Light nuclei as a probe of the QCD phase diagram

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- Introduction
- The coalescence model
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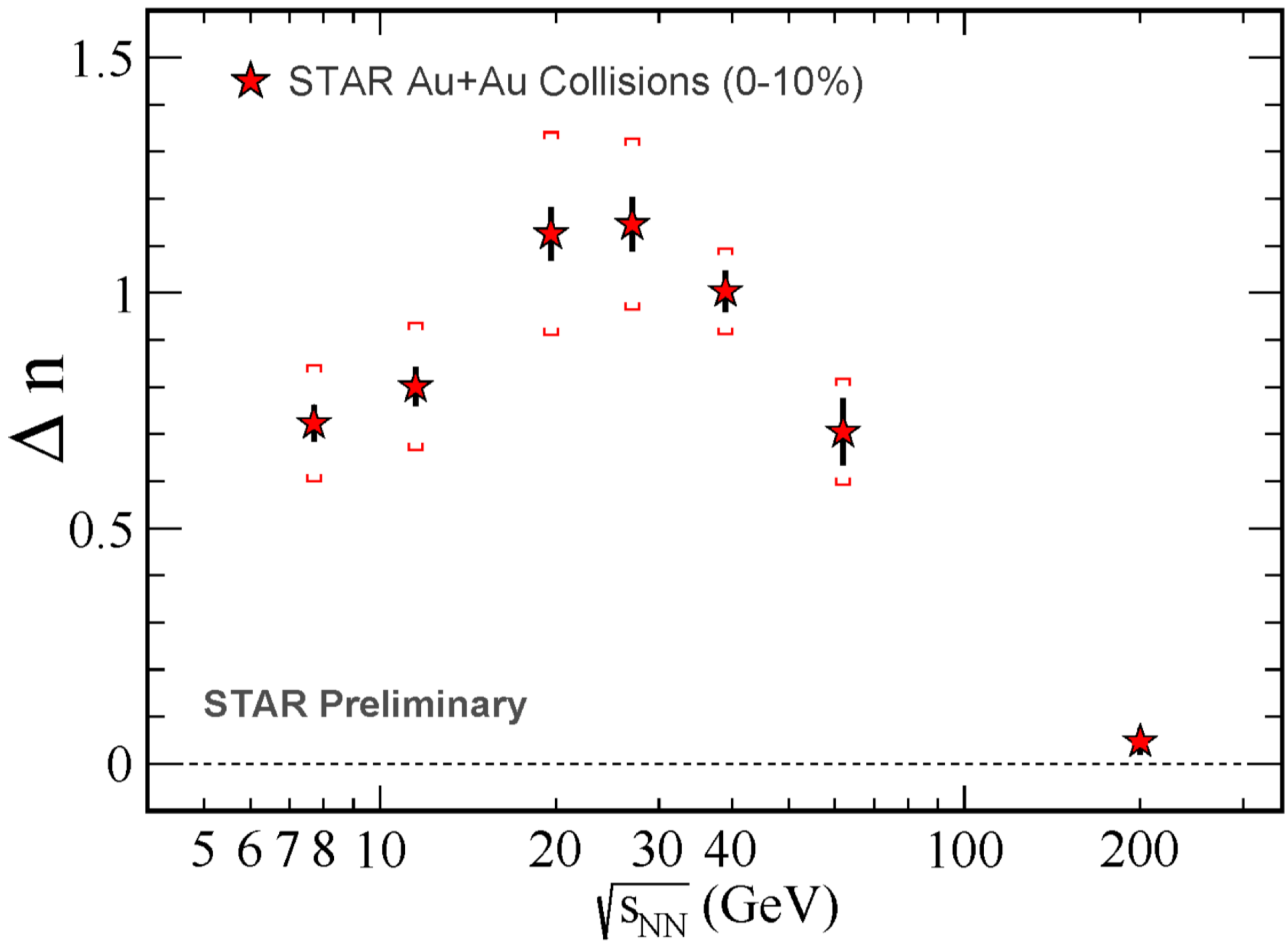
- Summary

Neutron relative density fluctuation from yield ratio of light nuclei

Dingwei Zhang for STAR Collaboration, NN2018

$$\mathcal{O}_{\mathrm{d-d-t}} = \frac{N_{^3H} N_p}{N_d^2} = g(1+\Delta n),\ \ g = \frac{4}{9}\left(\frac{3}{4}\right)^{3/2} \approx 0.29$$



Coalescence model in the sudden approximation

Wave functions for initial |i>=|1,2> and final |f>=|3> states

$$\langle \mathbf{r}_1, \mathbf{r}_2 | i \rangle = \phi_1(\mathbf{r}_1)\phi_2(\mathbf{r}_2)$$

$$\langle \mathbf{r}_1, \mathbf{r}_2 | f \rangle = \frac{1}{\sqrt{V}} e^{i\mathbf{K}\cdot(\mathbf{r}_1+\mathbf{r}_2)/2} \Phi(\mathbf{r}_1 - \mathbf{r}_2)$$

Probability for 1+2 -> 3 $\quad \mathcal{P} = |\langle f | i \rangle|^2$

Probability for particle 1 of momentum $\mathbf{k}_1$ and particle 2 of momentum $\mathbf{k}_2$ to coalescence to cluster 3 with momentum $\mathbf{K}$

$$\frac{dN}{d^3\mathbf{K}} = g \int d^3\mathbf{x}_1 d^3\mathbf{k}_1 d^3\mathbf{x}_2 d^3\mathbf{k}_2 W_1(\mathbf{x}_1, \mathbf{k}_1) W_2(\mathbf{x}_2, \mathbf{k}_2)$$

$$\times W(\mathbf{y}, \mathbf{k}) \delta^{(3)}(\mathbf{K} - \mathbf{k}_1 - \mathbf{k}_2), \qquad \mathbf{y} = \mathbf{x}_1 - \mathbf{x}_2, \quad \mathbf{k} = \frac{\mathbf{k}_1 - \mathbf{k}_2}{2}$$

Wigner functions $\quad W(\mathbf{x}, \mathbf{k}) = \int d^3\mathbf{y} \phi^* \left(\mathbf{x} - \frac{\mathbf{y}}{2}\right) \phi \left(\mathbf{x} + \frac{\mathbf{y}}{2}\right) e^{-i\mathbf{k}\cdot\mathbf{y}}$

For a system of particles 1 and 2 with phase-space distributions $f_i(\mathbf{x}_i,\mathbf{k}_i)$ normalized to $\int d^3\mathbf{x_i} d^3\mathbf{k}_i f_i(\mathbf{x}_i,\mathbf{k}_i) = N_i$, number of particle 3 produced from coalescence of N_1 of particle 1 and N_2 of particle 2

$$\frac{dN}{d^3\mathbf{K}} \approx g\int d^3\mathbf{x}_1 d^3\mathbf{k}_1 d^3\mathbf{x}_2 d^3\mathbf{k}_2 f_1(\mathbf{x}_1,\mathbf{k}_1) f_2(\mathbf{x}_2,\mathbf{k}_2)$$
$$\times \quad \overline{W}(\mathbf{y},\mathbf{k})\delta^{(3)}(\mathbf{K}-\mathbf{k}_1-\mathbf{k}_2)$$

$$\overline{W}(\mathbf{y},\mathbf{k}) = \int \frac{d^3\mathbf{x}_1' d^3\mathbf{k}_1'}{(2\pi)^3}\frac{d^3\mathbf{x}_2' d^3\mathbf{k}_2'}{(2\pi)^3} W_1(\mathbf{x}_1',\mathbf{k}_1') W_2(\mathbf{x}_2',\mathbf{k}_2') W(\mathbf{y}',\mathbf{k}')$$

Wigner function $W_i(\mathbf{x}_i',\mathbf{k}_i')$ centers around $\mathbf{x}_i$ and $\mathbf{k}_i$

$$g = \frac{2J+1}{(2J_1+1)(2J_2+1)}$$

Statistical factor for two particles of spin J_1 and J_2 to form a particle of spin J

The above formula can be straightforwardly generalized to multi-particle coalescence, but is usually used by taking particle Wigner functions as delta functions in space and momentum.

Gyulassy, Frankel, and Remler, NPA 402, 596 (1983): Generalized coalescence model using nucleon Wigner functions that are delta functions in space and momentum, i.e., evaluating

$$\overline{W}(\mathbf{y},\mathbf{k}) = \int \frac{d^3\mathbf{x}_1' d^3\mathbf{k}_1'}{(2\pi)^3} \frac{d^3\mathbf{x}_2' d^3\mathbf{k}_2'}{(2\pi)^3} W_1(\mathbf{x}_1',\mathbf{k}_1') W_2(\mathbf{x}_2',\mathbf{k}_2') W(\mathbf{y}',\mathbf{k}')$$

with $W_i(\mathbf{x}_i',\mathbf{k}_i') = (2\pi)^3\delta^3(\mathbf{x}_i'-\mathbf{x}_i)\delta^3(\mathbf{k}_i'-\mathbf{k}_i)$

$$\rightarrow \frac{dN}{d^3\mathbf{K}} \approx g\int d^3\mathbf{x}_1 d^3\mathbf{k}_1 d^3\mathbf{x}_2 d^3\mathbf{k}_2 f_1(\mathbf{x}_1,\mathbf{k}_1) f_2(\mathbf{x}_2,\mathbf{k}_2) \times W(\mathbf{y},\mathbf{k})\delta^{(3)}(\mathbf{K}-\mathbf{k}_1-\mathbf{k}_2)$$

It is later called by Kahana et al. the standard Wigner calculation in contrast to the general one which they called the quantum Wigner calculation.

Deuteron number in the coalescence model

In the coalescence model, the deuteron number is given by

$$N_d = g_d \int d^3\mathbf{x}_1 \int d^3\mathbf{k}_1 \int d^3\mathbf{x}_2 \int d^3\mathbf{k}_2 f_1(\mathbf{x}_1, \mathbf{k}_1) f_2(\mathbf{x}_2, \mathbf{k}_2) W_d(\mathbf{x}_1 - \mathbf{x}_2, (\mathbf{k}_1 - \mathbf{k}_2)/2).$$

In the above, the proton or neutron distribution function is given by

$$f(\mathbf{x}, \mathbf{k}) = \frac{2\gamma}{(2\pi)^3} \; e^{-\frac{k^2}{2mT}},$$

where T, m and γ are the temperature, nucleon mass, and fugacity, respectively, and is normalized to

$$N = \int d^3\mathbf{x} \int d^3\mathbf{k} f(\mathbf{x}, \mathbf{k}) = 2\gamma V \left(\frac{mT}{2\pi}\right)^{3/2}.$$

after using

$$\int d^3\mathbf{x} = V, \quad \int d^3\mathbf{k} \; e^{-ak^2} = \left(\frac{\pi}{a}\right)^{3/2},$$

with V being the volume. The deuteron Wigner function is given by

$$W_d(\mathbf{x}, \mathbf{k}) = 8 \; e^{-\frac{x^2}{\sigma^2}} \; e^{-\sigma^2 k^2},$$

and is normalized according to

$$\int d^3\mathbf{x} \int d^3\mathbf{k} \; W_d(\mathbf{x}, \mathbf{k}) = (2\pi)^3.$$

Deuteron number in the coalescence model (Continued)

Changing variables to

$$\begin{aligned} \mathbf{X} &= \frac{\mathbf{x}_1 + \mathbf{x}_2}{2}, \quad \mathbf{x} = \mathbf{x}_1 - \mathbf{x}_2, \\ \mathbf{K} &= \mathbf{k}_1 + \mathbf{k}_2, \quad \mathbf{k} = \frac{\mathbf{k}_1 - \mathbf{k}_2}{2}, \end{aligned}$$

then

$$\begin{aligned} N_d &= \frac{32 g_d \gamma_1 \gamma_2}{(2\pi)^6} \int d^3\mathbf{X} \int d^3\mathbf{x}\; e^{-\frac{x^2}{\sigma^2}} \int d^3\mathbf{K}\; e^{-\frac{K^2}{4mT}} \int d^3\mathbf{k}\; e^{-k^2(\sigma^2 + \frac{1}{mT})} \\ &= \frac{32 g_d \gamma_1 \gamma_2}{(2\pi)^6} V\, (\pi\sigma^2)^{3/2}\, (4\pi mT)^{3/2} \left(\frac{\pi}{\sigma^2 + \frac{1}{mT}}\right)^{3/2} \\ &= 2^{3/2} g_d \left(\frac{2\pi}{mT + \frac{1}{\sigma^2}}\right)^{3/2} \frac{N_1 N_2}{V} \\ &= \frac{3}{2^{1/2}} \left(\frac{2\pi}{mT}\right)^{3/2} \frac{1}{(1 + \frac{1}{mT\sigma^2})^{3/2}} \frac{N_1 N_2}{V}, \quad (g_d = 3/4) \\ &\approx \frac{3}{2^{1/2}} \left(\frac{2\pi}{mT}\right)^{3/2} \frac{N_1 N_2}{V}, \quad (mT \gg 1/\sigma^2) \end{aligned}$$

Effect of density fluctuations on deuteron number

For non-uniform distributions, the factor $F = N_1 N_2 / V$ is replaced by

$$F = \frac{1}{(\pi\sigma^2)^{3/2}} \int d^3\mathbf{x}_1 \int d^3\mathbf{x}_2 n_1(\mathbf{x}_1) n_2(\mathbf{x}_2) e^{-(\mathbf{x}_1-\mathbf{x}_2)^2/\sigma^2}.$$

It can be rewritten as

$$\begin{aligned}
F &= \frac{1}{(\pi\sigma^2)^{3/2}} \int d^3\mathbf{X} \int d^3\mathbf{x} e^{-\mathbf{x}^2/\sigma^2} n_1\left(\mathbf{X}+\frac{\mathbf{x}}{2}\right) n_2\left(\mathbf{X}-\frac{\mathbf{x}}{2}\right) \\
&\approx \frac{1}{(\pi\sigma^2)^{3/2}} \int d^3\mathbf{X} \int d^3\mathbf{x} e^{-\mathbf{x}^2/\sigma^2} \left[n_1(\mathbf{X}) + \nabla n_1(\mathbf{X}) \cdot \frac{\mathbf{x}}{2}\right] \left[n_2(\mathbf{X}) - \nabla n_2(\mathbf{X}) \cdot \frac{\mathbf{x}}{2}\right] \\
&= \frac{1}{(\pi\sigma^2)^{3/2}} \int d^3\mathbf{X} \int d^3\mathbf{x} e^{-\mathbf{x}^2/\sigma^2} \left\{n_1(\mathbf{X}) n_2(\mathbf{X}) + \frac{\mathbf{x}}{2} \cdot [n_2(\mathbf{X}) \nabla n_1(\mathbf{X}) - n_1(\mathbf{X}) \nabla n_2(\mathbf{X})] \right. \\
&\quad \left. - \left[\frac{\mathbf{x}}{2} \cdot \nabla n_1(\mathbf{X})\right] \left[\frac{\mathbf{x}}{2} \cdot \nabla n_2(\mathbf{X})\right]\right\} \\
&= \int d^3\mathbf{X} n_1(\mathbf{X}) n_2(\mathbf{X}) \\
&\quad + \frac{1}{(\pi\sigma^2)^{3/2}} \int d^3\mathbf{X} \int d^3\mathbf{x} e^{-\mathbf{x}^2/\sigma^2} \left[\frac{\mathbf{x}}{2} \cdot \nabla n_1(\mathbf{X})\right] \left[\frac{\mathbf{x}}{2} \cdot \nabla n_2(\mathbf{X})\right].
\end{aligned}$$

Assuming $\nabla\rho_n(\mathbf{X}) \sim \frac{\rho_n(\mathbf{X})}{a}\mathbf{e_n}, \quad \nabla\rho_{\mathbf{p}}(\mathbf{X}) \sim \frac{\rho_{\mathbf{p}}(\mathbf{X})}{a}\mathbf{e_p}$

where $\mathbf{e_n}$ and $\mathbf{e_p}$ are unit vectors along the density gradient of the neutron and proton spatial distributions, and *a* is the length over which they change appreciably, then the second term becomes

$$\begin{aligned} F_2 &\approx \int \mathrm{d}^3\mathbf{X}\rho_n(\mathbf{X})\rho_p(\mathbf{X})\frac{1}{(\pi\sigma^2)^{3/2}}\int \mathrm{d}^3\mathbf{x}e^{-\frac{\mathbf{x}^2}{\sigma^2}}\left[\frac{\mathbf{x}\cdot\mathbf{e_n}}{2a}\right]\left[\frac{\mathbf{x}\cdot\mathbf{e_p}}{2a}\right] \\ &\leq \int \mathrm{d}^3\mathbf{X}\rho_n(\mathbf{X})\rho_p(\mathbf{X})\frac{1}{(\pi\sigma^2)^{3/2}}\int \mathrm{d}^3\mathbf{x}e^{-\frac{\mathbf{x}^2}{\sigma^2}}\left(\frac{x}{2a}\right)^2 \\ &< \frac{3}{8}\left(\frac{\sigma}{a}\right)^2\int \mathrm{d}^3\mathbf{X}\rho_n(\mathbf{X})\rho_p(\mathbf{X}). \end{aligned}$$

If the directions of $\mathbf{e_n}$ and $\mathbf{e_p}$ are not strongly correlated and *a* is significantly larger than σ, then F_2 is much smaller than the first term, and

$$N_{\mathrm{d}} \approx \frac{3}{2^{1/2}}\left(\frac{2\pi}{mT}\right)^{3/2}\int \mathrm{d}^3\mathbf{x}\ \rho_n(\mathbf{x})\rho_p(\mathbf{x}).$$

Taking into account density fluctuations

$$\rho_n(\mathbf{x}) = \frac{1}{V}\int \rho_n(\mathbf{x})\mathrm{d}^3\mathbf{x} + \delta\rho_n(\mathbf{x}) = \langle\rho_n\rangle + \delta\rho_n(\mathbf{x}),$$
$$\rho_p(\mathbf{x}) = \frac{1}{V}\int \rho_p(\mathbf{x})\mathrm{d}^3\mathbf{x} + \delta\rho_p(\mathbf{x}) = \langle\rho_p\rangle + \delta\rho_p(\mathbf{x}).$$

then
$$N_\mathrm{d} \approx \frac{3}{2^{1/2}}\left(\frac{2\pi}{mT}\right)^{3/2} \int d^3\mathbf{x}((\langle\rho_n\rangle + \delta\rho_n)((\langle\rho_p\rangle + \delta\rho_p)$$
$$= \frac{3}{2^{1/2}}\left(\frac{2\pi}{mT}\right)^{3/2} \left[\int d^3\mathbf{x}\langle\rho_n\rangle\langle\rho_p\rangle + \int d^3\mathbf{x}(\delta\rho_n)((\delta\rho_p)\right]$$
$$= \frac{3}{2^{1/2}}\left(\frac{2\pi}{mT}\right)^{3/2} N_p\langle\rho_n\rangle(1 + C_\mathrm{np}),$$

where $\langle\rho_n\rangle, \langle\rho_p\rangle$: average neutron and proton densities

$C_\mathrm{np} = \langle\delta\rho_n\delta\rho_p\rangle/(\langle\rho_n\rangle\langle\rho_p\rangle)$: neutron and proton density correlation

For triton

$$
\begin{aligned}
N_{^3\mathrm{H}} &\approx \frac{3^{3/2}}{4}\left(\frac{2\pi}{mT}\right)^3 \int d^3\mathbf{x}\ \rho_n^2(\mathbf{x})\rho_p(\mathbf{x}) \\
&= \frac{3^{3/2}}{4}\left(\frac{2\pi}{mT}\right)^3 \int d^3\mathbf{x}\ (\langle\rho_n\rangle + \delta\rho_n)^2(\langle\rho_p\rangle + \delta\rho_p) \\
&= \frac{3^{3/2}}{4}\left(\frac{2\pi}{mT}\right)^3 \left[\int d^3\mathbf{x}\ \langle\rho_n\rangle^2\langle\rho_p\rangle \right. \\
&\quad \left. +2\langle\rho_n\rangle \int d^3\mathbf{x}\ (\delta\rho_n)(\delta\rho_p) + \langle\rho_p\rangle \int d^3\mathbf{x}\ (\delta\rho_n)^2\right] \\
&= \frac{3^{3/2}}{4}\left(\frac{2\pi}{mT}\right)^3 N_p\langle\rho_n\rangle^2(1 + \Delta\rho_n + 2C_{\mathrm{np}}),
\end{aligned}
$$

where

$$\Delta\rho_n = \langle(\delta\rho_n)^2\rangle/\langle\rho_n\rangle^2 : \text{relative neutron density fluctuation}$$

Define yield ratio

$$\mathcal{O}_{\mathrm{p-d}} \equiv \frac{N_{\mathrm{d}}}{N_{\mathrm{p}}^2} = \frac{3}{2^{1/2}} \left(\frac{2\pi}{mT}\right)^{3/2} \frac{\langle \rho_{\mathrm{n}} \rangle}{N_{\mathrm{p}}} (1 + C_{\mathrm{np}})$$

then

$$\begin{aligned} C_{\mathrm{np}} &= \frac{2^{1/2}}{3} \left(\frac{mT}{2\pi}\right)^{3/2} \frac{N_{\mathrm{p}}}{\langle \rho_{\mathrm{n}} \rangle} \mathcal{O}_{\mathrm{p-d}} - 1 \\ &= \frac{2^{1/2}}{3(2\pi)^3} [(2\pi mT)^{3/2} V] \left[\frac{N_{\mathrm{p}}/V}{\langle \rho_{\mathrm{n}} \rangle}\right] \mathcal{O}_{\mathrm{p-d}} - 1 \\ &= g_{\mathrm{p-d}} V_{\mathrm{ph}} R_{\mathrm{np}} \mathcal{O}_{\mathrm{p-d}} - 1 \end{aligned}$$

where

$$g_{\mathrm{p-d}} = \frac{2^{1/2}}{3(2\pi)^3} \approx 0.0019$$

$$V_{\mathrm{ph}} = (2\pi mT)^{3/2} V \text{ : effective phase volume of nucleons}$$

$$R_{\mathrm{np}} = \frac{N_{\mathrm{p}}}{N_{\mathrm{n}}} = \frac{\langle \rho_{\mathrm{p}} \rangle}{\langle \rho_{\mathrm{n}} \rangle}$$

Define yield ratio

$$\mathcal{O}_{\mathrm{p-d-t}} \equiv \frac{N_{^3\mathrm{H}} N_{\mathrm{p}}}{N_{\mathrm{d}}^2} = \frac{\frac{3^{3/2}}{4}\left(\frac{2\pi}{mT}\right)^3 N_p \langle\rho_n\rangle^2 (1 + \Delta\rho_n + 2C_{\mathrm{np}})}{\left[\frac{3}{2^{1/2}}\left(\frac{2\pi}{mT}\right)^{3/2} N_p \langle\rho_n\rangle (1 + C_{\mathrm{np}})\right]^2}$$

$$= \frac{3^{3/2}}{18} \frac{1 + \Delta\rho_n + 2C_{\mathrm{np}}}{(1 + C_{\mathrm{np}})^2}$$

then $\Delta\rho_{\mathrm{n}} = g_{\mathrm{p-d-t}}(1 + C_{\mathrm{np}})^2 \mathcal{O}_{\mathrm{p-d-t}} - 2C_{\mathrm{np}} - 1$

where $g_{\mathrm{p-d-t}} = \frac{9}{4}\left(\frac{4}{3}\right)^{3/2} \approx 3.5$

Define isospin density fluctuation

$$\Delta\rho_{\mathrm{I}} \equiv \frac{\langle(\delta\rho_{\mathrm{n}} - \delta\rho_{\mathrm{p}})^2\rangle}{(\langle\rho_{\mathrm{n}}\rangle + \langle\rho_{\mathrm{p}}\rangle)^2} = \frac{\langle(\delta\rho_{\mathrm{n}})^2 - 2(\delta\rho_{\mathrm{n}})(\delta\rho_{\mathrm{p}}) + (\delta\rho_{\mathrm{p}})^2\rangle}{\langle\rho_{\mathrm{n}}\rangle^2 (1 + R_{\mathrm{np}})^2}$$

$$= \frac{\Delta\rho_{\mathrm{n}} - 2R_{\mathrm{np}}C_{\mathrm{np}} + R_{\mathrm{np}}^2 \Delta\rho_{\mathrm{p}}}{(1 + R_{\mathrm{np}})^2}$$

Table 1

Yields dN/dy of p, d and ^{3}H at midrapidity, together with the yield ratio π^+/π^- measured in central Pb+Pb collisions at 20 AGeV ($0-7\%$ centrality, $\sqrt{s_{NN}}=6.3$ GeV), 30 AGeV ($0-7\%$ centrality, $\sqrt{s_{NN}}=7.6$ GeV), 40 AGeV ($0-7\%$ centrality, $\sqrt{s_{NN}}=8.8$ GeV), 80 AGeV ($0-7\%$ centrality, $\sqrt{s_{NN}}=12.3$ GeV), and 158 AGeV ($0-12\%$ centrality, $\sqrt{s_{NN}}=17.3$ GeV) by the NA49 Collaboration [31,41,42]. Also given are the chemical freeze-out temperature T_{ch} (GeV) and volume V_{ch} (fm^3), the derived yield ratios $\mathcal{O}_{\text{p-d}}$ and $\mathcal{O}_{\text{p-d-t}}$, and the extracted C_{np}, $\Delta\rho_n$ and $\Delta\rho_I$. In obtaining $\mathcal{O}_{\text{p-d}}$ and $\mathcal{O}_{\text{p-d-t}}$, the weak decay contributions to the yield of proton from hyperons are corrected by using results from the statistical model (see text for details).

$\sqrt{s_{NN}}$	p	d	^{3}H(10^{-3})	π^+/π^-	T_{ch}	V_{ch}	$\mathcal{O}_{\text{p-d}}(10^{-4})$	$\mathcal{O}_{\text{p-d-t}}$	C_{np}	$\Delta\rho_n$	$\Delta\rho_I$
6.3	46.1 ± 2.1	2.094 ± 0.168	$43.7(\pm6.4)$	0.86	0.131	1389	10.5 ± 0.11	0.444 ± 0.014	-0.636 ± 0.004	0.475 ± 0.007	0.556 ± 0.004
7.6	42.1 ± 2.0	1.379 ± 0.111	$22.3(\pm3.4)$	0.88	0.139	1212	8.78 ± 0.13	0.465 ± 0.019	-0.707 ± 0.004	0.551 ± 0.007	0.629 ± 0.004
8.8	41.3 ± 1.1	1.065 ± 0.086	$14.8(\pm2.6)$	0.90	0.144	1166	7.32 ± 0.20	0.500 ± 0.020	-0.749 ± 0.007	0.606 ± 0.045	0.677 ± 0.006
12.3	30.1 ± 1.0	0.543 ± 0.044	$4.49(\pm0.94)$	0.91	0.153	1231	7.70 ± 0.11	0.404 ± 0.034	-0.693 ± 0.004	0.518 ± 0.012	0.605 ± 0.006
17.3	23.9 ± 1.0	0.279 ± 0.023	$1.58(\pm0.31)$	0.93	0.159	1389	6.66 ± 0.01	0.415 ± 0.032	-0.681 ± 0.0004	0.507 ± 0.011	0.594 ± 0.006

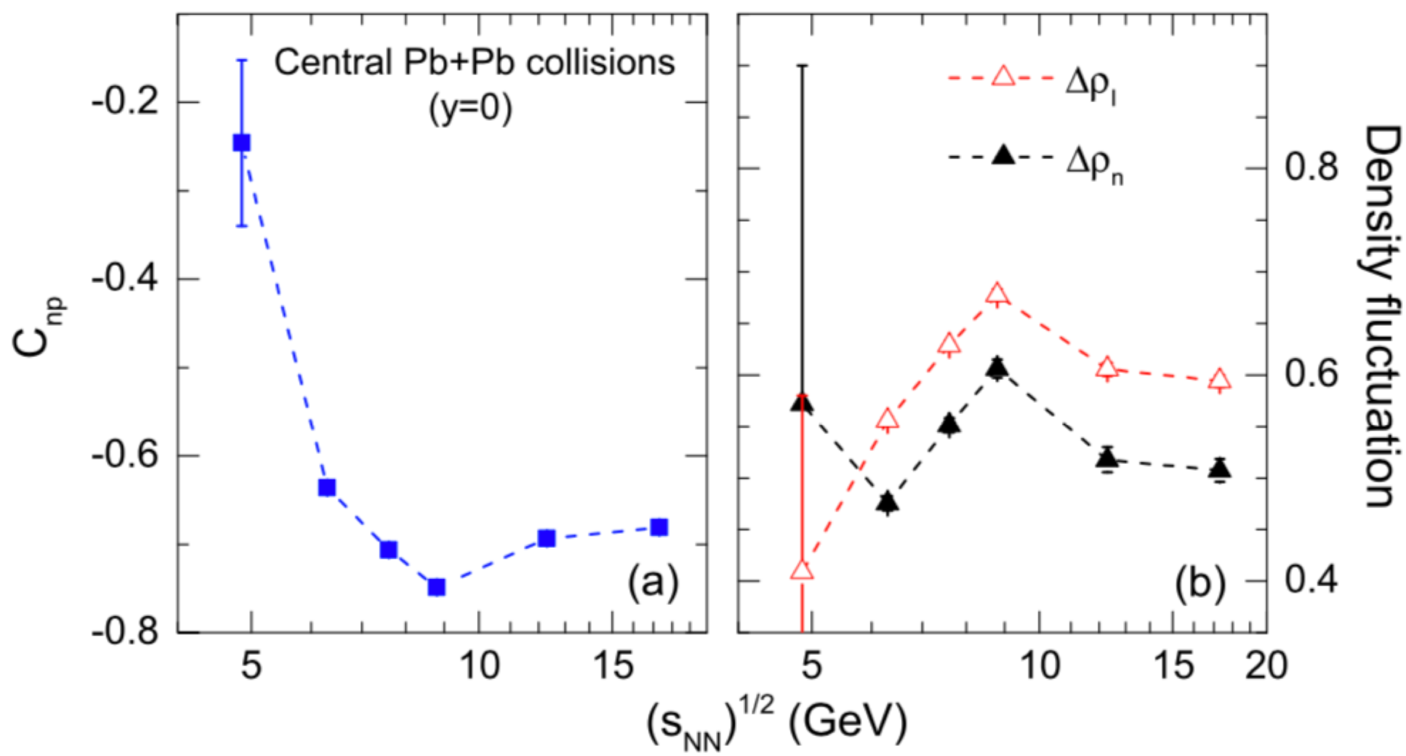


Fig. 2. Collision energy dependence of the neutron and proton density correlation C_{np} (a) and the neutron and isospin density fluctuations $\Delta\rho_n$ and $\Delta\rho_I$ (b) in central Pb+Pb collisions at SPS energies and Au+Au collisions at AGS energies.

Spinodal instability in Nambu-Jona-Lasinio model

- NJL model [Bratovic, Hatsuda & Weise, PLB 719, 131 (2013)]

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - M)\psi + \frac{G}{2}\sum_{a=0}^{8}\left[(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2\right]$$

Scalar-pseudoscalar

$$+ \sum_{a=0}^{8}\left[\frac{G_V}{2}(\bar{\psi}\gamma_\mu\lambda^a\psi)^2 + \frac{G_A}{2}(\bar{\psi}\gamma_\mu\gamma_5\lambda^a\psi)^2\right]$$

Vector-axial vector

$$- K\left[\det{}_f\left(\bar{\psi}(1+\gamma_5)\psi\right) + \det{}_f\left(\bar{\psi}(1-\gamma_5)\psi\right)\right],$$

Kobayashi-Maskawa-t'Hooft (KMT)

where $$\det{}_f(\bar{\psi}\Gamma\psi) = \sum_{i,j,k}\varepsilon_{ijk}(\bar{u}\Gamma q_i)(\bar{d}\Gamma q_j)(\bar{s}\Gamma q_k).$$

- Mean-field approximation

$$\mathcal{L} = \bar{\psi}\left(i\partial^\mu - \frac{2}{3}G_V\langle\bar{\psi}\gamma^\mu\psi\rangle\right)\gamma_\mu\psi - \bar{\psi}M^*\psi + \ldots$$

where $M^* = \mathrm{diag}(M_u, M_d, M_s)$ with

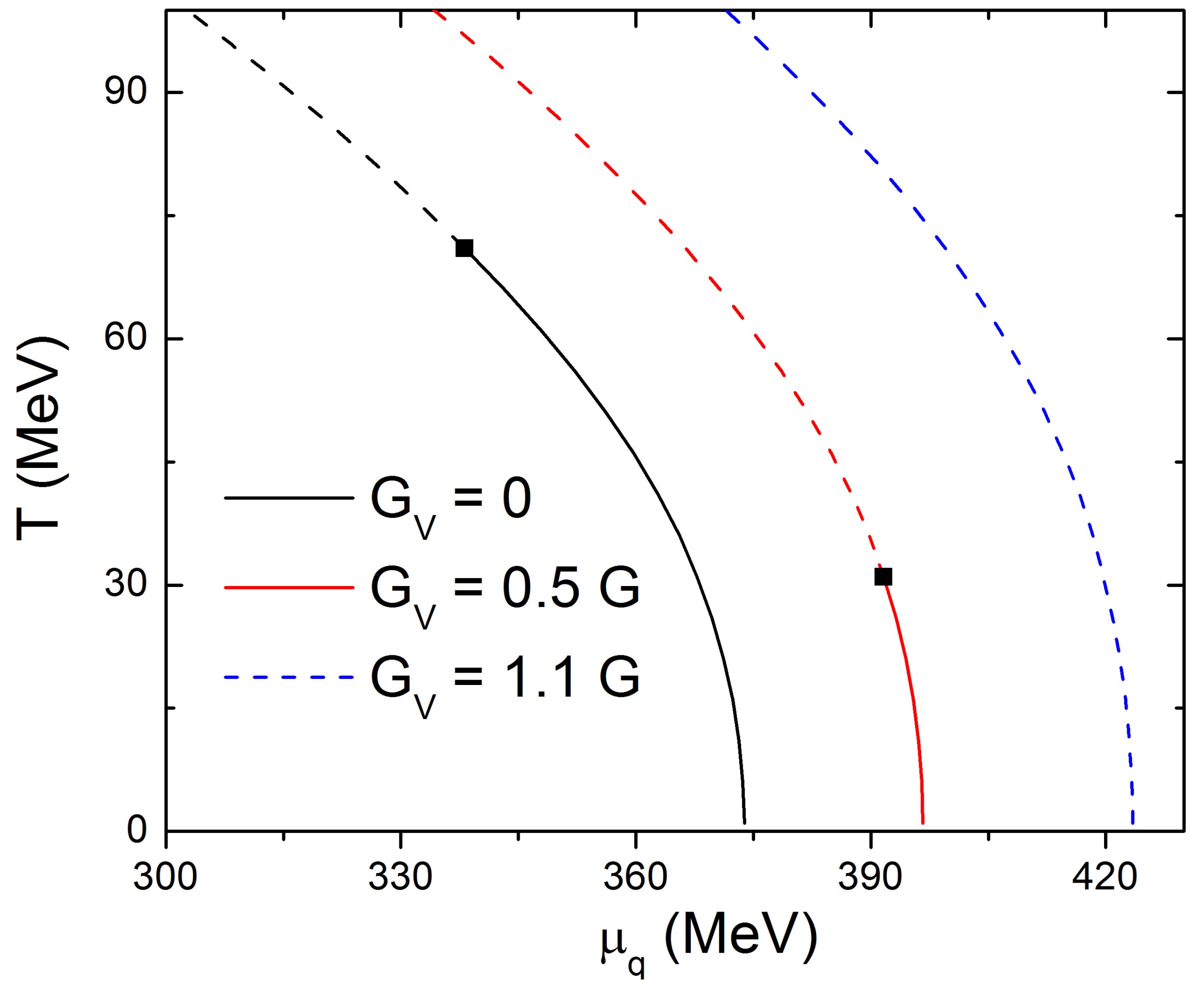
$$M_u = m_u - 2G\langle\bar{u}u\rangle + 2K\langle\bar{d}d\rangle\langle\bar{s}s\rangle$$
$$M_d = m_d - 2G\langle\bar{d}d\rangle + 2K\langle\bar{s}s\rangle\langle\bar{u}u\rangle$$
$$M_s = m_s - 2G\langle\bar{s}s\rangle + 2K\langle\bar{u}u\rangle\langle\bar{d}d\rangle$$

$$\langle\bar{q}_i q_i\rangle = -2M_i N_c\int\frac{d^3\mathbf{k}}{(2\pi)^3 E_i}\left[1 - f_i(k) - \bar{f}_i(k)\right]$$

$$\langle\bar{\psi}\gamma^\mu\psi\rangle = 2N_c\sum_{i=u,d,s}\int\frac{d^3\mathbf{k}}{(2\pi)^3 E_i}\, k^\mu[f_i(k) - \bar{f}_i(k)],$$

Effect of vector interaction on QCD phase diagram

Jun Xu, Song, Ko & Li, PRL 112, 012301 (2014)

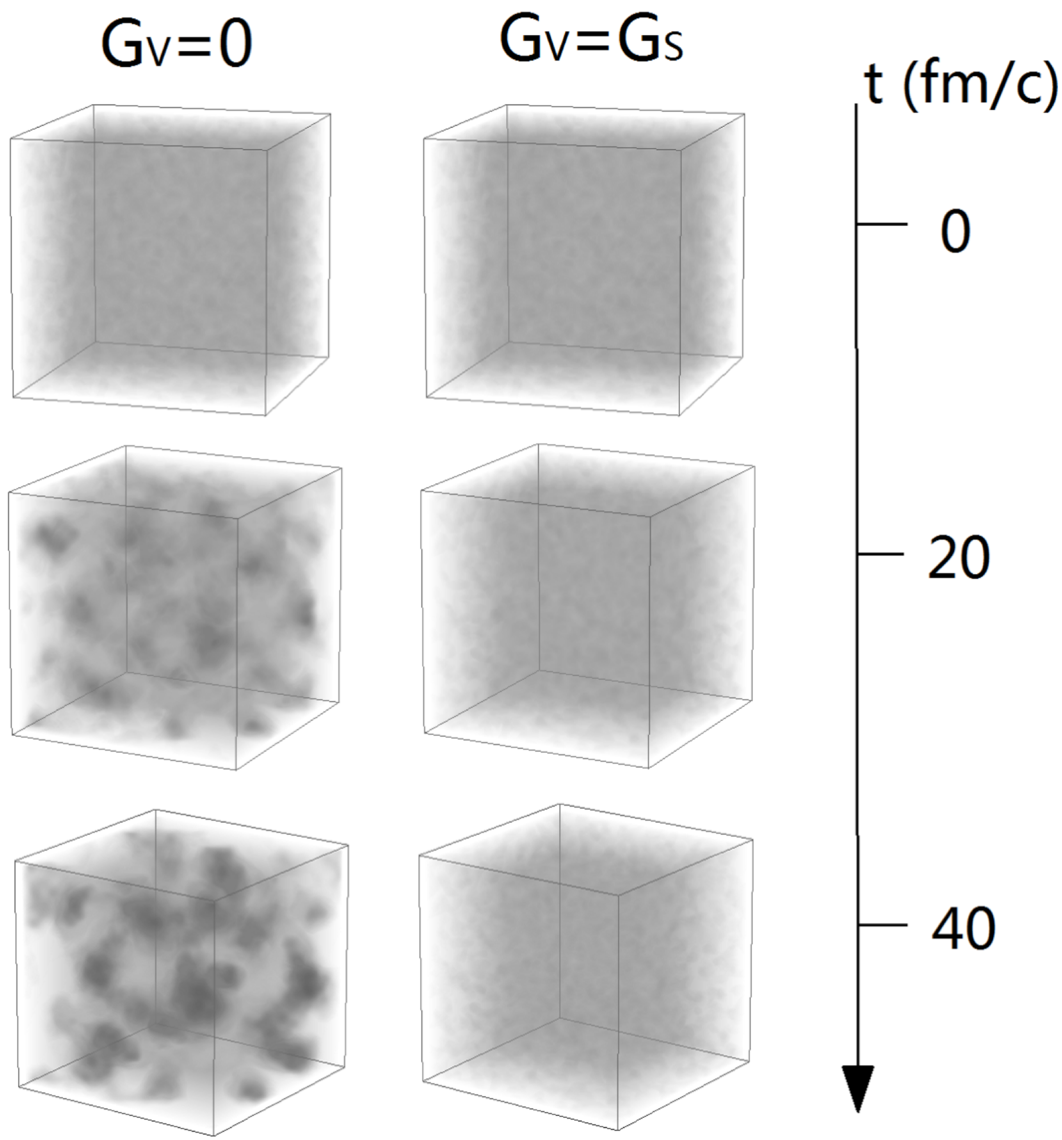


- Location of critical point depends strongly on G_V; moving to lower temperature and larger baryon chemical potential as G_V increases, and it disappears for $G_V > 0.6$ G.

Transport description of quark matter in a box

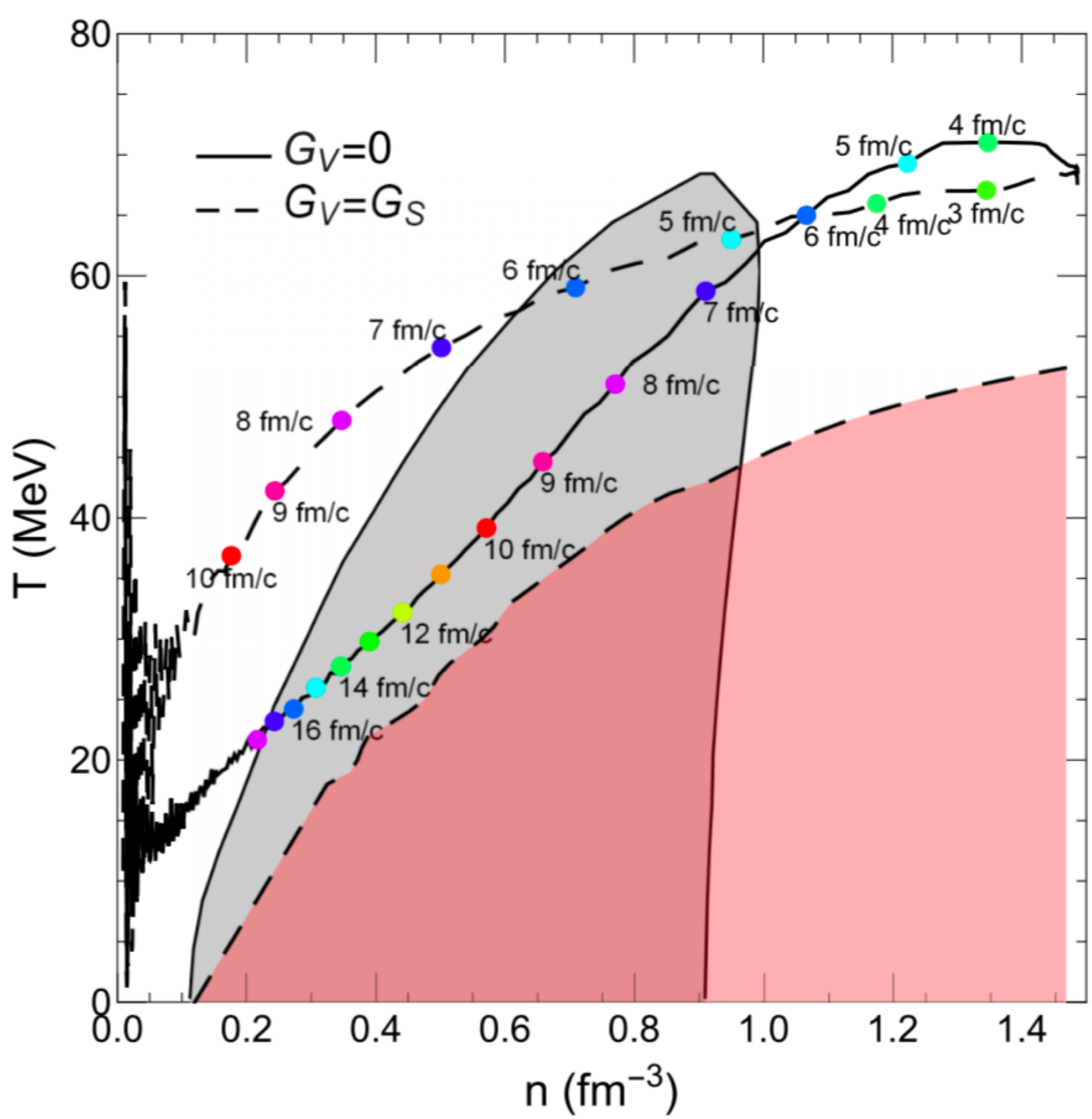
$$\partial_t f + \mathbf{p}/E \cdot \nabla f - \nabla H \cdot \nabla_p f = \mathcal{C}[f]$$

C[f] includes quark elastic scattering with cross section of 3 mb



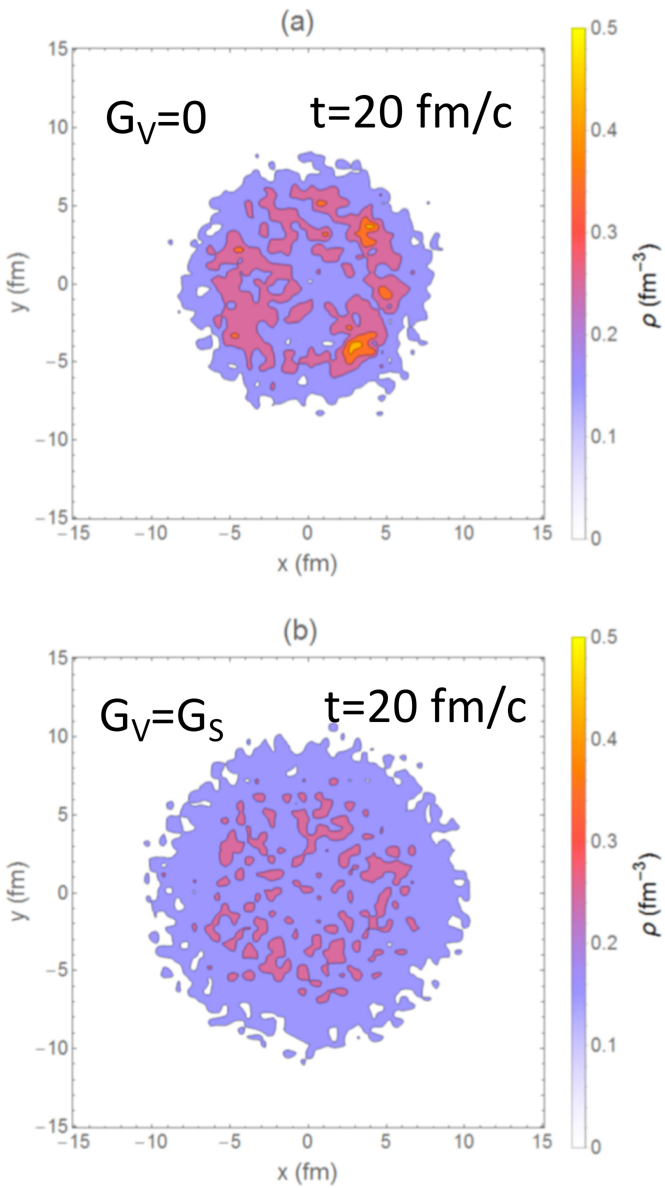
- $n_q = 0.5/fm^3$, T = 20 MeV
- Right: outside spinodal region
- Left: inside spinodal region; large density fluctuations appear due to growth of unstable modes

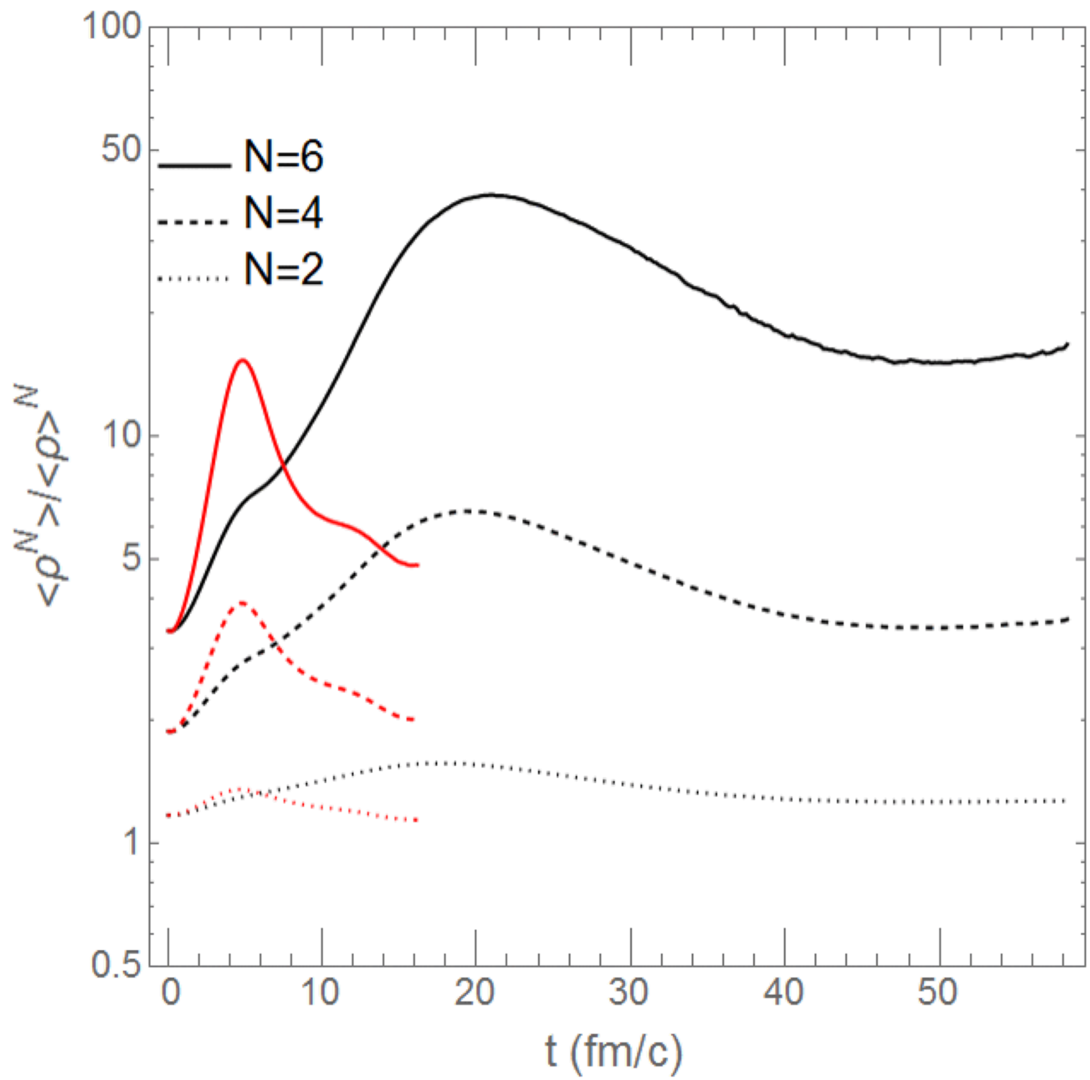
Trajectory of an expanding quark matter



- First-order phase transition results in a longer QGP phase of an expanding dense matter.

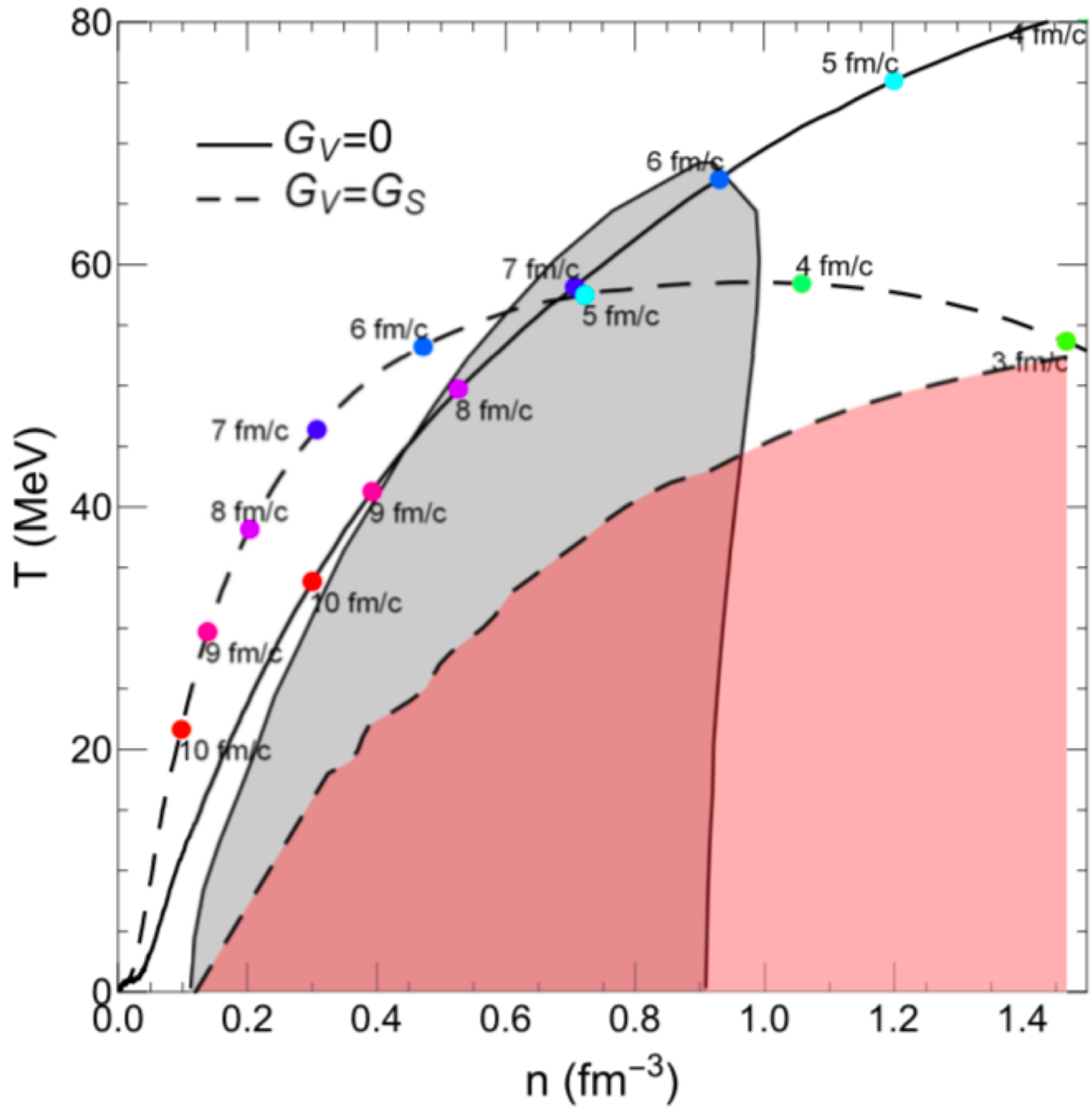
Time evolution of density moments



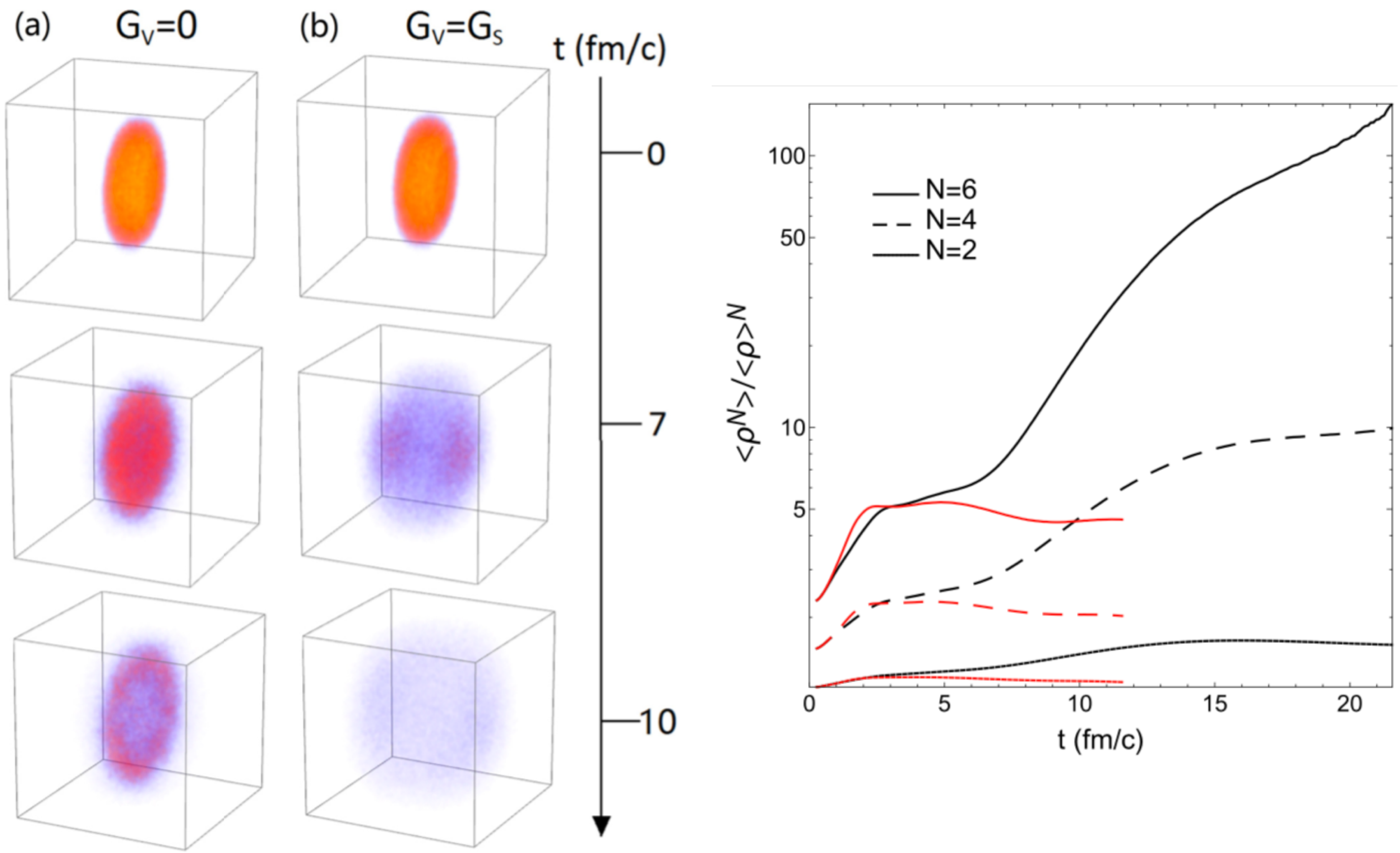


- Large density moments are generated by the spinodal instability.

Expanding quark matter: AMPT initial conditions ($\sqrt{s_{NN}}$ = 2. 5 GeV)



Time evolution of density moments



- Large density moments are generated by the spinodal instability.

Summary

- Experimentally observed enhancement of light nuclei production in beam energy scan program at RHIC can be explained by nucleon density fluctuations if they are produced via nucleon coalescence at kinetic freeze out.
- The spinodal instability in baryon-rich quark matter due to its first-order transition to the hadronic matter can result in large quark density fluctuation.
- How can quark density fluctuations due to the spinodal instability of baryon-rich quark matter survive during hadronic evolution remains a challenge.