclear EOS, and data are consistent with a soft one (K = 200 MeV)

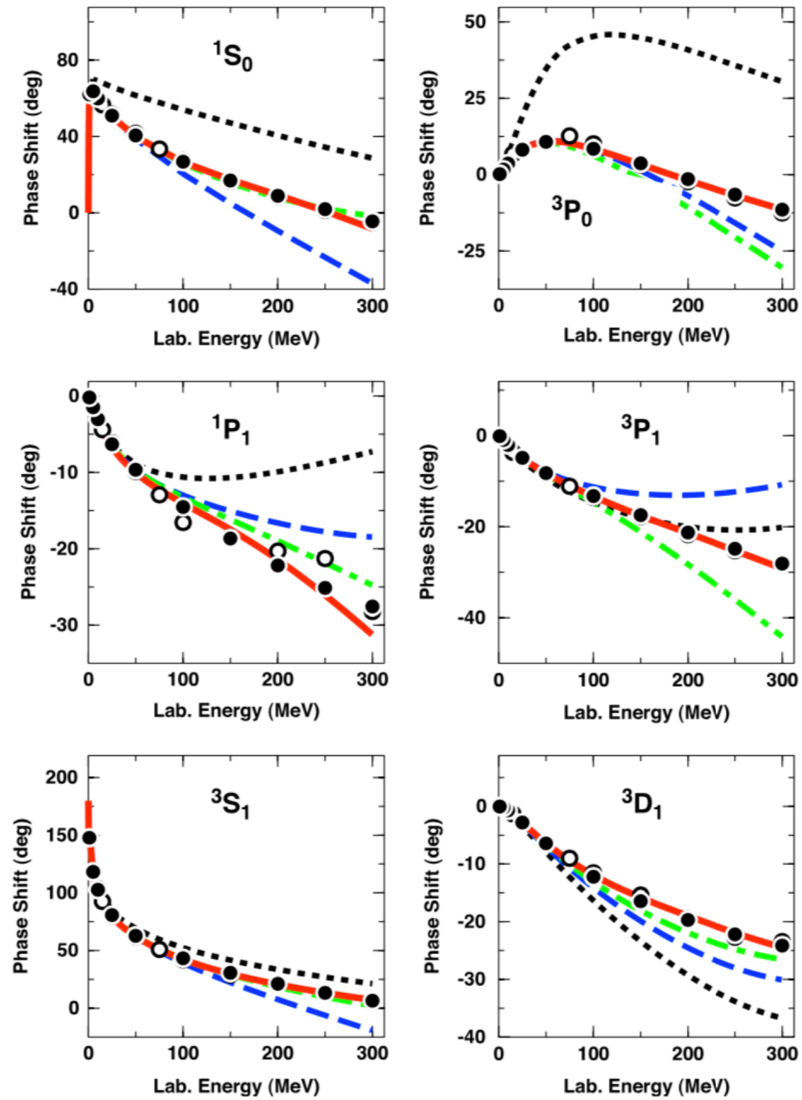
Conflicting results on symmetry energy from charged pion ratio



Chiral nucleon-nucleon forces

Machleidt & Entem, Physics reports 503, 1 (2011)





- Fit measured phase shifts well

Selfconsistent Green's function method

W. H. Dickhoff and C. Barbieri, Prog. Part. Nucl. Phys. **52**, 377 (2004).
A. Rios, A. Polls, and I. Vidaña, Phys. Rev. C **79**, 025802 (2009).

$$\text{Dyson's equation} \quad G(\mathbf{p},\omega) = G_0(\mathbf{p},\omega) + G_0(\mathbf{p},\omega)\Sigma^\star(\mathbf{p},\omega)G(\mathbf{p},\omega)$$

$$\text{Nucleon spectral function} \quad A(\mathbf{p},\omega) \sim \mathrm{Im}\Sigma^*(\mathbf{p},\omega)$$

$$\text{Nucleon occupation number} \quad n(\mathbf{p}) = \int \frac{\mathrm{d}\omega}{2\pi}\mathcal{A}(\mathbf{p},\omega)f(\omega)$$

Fermi-Dirac distribution

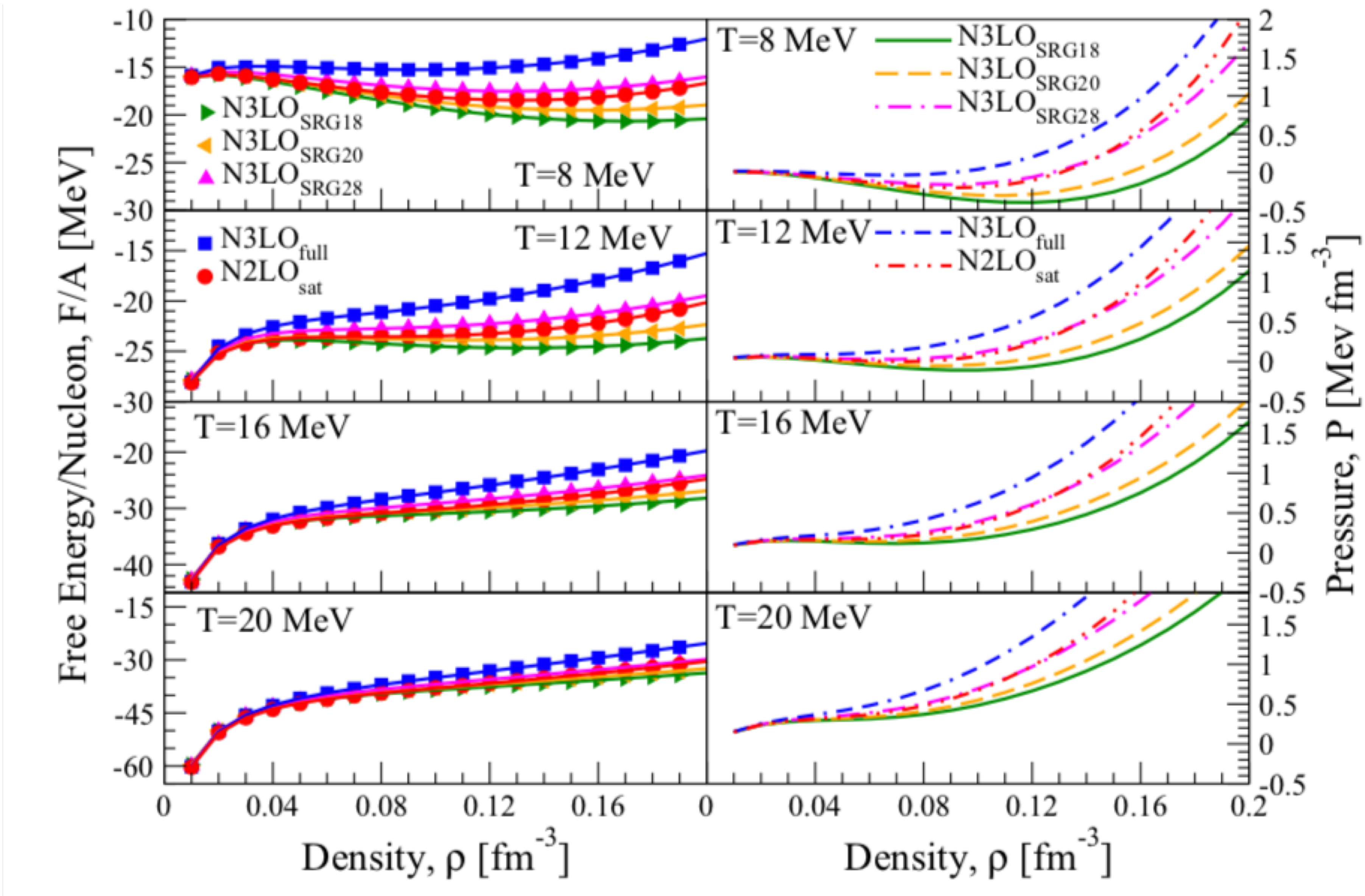
Energy per nucleon (Equation of state)

$$\frac{E}{A} = \frac{\nu}{\rho}\int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3}\int \frac{\mathrm{d}\omega}{2\pi}\frac{1}{2}\Big(\frac{p^2}{2m}+\omega\Big)\mathcal{A}(\mathbf{p},\omega)f(\omega) - \frac{1}{2}\langle\hat{W}\rangle$$

Three-body contribution

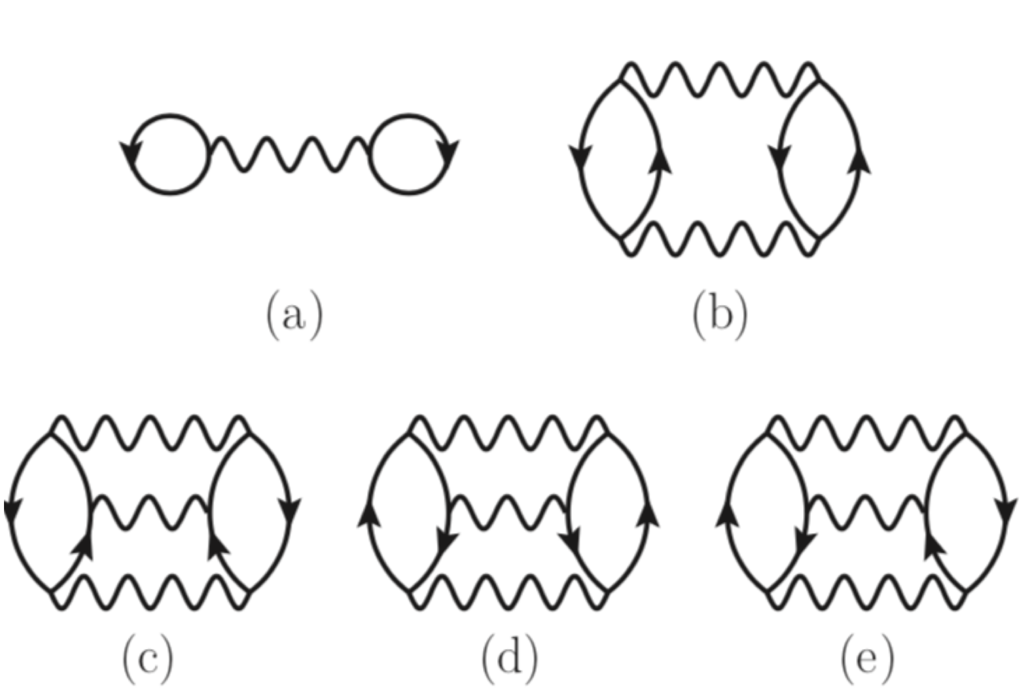
Nuclear equation of state at different temperatures

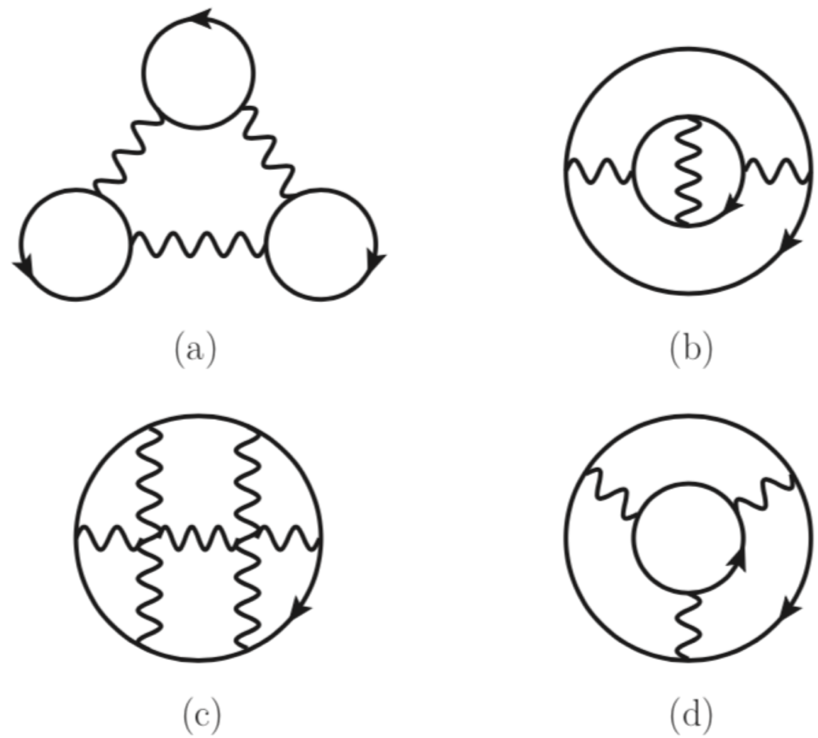
Carbone, Polls & Rios, PRC 98, 025804 (2018), SCGF approach



Chiral effective many-body perturbation theory

J. W. Holt and N. Kaiser, PRC 95, 034326 (2017)

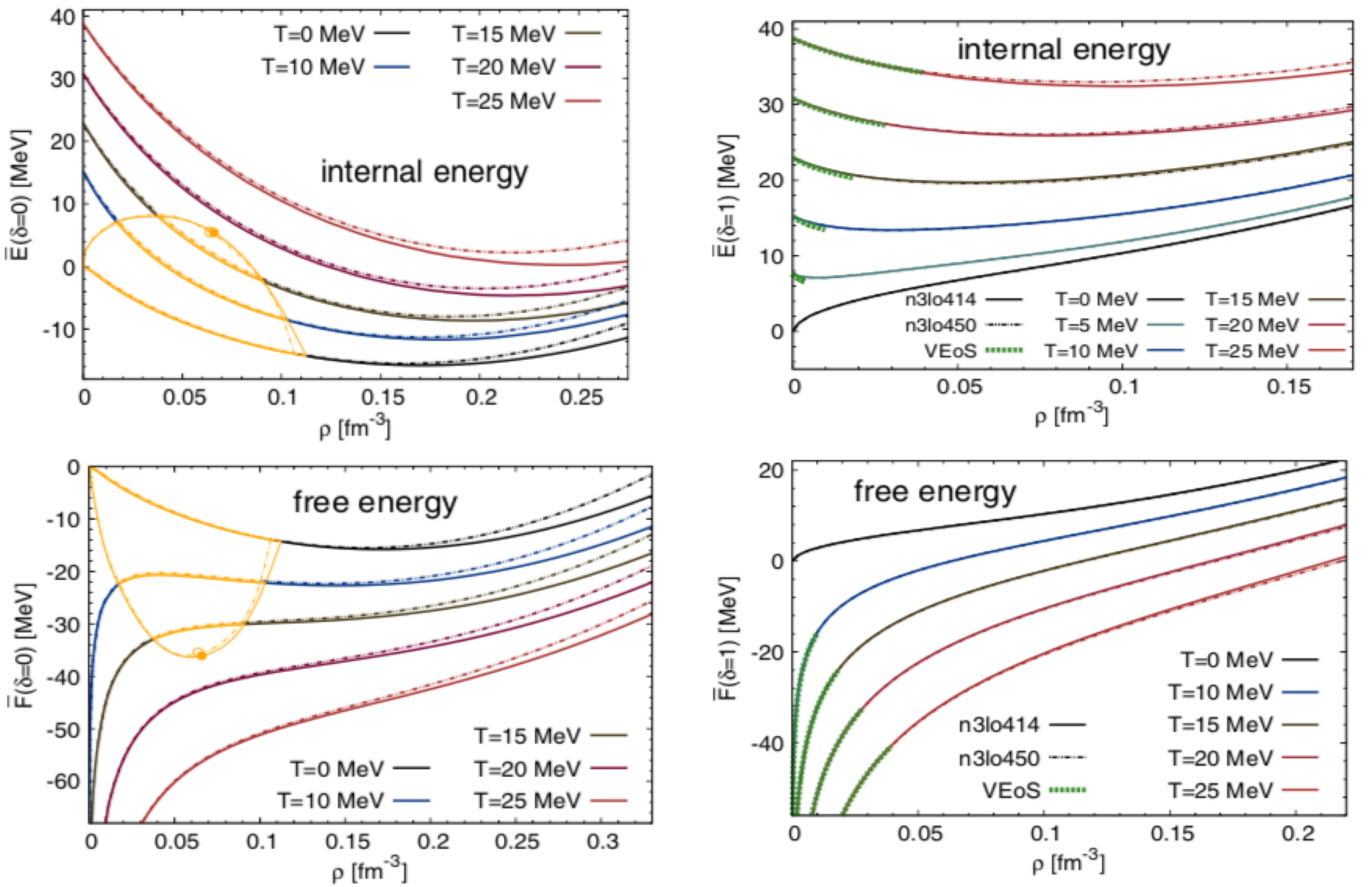




- The wavy line includes the (antisymmetrized) density-dependent NN interaction derived from the chiral three-body force at N2LO.

Nuclear equation of state at different temperatures

Wellenhofer, Holt & Kaiser, PRC 92, 015801 (2015), Perturbative approach with chiral NN interactions



Improved momentum-dependent density functional

J. Xu, L. W. Chen, and B. A. Li, Phys. Rev. C **91**, 014611 (2015).

$$
\begin{aligned}
V_{\mathrm{ImMDI}} &= \frac{A_u \rho_n \rho_p}{\rho_0} + \frac{A_l}{2\rho_0}(\rho_n^2 + \rho_p^2) + \frac{B}{\sigma+1}\frac{\rho^{\sigma+1}}{\rho_0^\sigma}(1 - x\delta^2) \\
&+ \frac{1}{\rho_0}\sum_{q,q'} C_{q,q'} \int\int d^3p d^3p' \frac{f_q(\vec{r},\vec{p}) f_{q'}(\vec{r},\vec{p}')}{1 + (\vec{p} - \vec{p}')^2/\Lambda^2}
\end{aligned}
$$

$$
\begin{aligned}
A_l(x,y) &= A_0 + y + x\frac{2B}{\sigma+1}, \quad A_u(x,y) = A_0 - y - x\frac{2B}{\sigma+1} \\
C_{q,q}(y) &= C_{l0} - 2(y - 2z)\frac{p_{f0}^2}{\Lambda^2 \ln[(4p_{f0}^2 + \Lambda^2)/\Lambda^2]} \\
C_{q,-q}(y) &= C_{u0} + 2(y - 2z)\frac{p_{f0}^2}{\Lambda^2 \ln[(4p_{f0}^2 + \Lambda^2)/\Lambda^2]}
\end{aligned}
$$

- Fit to empirical energy dependence of single nucleon potential from Hama et al. [PRC 41, 2737 (1990), 47, 297 (1993)].

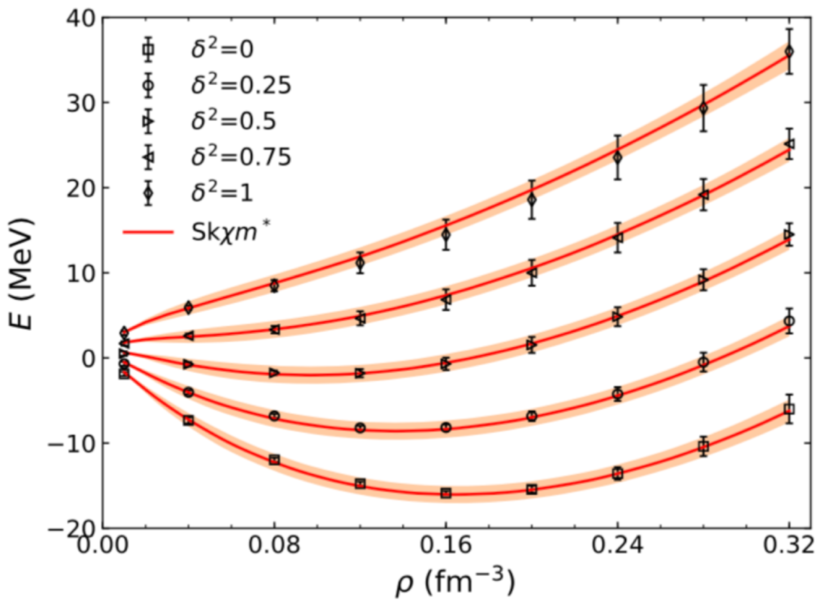
Chiral effective theory inspired Skyrme density functional

- Skχm* interaction (K_0=230 MeV, E_{sym}=31 MeV, L=45.6 MeV)

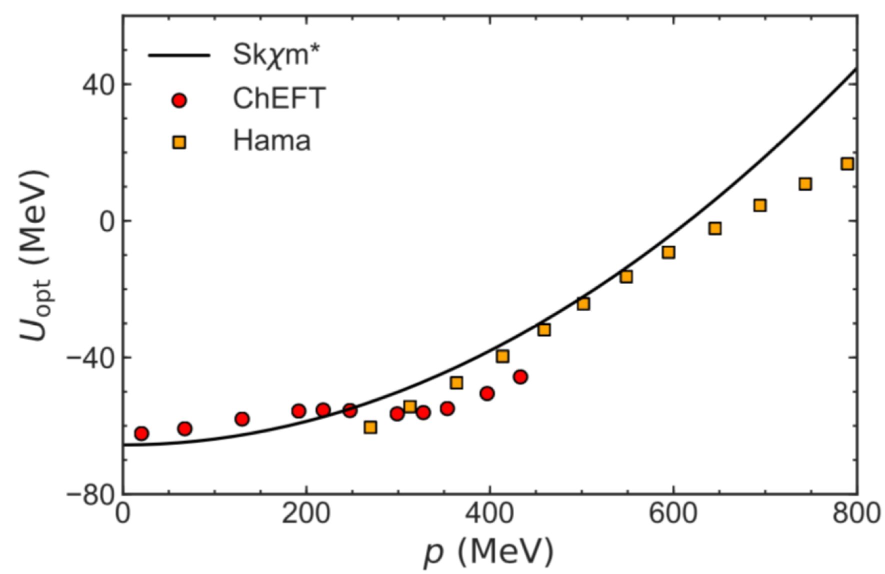
Zhang, Liam, Holt & Ko, PLB 777, 73 (2018), Zhang & Ko, PRC 98, 054614 (2018)

$$
\begin{aligned}
V_{\mathrm{SHF}} &= t_0[(2+x_0)\rho^2-(2x_0+1)(\rho_p^2+\rho_n^2)]/4 \\
&+ [t_1(2+x_1)+t_2(2+x_2)]\tau\rho/8 \\
&+ [t_2(2x_2+1)-t_1(2x_1+1)](\tau_n\rho_n+\tau_p\rho_p)/8 \\
&+ t_3\rho^\sigma[(2+x_3)\rho^2-(2x_3+1)(\rho_p^2+\rho_n^2)]/24
\end{aligned}
$$

- Parameters fitted to EOS from chiral effective theory and binding energies of 7 doubly magic nuclei

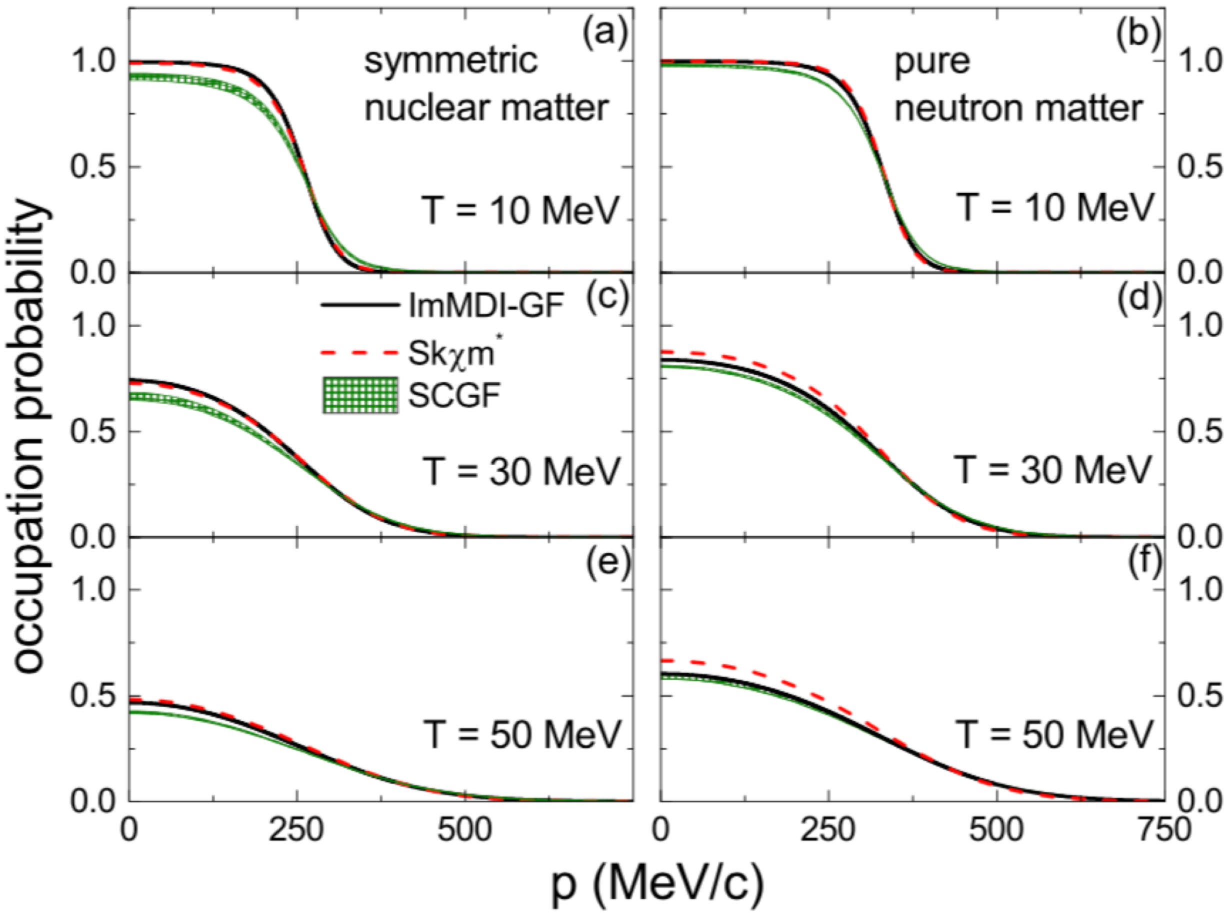


- Good description of momentum dependence of optical potential below 600 MeV, dipole polarizability, and neutron skin thickness



	ImMDI-GF		Skχm*
A_0 (MeV)	-66.963	t_0 (MeVfm^3)	-2260.7
B (MeV)	141.963	x_0	0.327488
C_{u0} (MeV)	-99.70	t_1 (MeVfm^5)	433.189
C_{l0} (MeV)	-60.49	x_1	-1.088968
σ	1.2652	t_2 (MeVfm^5)	274.553
Λ (p_{f0})	2.424	x_2	-1.822404
x	0.5	t_3 (MeVfm$^{3+3\alpha}$)	12984.4
y (MeV)	-60	x_3	0.442900
z (MeV)	-2.5	α	0.198029
ρ_{sat} (fm^{-3})	0.16	ρ_{sat} (fm^{-3})	0.1651
$E_0(\rho_{\mathrm{sat}})$ (MeV)	-16	$E_0(\rho_{\mathrm{sat}})$ (MeV)	-16.07
K_0 (MeV)	230	K_0 (MeV)	230.4
U_0^∞ (MeV)	75	U_0^∞ (MeV)	N/A
m_s^* (m)	0.70	m_s^* (m)	0.750
$E_{\mathrm{sym}}(\rho_{\mathrm{sat}})$ (MeV)	30	$E_{\mathrm{sym}}(\rho_{\mathrm{sat}})$ (MeV)	30.94
L (MeV)	40	L (MeV)	45.6
m_v^* (m)	0.59	m_v^* (m)	0.694
G_S (MeVfm^5)	N/A	G_S (MeVfm^5)	141.5
G_V (MeVfm^5)	N/A	G_V (MeVfm^5)	-70.5

Nucleon number occupation number at normal density

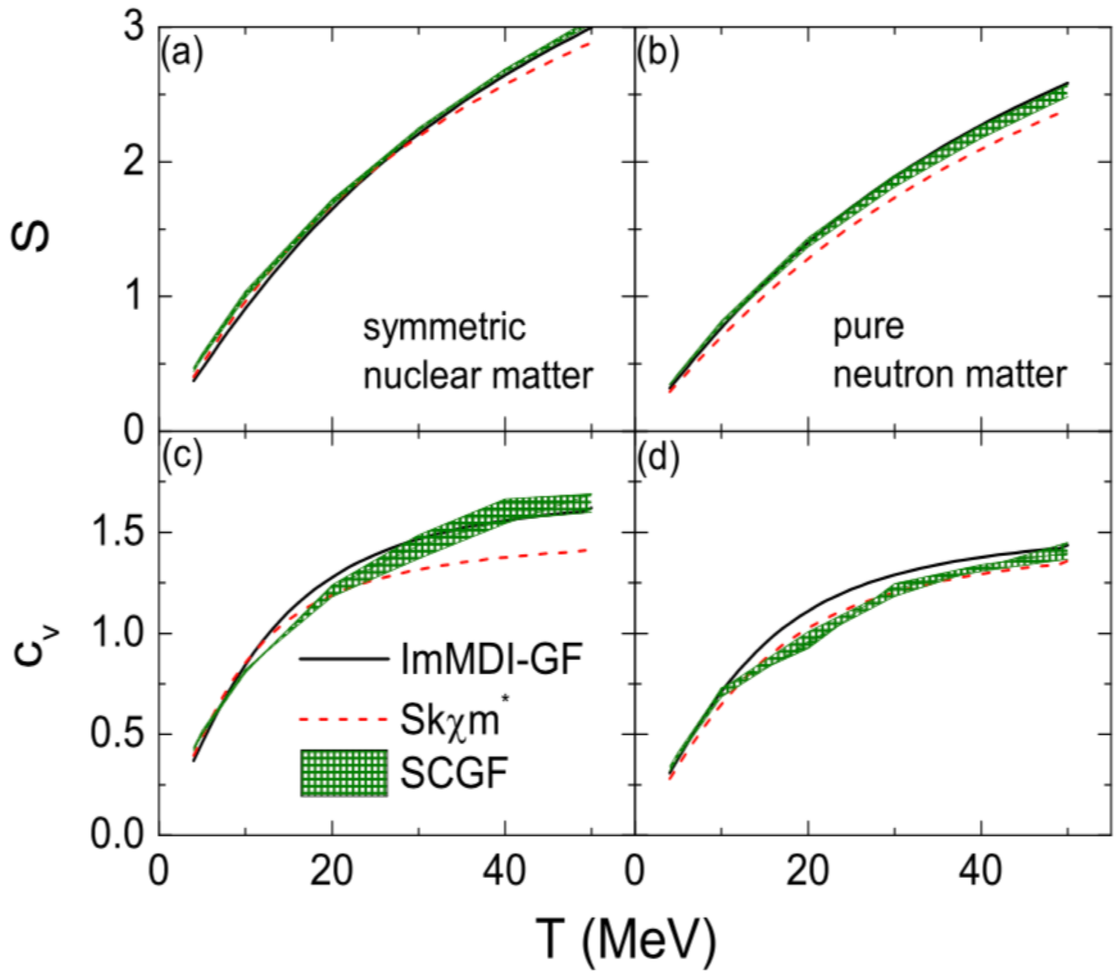


- Depletion of low momentum nucleons in SNM from SCGF due to correlation effects.
- More low momentum nucleons from Skχm* in PNM at high temperature due to strong quadratic momentum dependence.

Entropy per nucleon and heat capacity

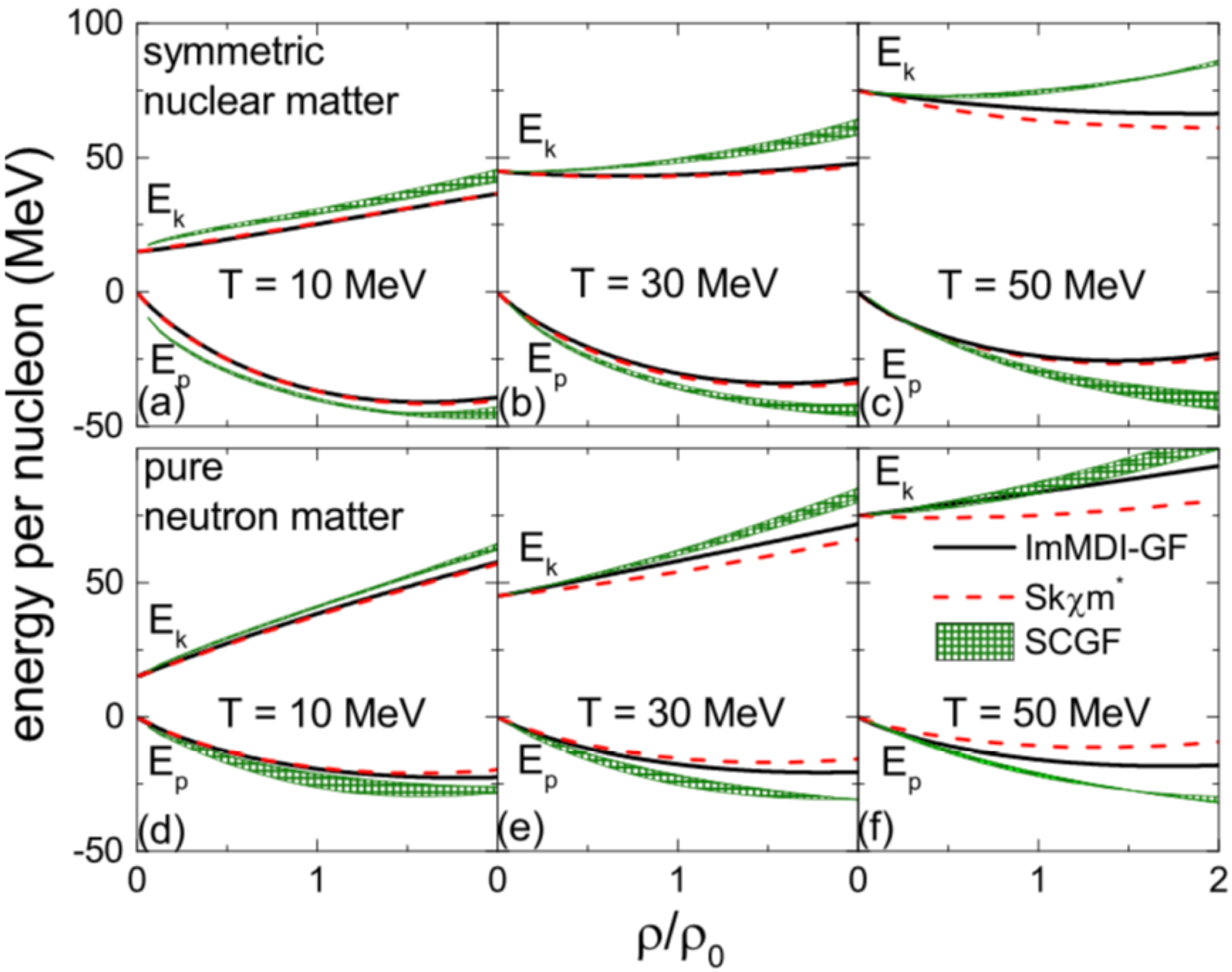
$$\text{Entropy}\quad S = -\frac{\nu}{\rho}\int \frac{d^3p}{(2\pi)^3}\left[n\ln n + (1-n)\ln(1-n)\right]$$

$$\text{Heat capacity}\quad c_v = T\left(\frac{\partial S}{\partial T}\right)_{\delta,\rho}$$



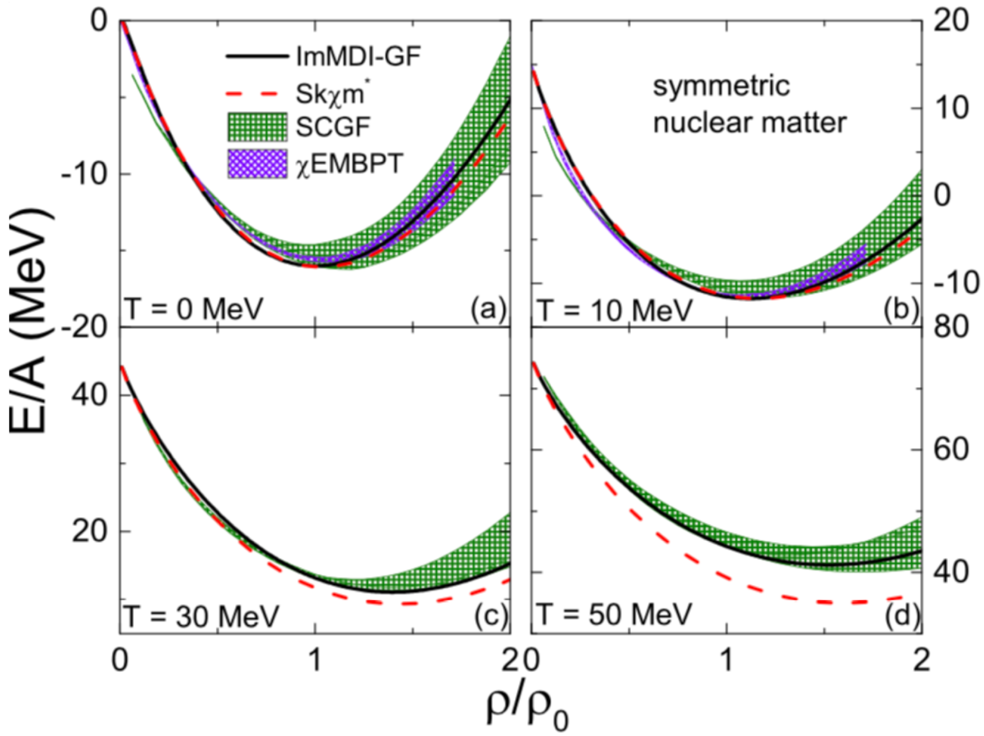
- Larger entropy from SCGF due to correlations.
- Smaller entropy from Skχm* due to sharper nucleon momentum distribution, and thus smaller heat capacity at larger momentum.

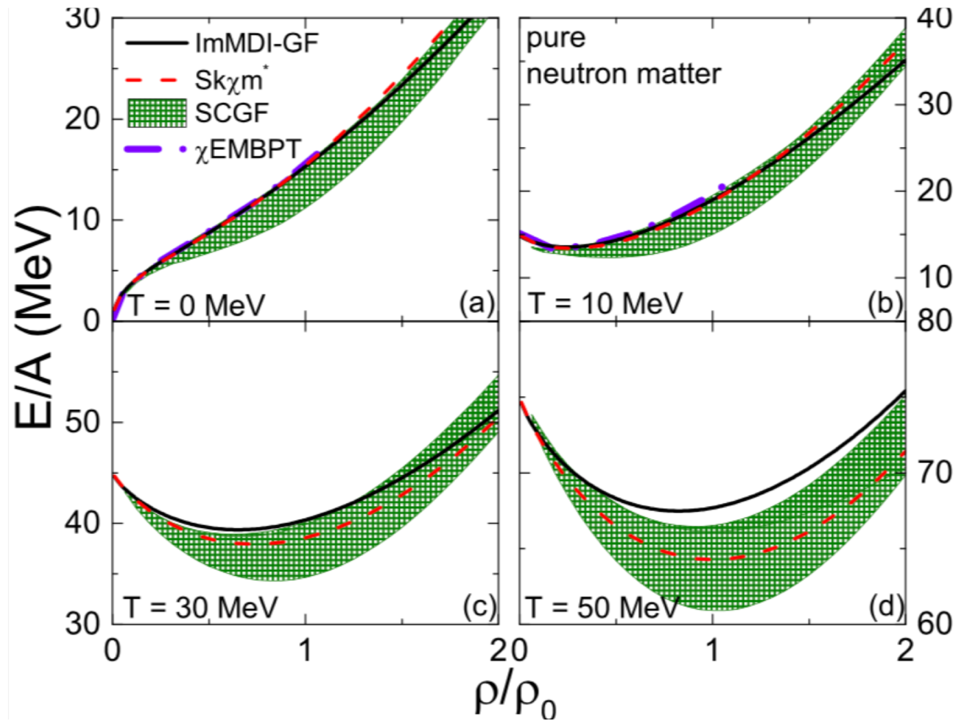
Kinetic energy and potential energy per nucleon



- SCGF has a larger kinetic energy due to smaller occupation number at low momentum as a result of correlations.
- ImMDI-GF and Skχm* have similar potential energy in SNM but differ in PNM due to different occupation numbers, and both higher than SCGF.

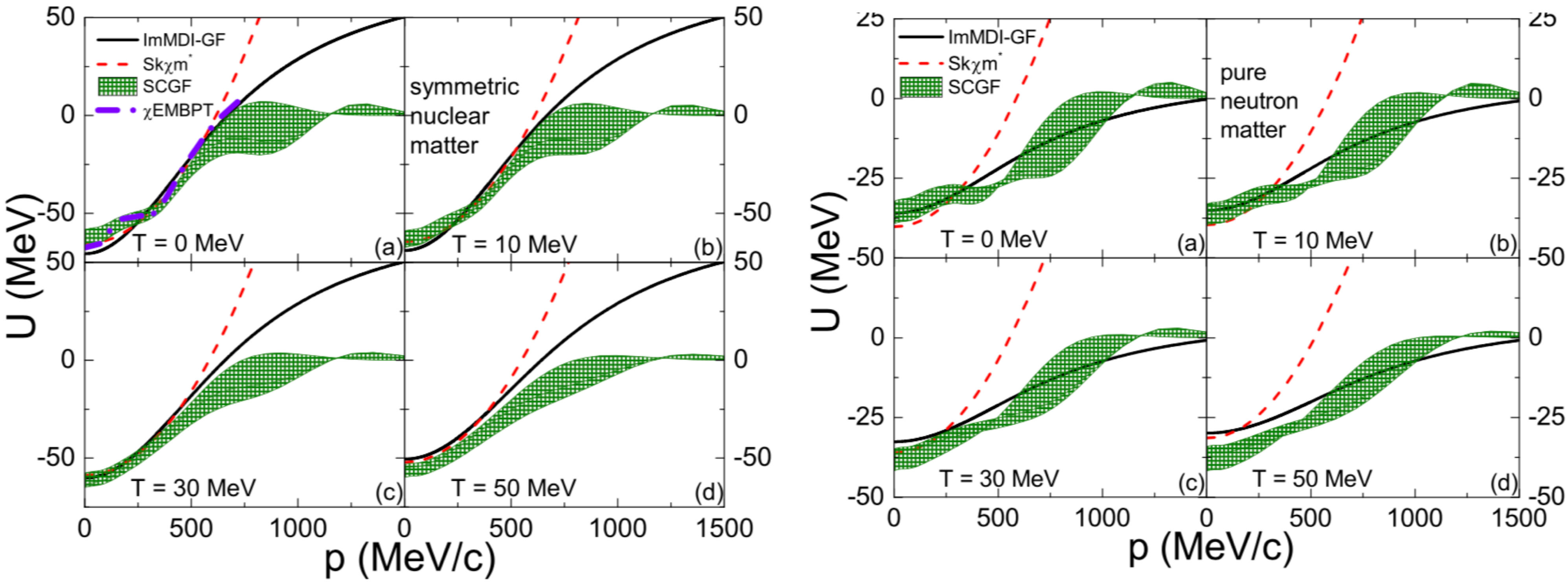
Nuclear matter equation of state at different temperatures





- In SNM, ImMDI-GF agrees with both SCGF and χEMBPT, which are consistent within their uncertainties, at all temperatures but Skχm* starts to deviate as temperature of SNM increases.
- In PNM, Skχm* does better at high temperature.

Momentum dependence of mean-field potential in nuclear matter at different temperatures



- In SNM, both ImMDI-GF and Skχm* have too strong momentum dependence than both SCGF and χEMBPT, with Skχm* even stronger due to its quadratic dependence.
- In PNM, ImMDI-GF agrees with both SCGF and χEMBPT but Skχm* remains too strong.

Summary

- Two representative nuclear effective interactions ImMDI-GF and Skχm*, with former fitted to the EOS of cold PNM from SCGF and latter fitted to that of cold SNM and ANM from $\chi\mathrm{EMPT}$ using chiral forces, are considered.
- For SNM at finite temperatures, EOS from ImMDI-GF agrees with those from both SCGF and χEMPT, while that from Skχm* is softer at high temperature.
- For PNM at finite temperature, EOS from Skχm* agrees with those from SCGF and χEMPT, while that from ImMDI-GF is slightly stiffer at high temperature.
- Momentum dependence in nucleon potential for SNM from both ImMDI-GF and Skχm* are too strong compared to those from SCGF and χEMPT. In PNM, ImMDI-GF has similar momentum dependence as in SCGF and χEMPT, while that in Skχm*remains too strong.
- Generally, correct momentum dependence in effective interactions help to model the temperature dependence in SCGF and χEMPT.

All effective nucleon-nucleon interactions or nuclear density functional used in transport model studies of heavy ion collisions should compare the resulting nuclear matter equation of states with those from ab initio theories such as SCGF and χEMBPT.

Thanks!