Light nuclei yield ratio and nucleon density fluctuations

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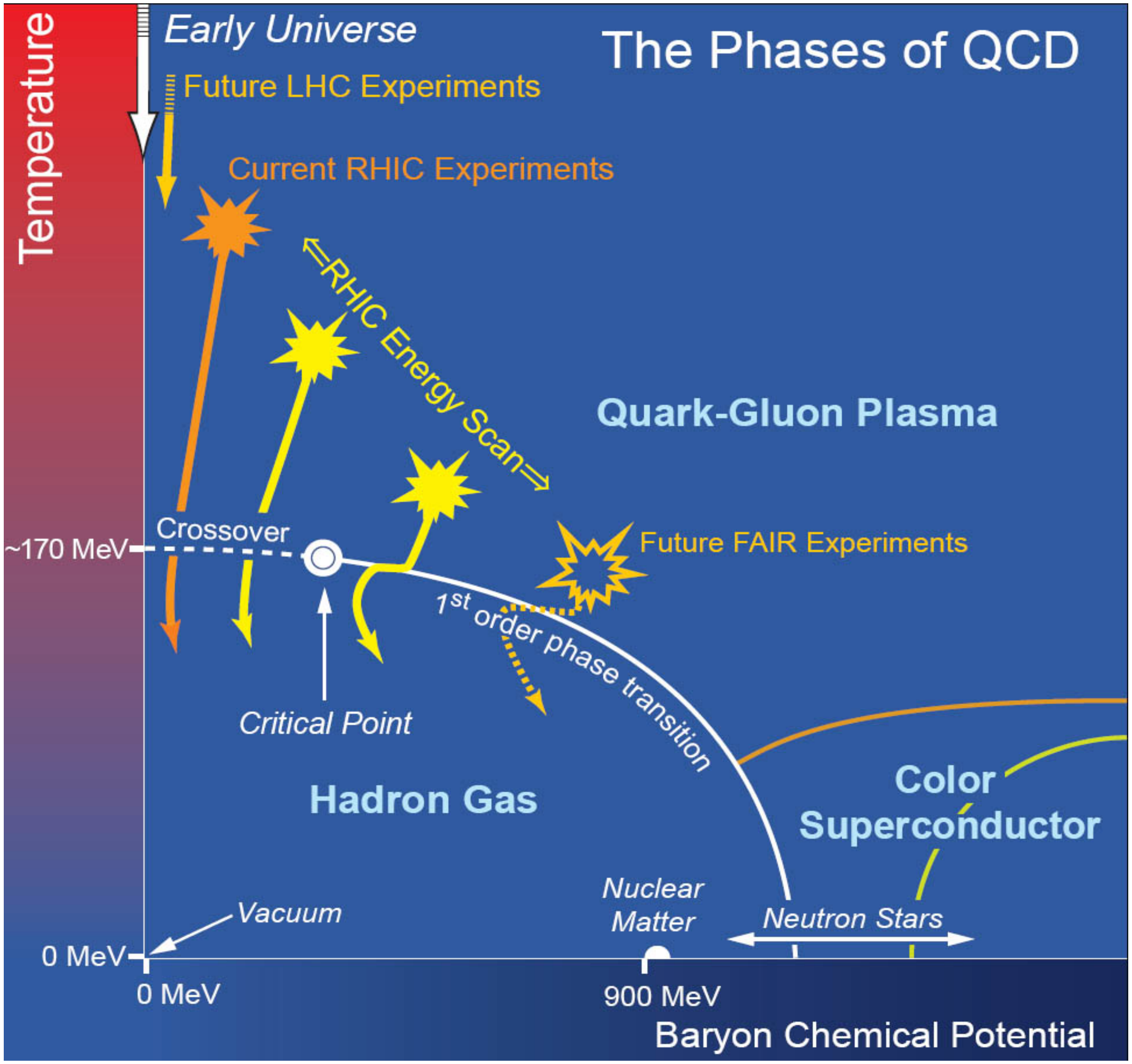
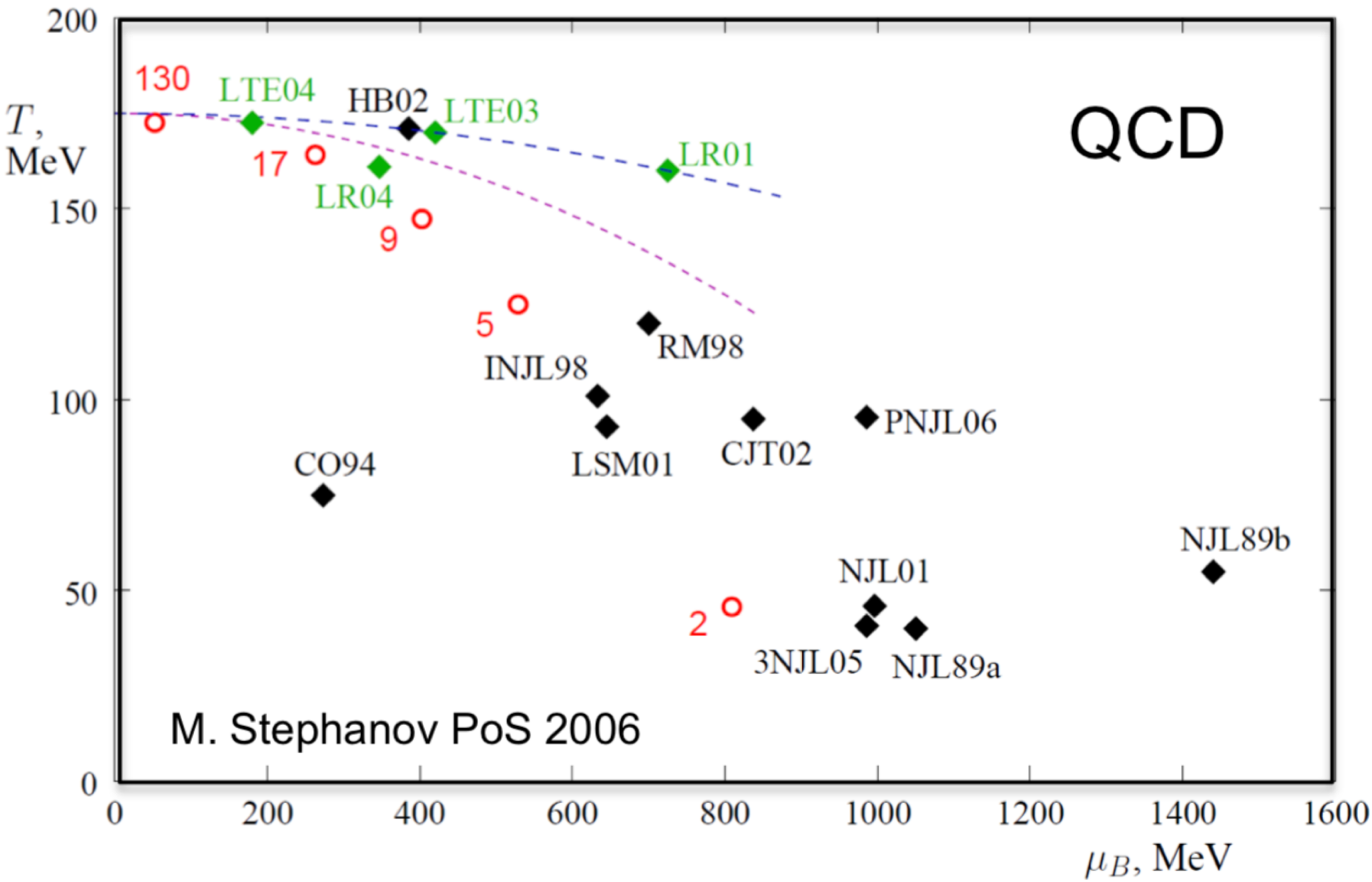
- ❑ Introduction
- ❑ The coalescence model
- ❑ Effect of density fluctuations
- ❑ Summary

QCD phase diagram



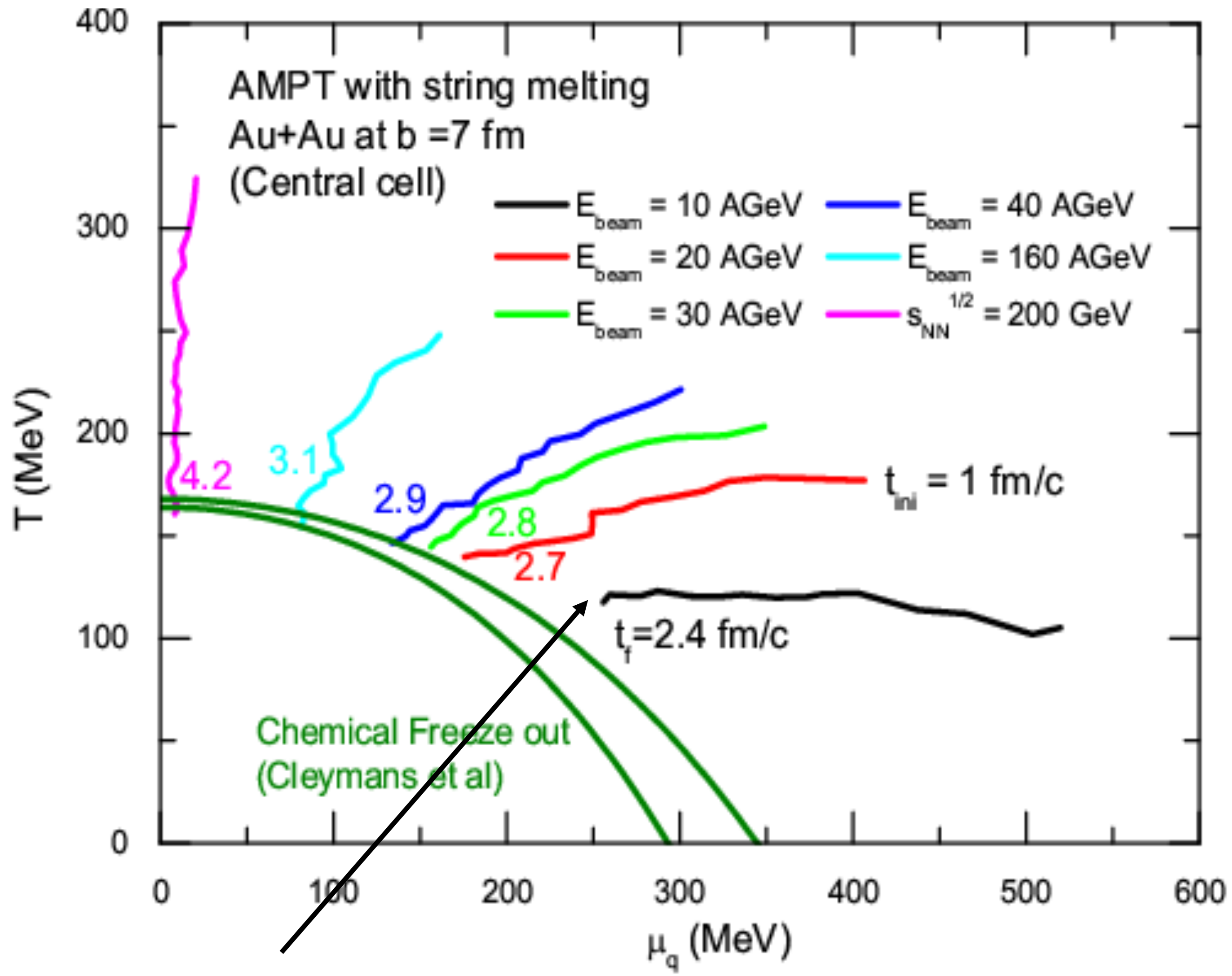
Critical point in QCD phase diagram



- Large uncertainties in theoretical predictions on the critical point.

Mapping the phase diagram via heavy ion collisions

Chen, Ko, Liu & Zhang, PoS (CPOD 2009) 034 (2009)



Require first-order phase transition ?

Collision energy dependence of density fluctuations

Sun, Chen, Ko, Pu & Xu, PLB 781, 499 (2018)

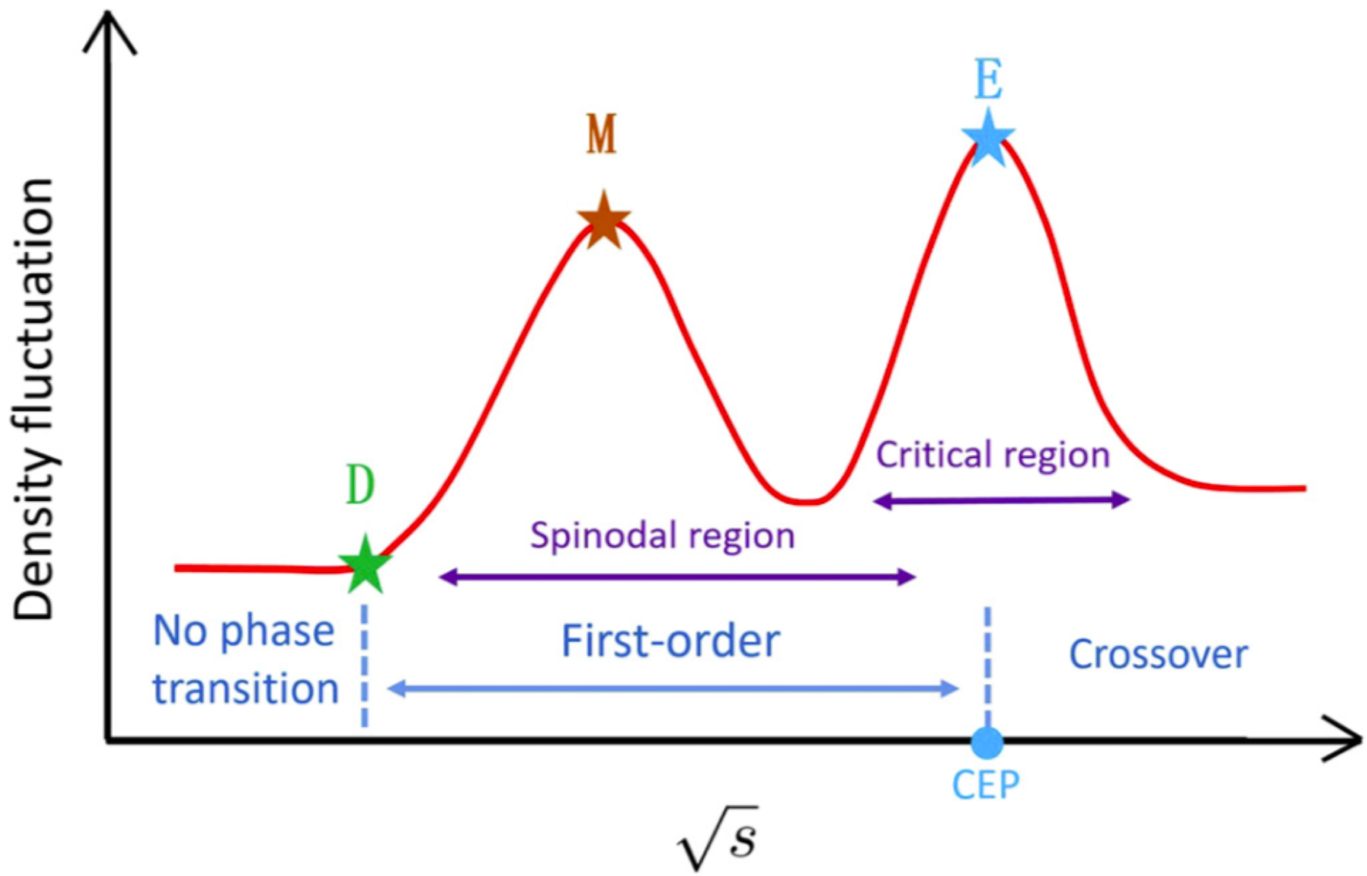
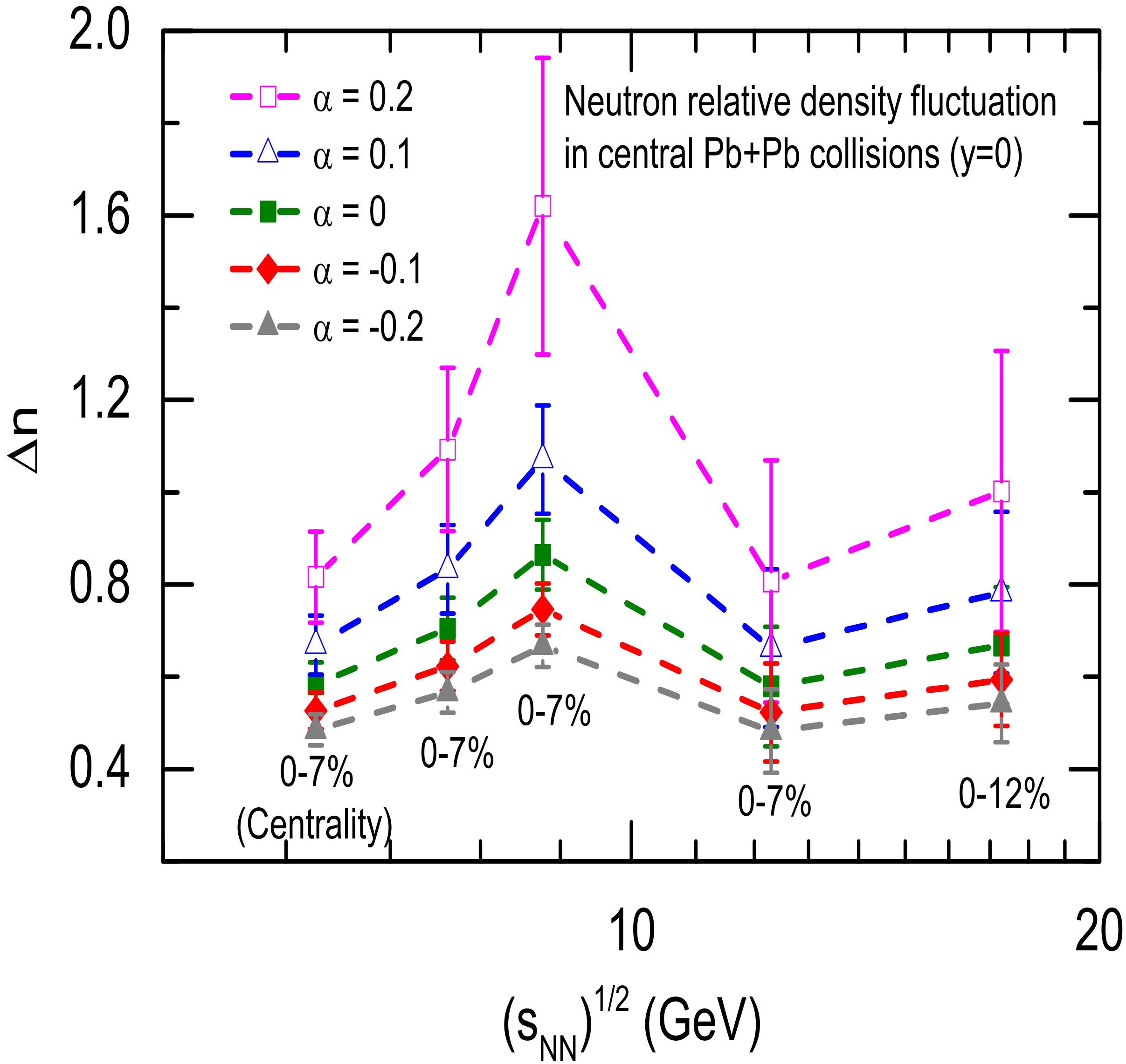


Fig. 1. Schematic depiction of the collision energy dependence of density fluctuations in heavy-ion collisions together with the corresponding phase regions in the QCD phase diagram. Point 'D' indicates the beginning of first-order phase transition, 'M' denotes the maximum caused by the spinodal instability and 'E' denotes the maximum due to the CEP.

Neutron relative density fluctuation from yield ratio of light nuclei

Sun, Chen, Ko & Xu, PLB 774, 103 (2017)



$$\mathcal{O}_{\text{p-d-t}} = \frac{N_{^3\mathrm{H}} N_p}{N_{\mathrm{d}}^2} = g\frac{1+(1+2\alpha)\Delta n}{(1+\alpha\Delta n)^2}$$

$$\Delta n = \frac{\langle(\delta n)^2\rangle}{\langle n\rangle^2}$$

$$\langle \delta n \delta n_p\rangle = \alpha\frac{\langle n_p\rangle}{\langle n\rangle}\langle(\delta n)^2\rangle$$

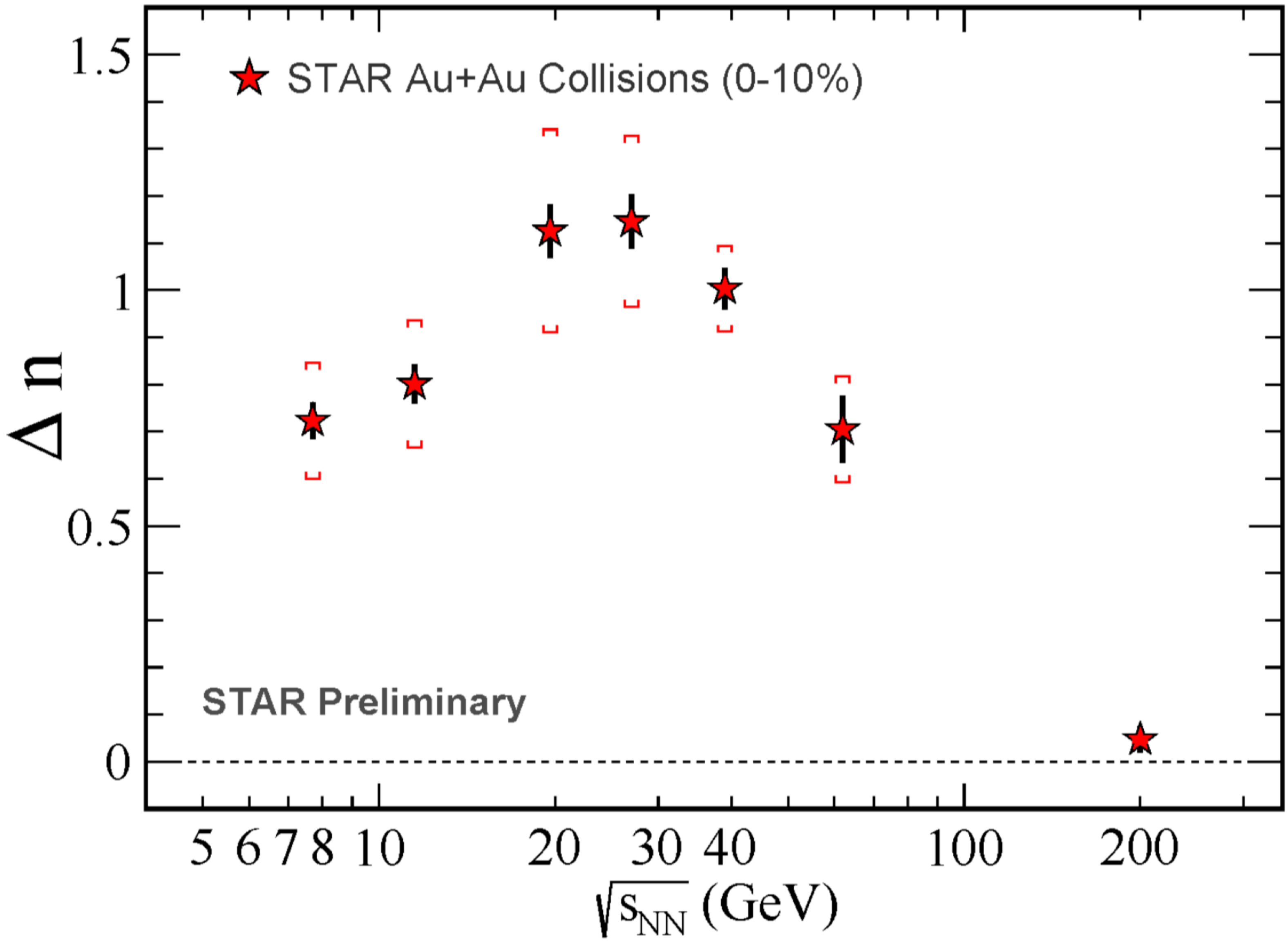
α: correlation factor

$T_C \approx 144$ MeV
$\mu_C \approx 385$ MeV

- Expect a similar behavior for $\frac{pK_0}{\pi^+\Lambda}$ from u-quark density fluctuation.

Neutron relative density fluctuation from yield ratio of light nuclei

Dingwei Zhang for STAR Collaboration, NN2018



Deuteron number in the coalescence model

In the coalescence model, the deuteron number is given by

$$N_d = g_d \int d^3\mathbf{x}_1 \int d^3\mathbf{k}_1 \int d^3\mathbf{x}_2 \int d^3\mathbf{k}_2 f_1(\mathbf{x}_1,\mathbf{k}_1) f_2(\mathbf{x}_2,\mathbf{k}_2) W_d(\mathbf{x}_1-\mathbf{x}_2,(\mathbf{k}_1-\mathbf{k}_2)/2).$$

In the above, the proton or neutron distribution function is given by

$$f(\mathbf{x},\mathbf{k}) = \frac{2\gamma}{(2\pi)^3}\, e^{-\frac{k^2}{2mT}},$$

where T, m and γ are the temperature, nucleon mass, and fugacity, respectively, and is normalized to

$$N = \int d^3\mathbf{x} \int d^3\mathbf{k} f(\mathbf{x},\mathbf{k}) = 2\gamma V \left(\frac{mT}{2\pi}\right)^{3/2}.$$

after using

$$\int d^3\mathbf{x} = V, \quad \int d^3\mathbf{k}\; e^{-ak^2} = \left(\frac{\pi}{a}\right)^{3/2},$$

with V being the volume. The deuteron Wigner function is given by

$$W_d(\mathbf{x},\mathbf{k}) = 8\, e^{-\frac{x^2}{\sigma^2}}\, e^{-\sigma^2 k^2},$$

and is normalized according to

$$\int d^3\mathbf{x} \int d^3\mathbf{k}\; W_d(\mathbf{x},\mathbf{k}) = (2\pi)^3.$$

Deuteron number in the coalescence model (Continued)

Changing variables to

$$\begin{aligned}\mathbf{X} &= \frac{\mathbf{x}_1+\mathbf{x}_2}{2}, \quad \mathbf{x}=\mathbf{x}_1-\mathbf{x}_2,\\ \mathbf{K} &= \mathbf{k}_1+\mathbf{k}_2, \quad \mathbf{k}=\frac{\mathbf{k}_1-\mathbf{k}_2}{2},\end{aligned}$$

then

$$\begin{aligned}N_d &= \frac{32 g_d \gamma_1\gamma_2}{(2\pi)^6}\int d^3\mathbf{X}\int d^3\mathbf{x}\; e^{-\frac{x^2}{\sigma^2}}\int d^3\mathbf{K}\; e^{-\frac{K^2}{4mT}}\int d^3\mathbf{k}\; e^{-k^2(\sigma^2+\frac{1}{mT})}\\ &= \frac{32 g_d \gamma_1\gamma_2}{(2\pi)^6}\, V\,(\pi\sigma^2)^{3/2}\,(4\pi mT)^{3/2}\left(\frac{\pi}{\sigma^2+\frac{1}{mT}}\right)^{3/2}\\ &= 2^{3/2} g_d\left(\frac{2\pi}{mT+\frac{1}{\sigma^2}}\right)^{3/2}\frac{N_1N_2}{V}\\ &= \frac{3}{2^{1/2}}\left(\frac{2\pi}{mT}\right)^{3/2}\frac{1}{(1+\frac{1}{mT\sigma^2})^{3/2}}\frac{N_1N_2}{V}, \quad (g_d=3/4)\\ &\approx \frac{3}{2^{1/2}}\left(\frac{2\pi}{mT}\right)^{3/2}\frac{N_1N_2}{V}, \quad (mT\gg 1/\sigma^2)\end{aligned}$$

In this limit, the coalescence model gives

$$
\begin{aligned}
N_{\mathrm{d}}^{\mathrm{coal}} &\approx \frac{3}{2^{1/2}}\left(\frac{2\pi}{mT}\right)^{3/2}\frac{N_1 N_2}{V} = \frac{3}{2^{1/2}}4V\gamma_1\gamma_2\left(\frac{mT}{2\pi}\right)^{3/2} \\
&= 3V\gamma_1\gamma_2\left(\frac{mT}{\pi}\right)^{3/2}
\end{aligned}
$$

Compared with the thermal model,

$$
\begin{aligned}
N_{\mathrm{d}}^{\mathrm{thermal}} &\approx \frac{3V\gamma_1\gamma_2}{(2\pi)^3}\int d^3\mathrm{k}e^{-\frac{k^2}{4mT}}e^{\frac{B_{\mathrm{d}}}{T}} \\
&= 3V\gamma_1\gamma_2\left(\frac{mT}{\pi}\right)^{3/2}
\end{aligned}
$$

so $$N_{\mathrm{d}}^{\mathrm{coal}} \approx N_{\mathrm{d}}^{\mathrm{thermal}} \text{ if } T \gg B_{\mathrm{d}} \text{ and } mT \gg 1/\sigma^2$$

Why $N_{\mathrm{d}}^{\mathrm{thermal}}(\mathrm{T_C}) = N_{\mathrm{d}}^{\mathrm{thermal}}(\mathrm{T_K})$?

Entropy per baryon and the d/p ratio

Siemens & Kapusta,
PRL 43, 1486 (1979)

$$\frac{S}{N} = \frac{E}{T} + \ln Z = \frac{3}{2}N + \ln\frac{(4V/\lambda_{\rm th}^3)^N}{N!}, \qquad \lambda_{\rm th} = \left(\frac{2\pi}{mT}\right)$$

$$\approx N\ln\left[\frac{(4V/N)}{\lambda_{\rm th}^3}e^{5/2}\right], \qquad \ln N! \approx N\ln N - N = N\ln(N/e)$$

$$= \frac{5}{2} - \ln\left[\frac{N}{4V}\lambda_{\rm th}^3\right]$$

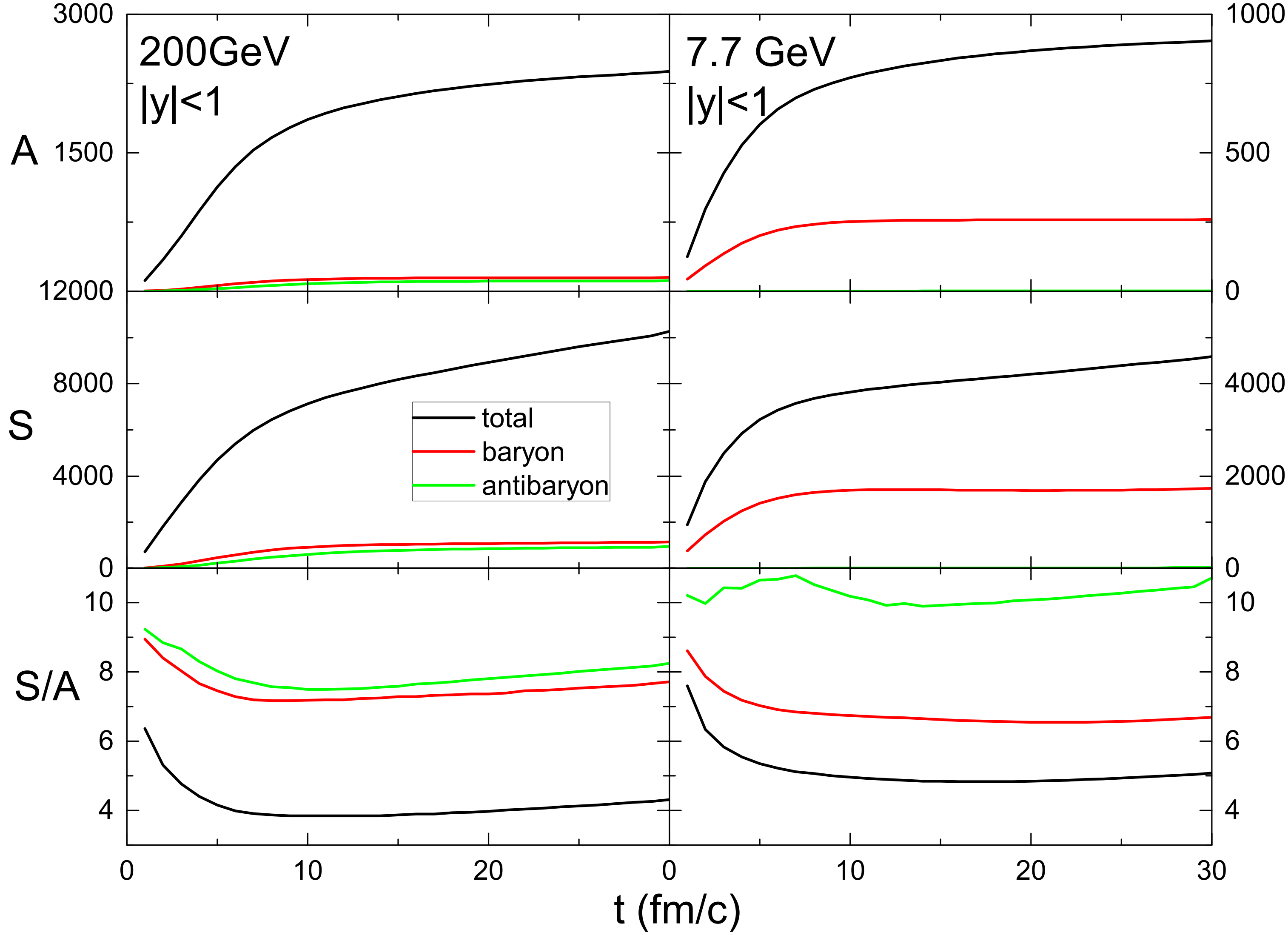
Since $R_{\rm dp} \equiv \dfrac{N_{\rm d}}{N_{\rm p}} = \dfrac{3V\gamma_{\rm n}\gamma_{\rm p}2^{3/2}/\lambda_{\rm th}^3}{2V\gamma_{\rm p}/\lambda_{\rm th}^3} = 3\sqrt{2}\gamma_{\rm n}$

$$= 3\sqrt{2}\left(\frac{N_{\rm n}}{2V}\right)\lambda_{\rm th}^3 = 3\sqrt{2}\left(\frac{N}{4V}\right)\lambda_{\rm th}^3, \qquad N = 2N_{\rm n}$$

so $\dfrac{S}{N} = \dfrac{5}{2} - \ln\left[\dfrac{N}{4V}\lambda_{\rm th}^3\right] = \dfrac{5}{2} - \ln\left(\dfrac{R_{\rm dp}}{3\sqrt{2}}\right)$

$$= \frac{5}{2} + \ln(3\sqrt{2}) - \ln R_{\rm dp} \approx 3.95 - \ln R_{\rm dp}$$

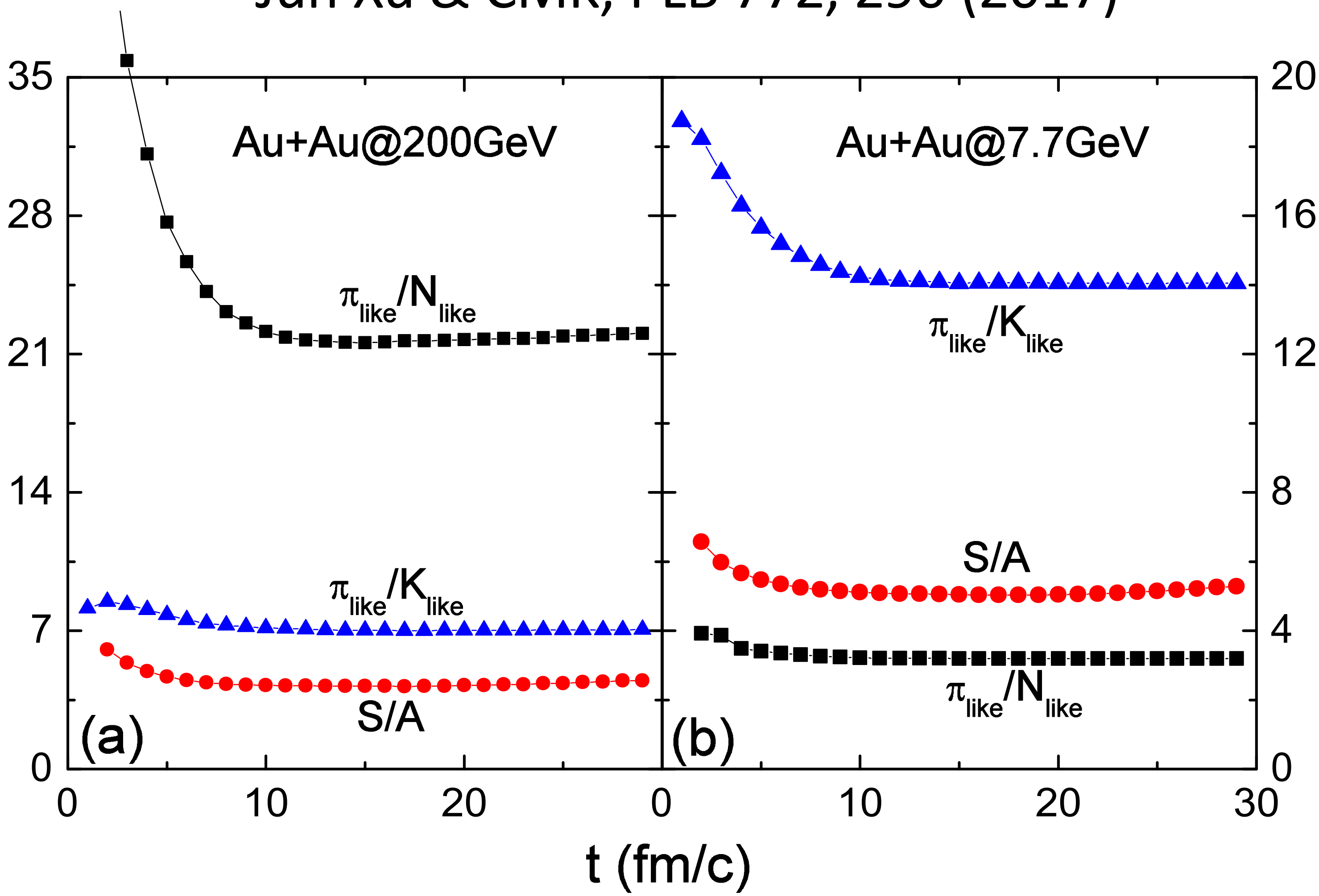
Time evolution of baryon entropy in relativistic heavy ion collisions From the hadronic phase of AMPT



- Baryon entropy per baryon remains essentially constant during hadronic evolution.

Chemical freeze-out in relativistic heavy ion collisions

Jun Xu & CMK, PLB 772, 290 (2017)



- Both ratio of effective particle numbers and entropy per particle remain essentially constant from chemical to kinetic freeze-out.

Effect of density fluctuations on deuteron number

For non-uniform distributions, the factor $F = N_1 N_2 / V$ is replaced by

$$F = \frac{1}{(\pi\sigma^2)^{3/2}} \int d^3\mathbf{x}_1 \int d^3\mathbf{x}_2 n_1(\mathbf{x}_1) n_2(\mathbf{x}_2) e^{-(\mathbf{x}_1 - \mathbf{x}_2)^2/\sigma^2}.$$

It can be rewritten as

$$\begin{aligned}
F &= \frac{1}{(\pi\sigma^2)^{3/2}} \int d^3\mathbf{X} \int d^3\mathbf{x} e^{-\mathbf{x}^2/\sigma^2} n_1\left(\mathbf{X} + \frac{\mathbf{x}}{2}\right) n_2\left(\mathbf{X} - \frac{\mathbf{x}}{2}\right) \\
&\approx \frac{1}{(\pi\sigma^2)^{3/2}} \int d^3\mathbf{X} \int d^3\mathbf{x} e^{-\mathbf{x}^2/\sigma^2} \left[n_1(\mathbf{X}) + \nabla n_1(\mathbf{X}) \cdot \frac{\mathbf{x}}{2}\right] \left[n_2(\mathbf{X}) - \nabla n_2(\mathbf{X}) \cdot \frac{\mathbf{x}}{2}\right] \\
&= \frac{1}{(\pi\sigma^2)^{3/2}} \int d^3\mathbf{X} \int d^3\mathbf{x} e^{-\mathbf{x}^2/\sigma^2} \left\{n_1(\mathbf{X}) n_2(\mathbf{X}) + \frac{\mathbf{x}}{2} \cdot [n_2(\mathbf{X}) \nabla n_1(\mathbf{X}) - n_1(\mathbf{X}) \nabla n_2(\mathbf{X})]\right. \\
&\quad \left. - \left[\frac{\mathbf{x}}{2} \cdot \nabla n_1(\mathbf{X})\right] \left[\frac{\mathbf{x}}{2} \cdot \nabla n_2(\mathbf{X})\right]\right\} \\
&= \int d^3\mathbf{X} n_1(\mathbf{X}) n_2(\mathbf{X}) \\
&\quad + \frac{1}{(\pi\sigma^2)^{3/2}} \int d^3\mathbf{X} \int d^3\mathbf{x} e^{-\mathbf{x}^2/\sigma^2} \left[\frac{\mathbf{x}}{2} \cdot \nabla n_1(\mathbf{X})\right] \left[\frac{\mathbf{x}}{2} \cdot \nabla n_2(\mathbf{X})\right].
\end{aligned}$$

Assuming $\nabla\rho_n(\mathbf{X}) \sim \frac{\rho_n(\mathbf{X})}{a}\mathbf{e_n}, \quad \nabla\rho_\mathbf{p}(\mathbf{X}) \sim \frac{\rho_\mathbf{p}(\mathbf{X})}{a}\mathbf{e_p}$

where $\mathbf{e_n}$ and $\mathbf{e_p}$ are unit vectors along the density gradient of the neutron and proton spatial distributions, and *a* is the length over which they change appreciably, then the second term becomes

$$
\begin{aligned}
F_2 &\approx \int \mathrm{d}^3\mathbf{X}\rho_n(\mathbf{X})\rho_p(\mathbf{X})\frac{1}{(\pi\sigma^2)^{3/2}}\int \mathrm{d}^3\mathbf{x}e^{-\frac{\mathbf{x}^2}{\sigma^2}}\left[\frac{\mathbf{x}\cdot\mathbf{e_n}}{2a}\right]\left[\frac{\mathbf{x}\cdot\mathbf{e_p}}{2a}\right] \\
&\leq \int \mathrm{d}^3\mathbf{X}\rho_n(\mathbf{X})\rho_p(\mathbf{X})\frac{1}{(\pi\sigma^2)^{3/2}}\int \mathrm{d}^3\mathbf{x}e^{-\frac{\mathbf{x}^2}{\sigma^2}}\left(\frac{x}{2a}\right)^2 \\
&< \frac{3}{8}\left(\frac{\sigma}{a}\right)^2\int \mathrm{d}^3\mathbf{X}\rho_n(\mathbf{X})\rho_p(\mathbf{X}).
\end{aligned}
$$

If the directions of $\mathbf{e_n}$ and $\mathbf{e_p}$ are not strongly correlated and *a* is significantly larger than σ, then F_2 is much smaller than the first term, and

$$
N_\mathrm{d} \approx \frac{3}{2^{1/2}}\left(\frac{2\pi}{mT}\right)^{3/2}\int \mathrm{d}^3\mathbf{x}\ \rho_n(\mathbf{x})\rho_p(\mathbf{x}).
$$

Taking into account density fluctuations

$$\rho_n(\mathbf{x}) = \frac{1}{V}\int \rho_n(\mathbf{x})\mathrm{d}^3\mathbf{x} + \delta\rho_n(\mathbf{x}) = \langle\rho_n\rangle + \delta\rho_n(\mathbf{x}),$$
$$\rho_p(\mathbf{x}) = \frac{1}{V}\int \rho_p(\mathbf{x})\mathrm{d}^3\mathbf{x} + \delta\rho_p(\mathbf{x}) = \langle\rho_p\rangle + \delta\rho_p(\mathbf{x}).$$

then $$N_\mathrm{d} \approx \frac{3}{2^{1/2}}\left(\frac{2\pi}{mT}\right)^{3/2}\int d^3\mathbf{x}(\langle\rho_n\rangle + \delta\rho_n)(\langle\rho_p\rangle + \delta\rho_p)$$
$$= \frac{3}{2^{1/2}}\left(\frac{2\pi}{mT}\right)^{3/2}\left[\int d^3\mathbf{x}\langle\rho_n\rangle\langle\rho_p\rangle + \int d^3\mathbf{x}(\delta\rho_n)((\delta\rho_p)\right]$$
$$= \frac{3}{2^{1/2}}\left(\frac{2\pi}{mT}\right)^{3/2} N_p\langle\rho_n\rangle(1 + C_\mathrm{np}),$$

where $\langle\rho_n\rangle, \langle\rho_p\rangle$: average neutron and proton densities

$C_\mathrm{np} = \langle\delta\rho_n\delta\rho_p\rangle/(\langle\rho_n\rangle\langle\rho_p\rangle)$: neutron and proton density correlation

For triton

$$
\begin{aligned}
N_{^3\mathrm{H}} &\approx \frac{3^{3/2}}{4}\left(\frac{2\pi}{mT}\right)^3 \int d^3\mathbf{x}\ \rho_n^2(\mathbf{x})\rho_p(\mathbf{x}) \\
&= \frac{3^{3/2}}{4}\left(\frac{2\pi}{mT}\right)^3 \int d^3\mathbf{x}\ (\langle\rho_n\rangle+\delta\rho_n)^2(\langle\rho_p\rangle+\delta\rho_p) \\
&= \frac{3^{3/2}}{4}\left(\frac{2\pi}{mT}\right)^3 \left[\int d^3\mathbf{x}\ \langle\rho_n\rangle^2\langle\rho_p\rangle\right. \\
&\quad \left. +2\langle\rho_n\rangle\int d^3\mathbf{x}\ (\delta\rho_n)(\delta\rho_p) + \langle\rho_p\rangle\int d^3\mathbf{x}\ (\delta\rho_n)^2\right] \\
&= \frac{3^{3/2}}{4}\left(\frac{2\pi}{mT}\right)^3 N_p\langle\rho_n\rangle^2(1+\Delta\rho_n+2C_{\mathrm{np}}),
\end{aligned}
$$

where

$$\Delta\rho_n = \langle(\delta\rho_n)^2\rangle/\langle\rho_n\rangle^2 : \text{relative neutron density fluctuation}$$

Define yield ratio

$$\mathcal{O}_{\mathrm{p-d}} \equiv \frac{N_{\mathrm{d}}}{N_{\mathrm{p}}^2} = \frac{3}{2^{1/2}} \left(\frac{2\pi}{mT}\right)^{3/2} \frac{\langle \rho_{\mathrm{n}} \rangle}{N_{\mathrm{p}}} (1 + C_{\mathrm{np}})$$

then

$$\begin{aligned} C_{\mathrm{np}} &= \frac{2^{1/2}}{3} \left(\frac{mT}{2\pi}\right)^{3/2} \frac{N_{\mathrm{p}}}{\langle \rho_{\mathrm{n}} \rangle} \mathcal{O}_{\mathrm{p-d}} - 1 \\ &= \frac{2^{1/2}}{3(2\pi)^3} [(2\pi mT)^{3/2} V] \left[\frac{N_{\mathrm{p}}/V}{\langle \rho_{\mathrm{n}} \rangle}\right] \mathcal{O}_{\mathrm{p-d}} - 1 \\ &= g_{\mathrm{p-d}} V_{\mathrm{ph}} R_{\mathrm{np}} \mathcal{O}_{\mathrm{p-d}} - 1 \end{aligned}$$

where

$$g_{\mathrm{p-d}} = \frac{2^{1/2}}{3(2\pi)^3} \approx 0.0019$$

$V_{\mathrm{ph}} = (2\pi mT)^{3/2} V$: effective phase volume of nucleons

$$R_{\mathrm{np}} = \frac{N_{\mathrm{p}}}{N_{\mathrm{n}}} = \frac{\langle \rho_{\mathrm{p}} \rangle}{\langle \rho_{\mathrm{n}} \rangle}$$

Define yield ratio

$$\mathcal{O}_{\mathrm{p-d-t}} \equiv \frac{N_{^3\mathrm{H}} N_{\mathrm{p}}}{N_{\mathrm{d}}^2} = \frac{\frac{3^{3/2}}{4}\left(\frac{2\pi}{mT}\right)^3 N_p \langle \rho_n \rangle^2 (1 + \Delta\rho_n + 2C_{\mathrm{np}})}{\left[\frac{3}{2^{1/2}}\left(\frac{2\pi}{mT}\right)^{3/2} N_p \langle \rho_n \rangle (1 + C_{\mathrm{np}})\right]^2}$$

$$= \frac{3^{3/2}}{18} \frac{1 + \Delta\rho_n + 2C_{\mathrm{np}}}{(1 + C_{\mathrm{np}})^2}$$

then $$\Delta\rho_{\mathrm{n}} = g_{\mathrm{p-d-t}}(1 + C_{\mathrm{np}})^2 \mathcal{O}_{\mathrm{p-d-t}} - 2C_{\mathrm{np}} - 1$$

where $$g_{\mathrm{p-d-t}} = \frac{9}{4}\left(\frac{4}{3}\right)^{3/2} \approx 3.5$$

Define isospin density fluctuation

$$\Delta\rho_{\mathrm{I}} \equiv \frac{\langle(\delta\rho_{\mathrm{n}} - \delta\rho_{\mathrm{p}})^2\rangle}{(\langle\rho_{\mathrm{n}}\rangle + \langle\rho_{\mathrm{p}}\rangle)^2} = \frac{\langle(\delta\rho_{\mathrm{n}})^2 - 2(\delta\rho_{\mathrm{n}})(\delta\rho_{\mathrm{p}}) + (\delta\rho_{\mathrm{p}})^2\rangle}{\langle\rho_{\mathrm{n}}\rangle^2 (1 + R_{\mathrm{np}})^2}$$

$$= \frac{\Delta\rho_{\mathrm{n}} - 2R_{\mathrm{np}}C_{\mathrm{np}} + R_{\mathrm{np}}^2 \Delta\rho_{\mathrm{p}}}{(1 + R_{\mathrm{np}})^2}$$

Table 1

Yields dN/dy of p, d and ^{3}H at midrapidity, together with the yield ratio π^+/π^- measured in central Pb+Pb collisions at 20 AGeV ($0-7\%$ centrality, $\sqrt{s_{NN}}=6.3$ GeV), 30 AGeV ($0-7\%$ centrality, $\sqrt{s_{NN}}=7.6$ GeV), 40 AGeV ($0-7\%$ centrality, $\sqrt{s_{NN}}=8.8$ GeV), 80 AGeV ($0-7\%$ centrality, $\sqrt{s_{NN}}=12.3$ GeV), and 158 AGeV ($0-12\%$ centrality, $\sqrt{s_{NN}}=17.3$ GeV) by the NA49 Collaboration [31,41,42]. Also given are the chemical freeze-out temperature $T_{\rm ch}$ (GeV) and volume $V_{\rm ch}$ (fm^3), the derived yield ratios $\mathcal{O}_{\text{p-d}}$ and $\mathcal{O}_{\text{p-d-t}}$, and the extracted $C_{\rm np}$, $\Delta\rho_n$ and $\Delta\rho_I$. In obtaining $\mathcal{O}_{\text{p-d}}$ and $\mathcal{O}_{\text{p-d-t}}$, the weak decay contributions to the yield of proton from hyperons are corrected by using results from the statistical model (see text for details).

$\sqrt{s_{NN}}$	p	d	^{3}H(10^{-3})	π^+/π^-	$T_{\rm ch}$	$V_{\rm ch}$	$\mathcal{O}_{\text{p-d}}(10^{-4})$	$\mathcal{O}_{\text{p-d-t}}$	$C_{\rm np}$	$\Delta\rho_n$	$\Delta\rho_I$
6.3	46.1 ± 2.1	2.094 ± 0.168	43.7($\pm$6.4)	0.86	0.131	1389	10.5 ± 0.11	0.444 ± 0.014	-0.636 ± 0.004	0.475 ± 0.007	0.556 ± 0.004
7.6	42.1 ± 2.0	1.379 ± 0.111	22.3($\pm$3.4)	0.88	0.139	1212	8.78 ± 0.13	0.465 ± 0.019	-0.707 ± 0.004	0.551 ± 0.007	0.629 ± 0.004
8.8	41.3 ± 1.1	1.065 ± 0.086	14.8($\pm$2.6)	0.90	0.144	1166	7.32 ± 0.20	0.500 ± 0.020	-0.749 ± 0.007	0.606 ± 0.045	0.677 ± 0.006
12.3	30.1 ± 1.0	0.543 ± 0.044	4.49($\pm$0.94)	0.91	0.153	1231	7.70 ± 0.11	0.404 ± 0.034	-0.693 ± 0.004	0.518 ± 0.012	0.605 ± 0.006
17.3	23.9 ± 1.0	0.279 ± 0.023	1.58($\pm$0.31)	0.93	0.159	1389	6.66 ± 0.01	0.415 ± 0.032	-0.681 ± 0.0004	0.507 ± 0.011	0.594 ± 0.006

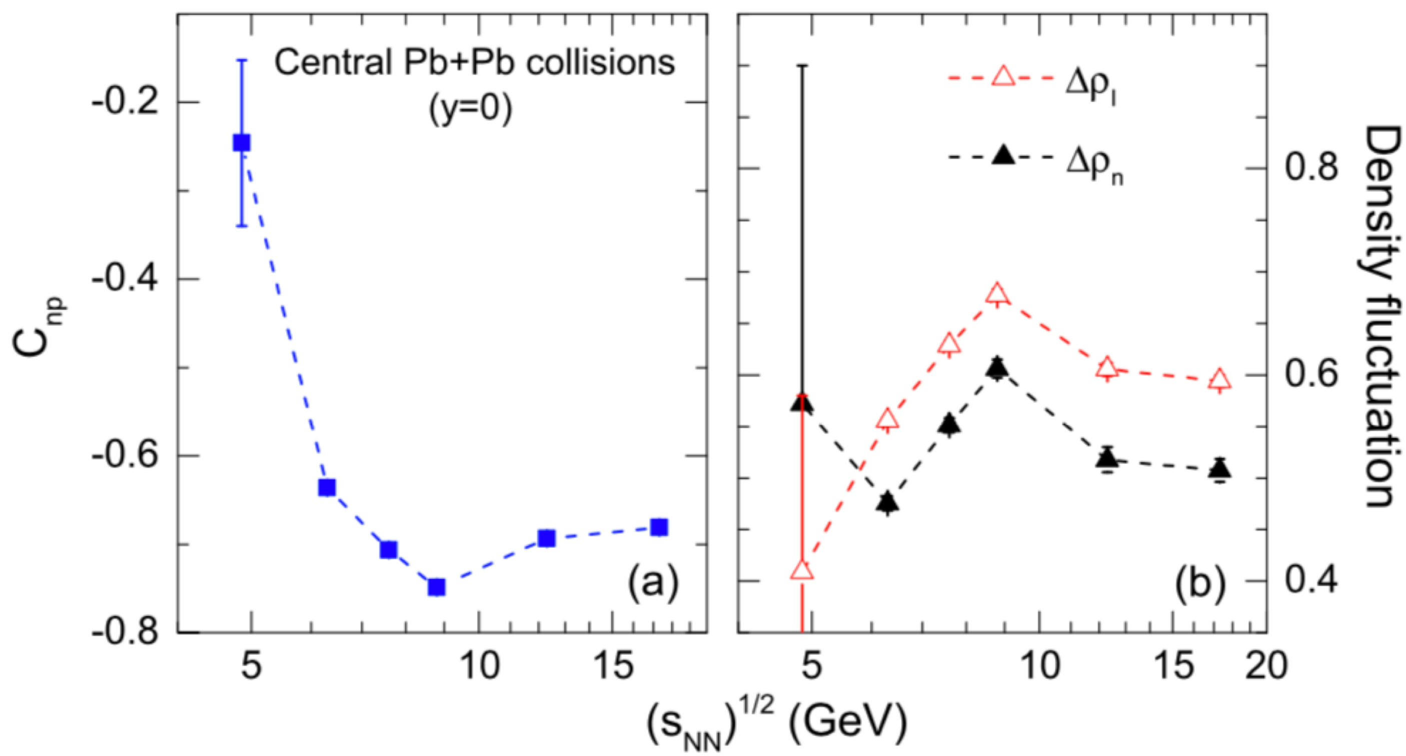


Fig. 2. Collision energy dependence of the neutron and proton density correlation $C_{\rm np}$ (a) and the neutron and isospin density fluctuations $\Delta\rho_n$ and $\Delta\rho_I$ (b) in central Pb+Pb collisions at SPS energies and Au+Au collisions at AGS energies.

Summary

- Both the first-order QGP to hadronic matter phase transition and the presence of the critical point are expected to give rise to large density fluctuations in the matter produced in relativistic heavy ion collisions.
- Without density fluctuations, the coalescence model and the thermal model give similar yields of light nuclei.
- Density fluctuations in produced matter affect the yield of light nuclei in coalescence model.
- The yield ratio $\mathrm{N_{^3H} N_p / N_d^2}$ is sensitive to the neutron density fluctuation.
- Studying the energy dependence of the yield ratio $\mathrm{N_{^3H} N_p / N_d^2}$ provides the possibility to extract information on the critical point and the region of first-order phase transition in QCD phase diagram.