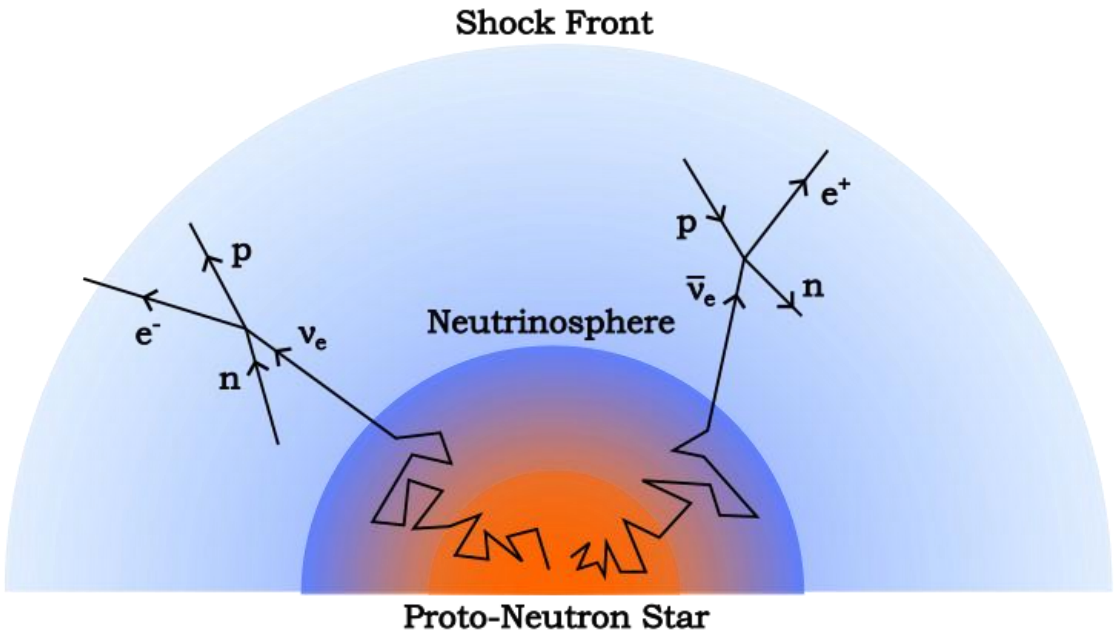
Microscopic modeling of hot, dense nuclear matter in core-collapse supernovae



David Friedenberg, Jeremy W. Holt
Cyclotron Institute, Texas A&M University
INT Workshop 26-96W, June 16th, 2026

Outline

- **Motivation**
  - Core-Collapse Supernova
  - Proto-Neutron Stars and the nucleon effective mass
  - Neutrino opacity in the neutrinosphere
- **Methodology**
  - Chiral Effective Field Theory
  - Nucleon Self-Energy in MBPT
- **Results**
- **Outlook**
  - Urca Cooling and Nucleon Width Approximation

Core-Collapse Supernova

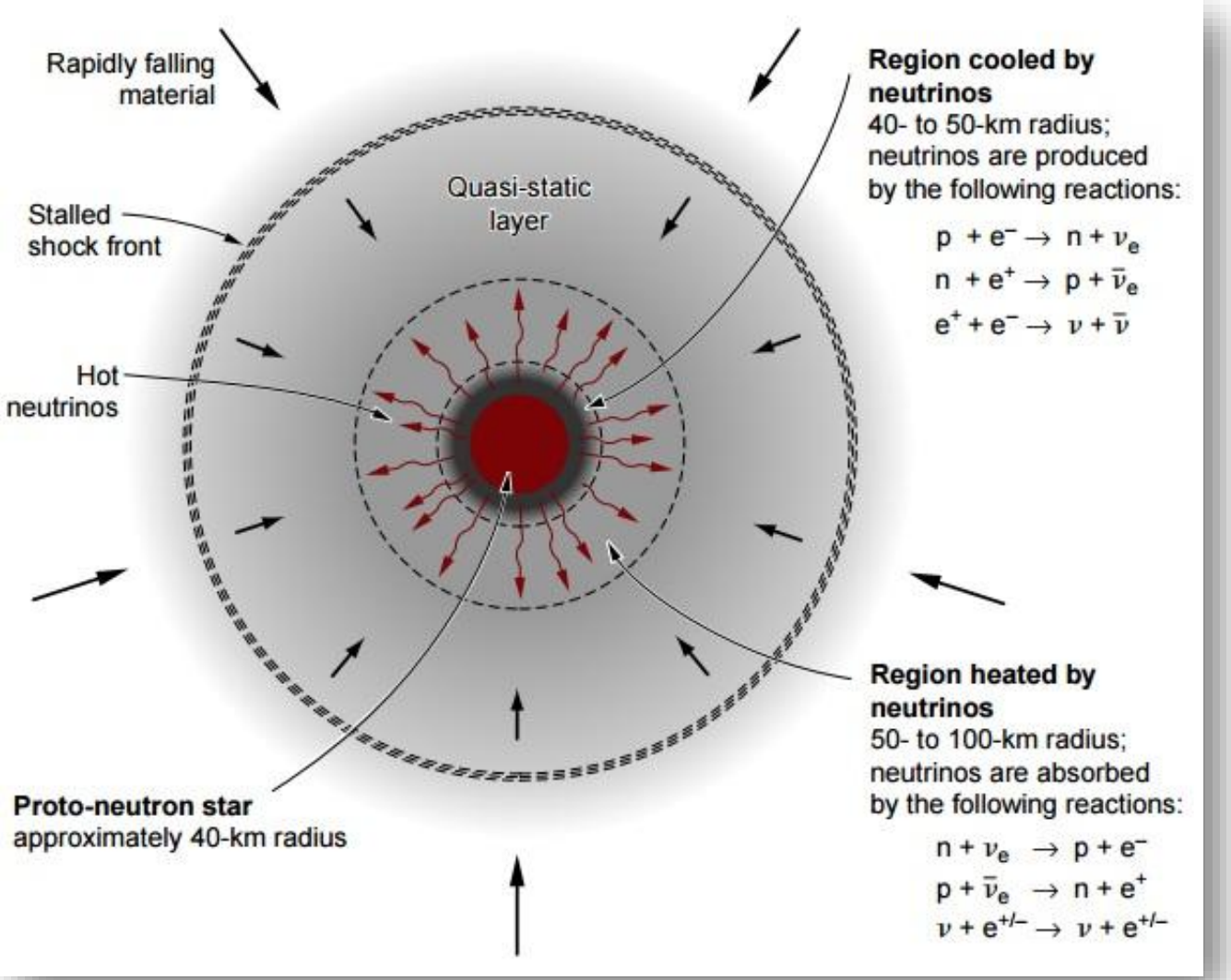


Image Credit: Herant et al, Los Alamos Sci. (1997)

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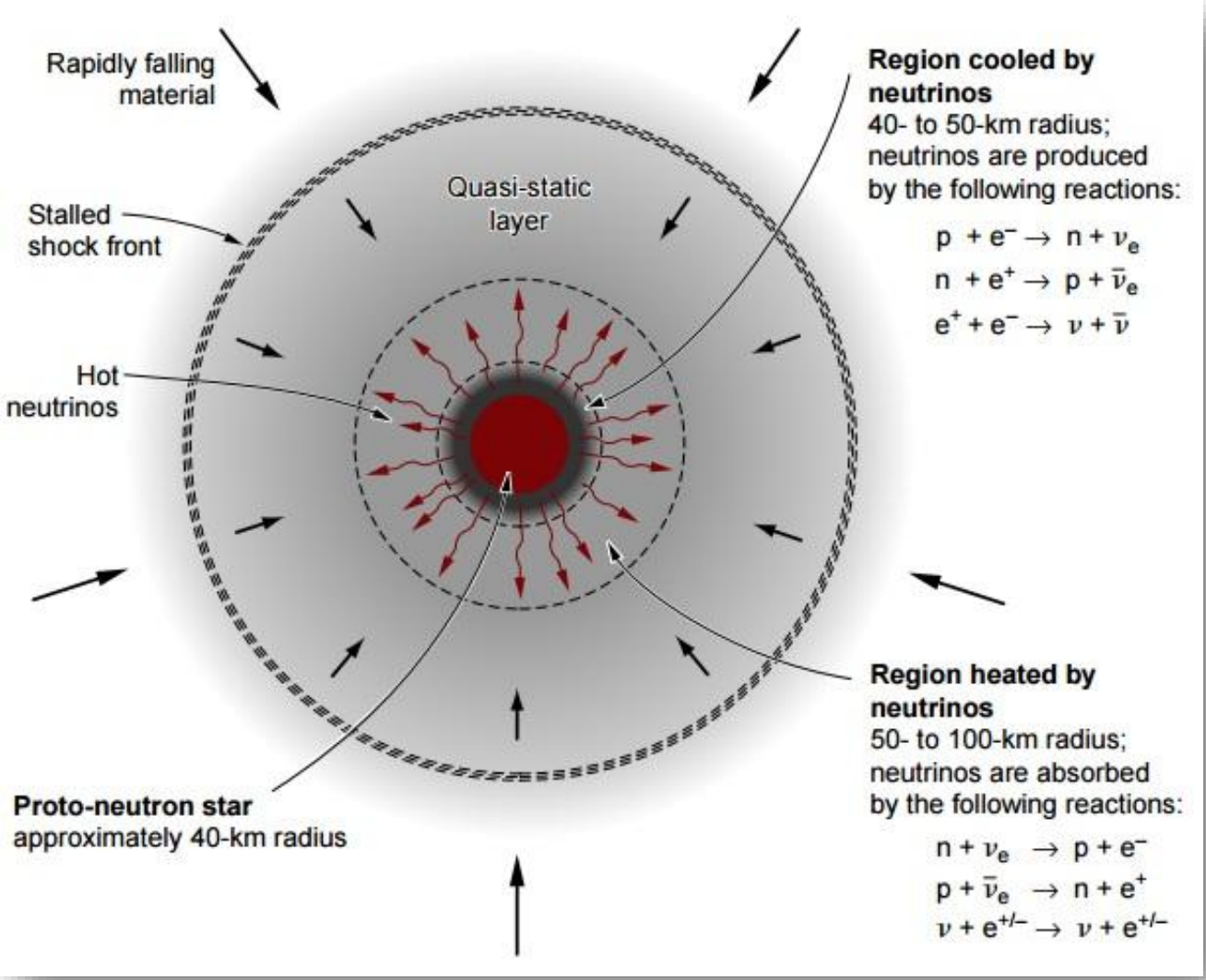

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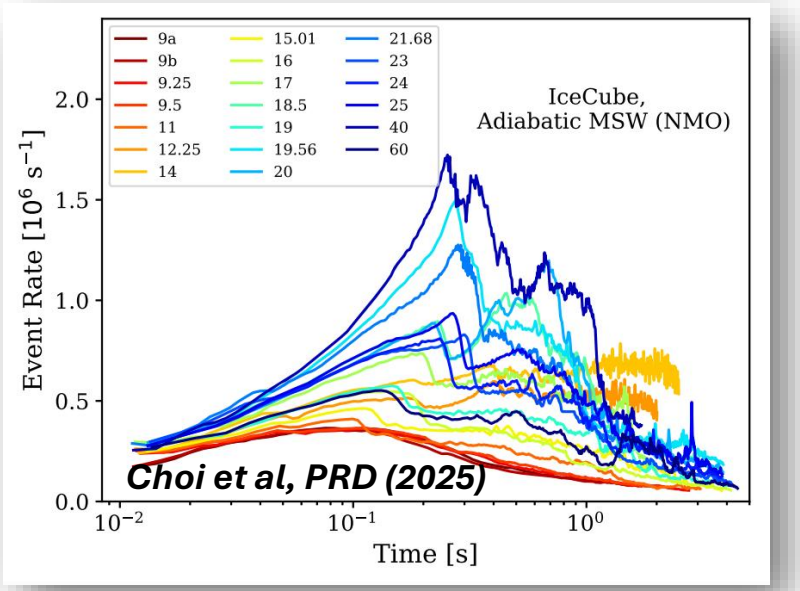
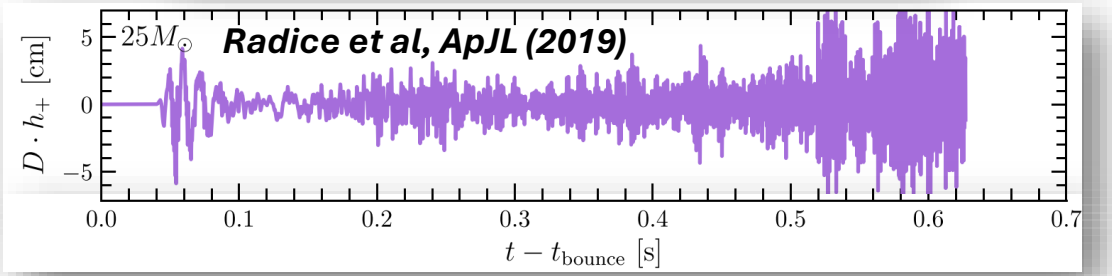
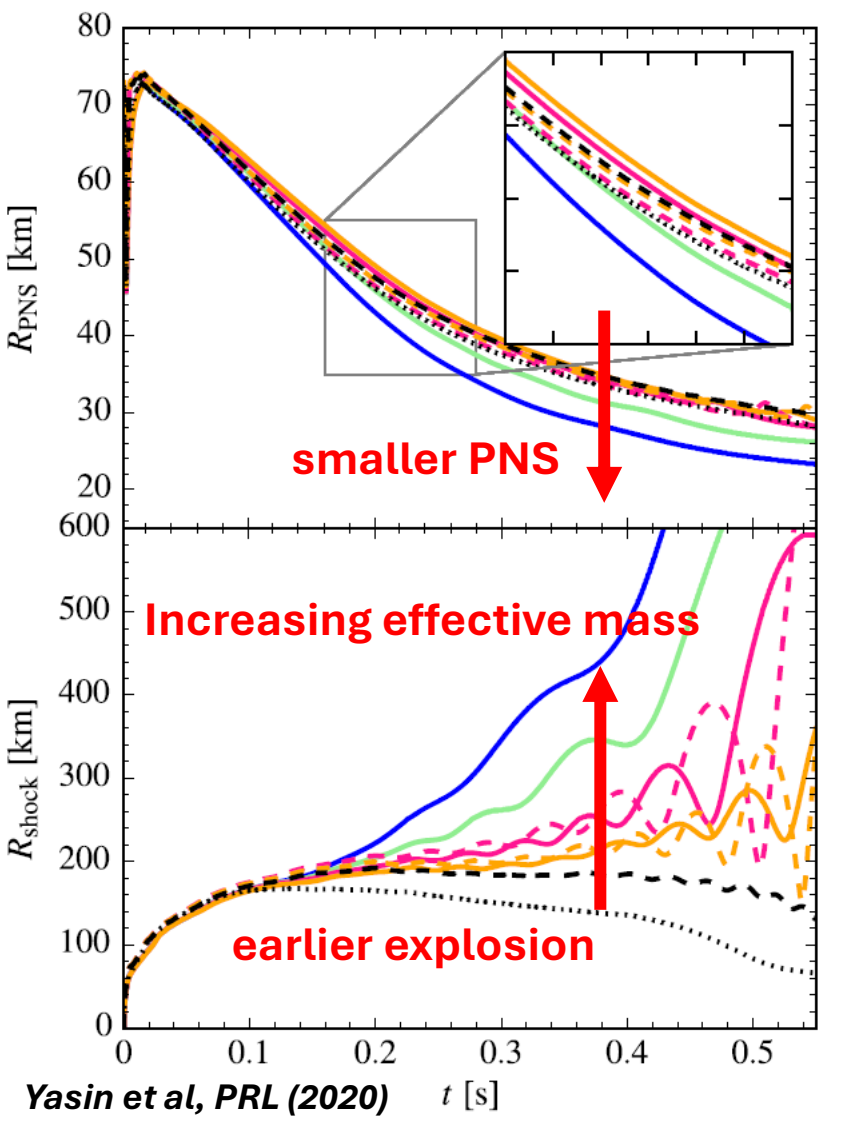
The Future of Multi-Messenger Astrophysics





Proto-Neutron Stars and the nucleon effective mass



Yasin et al, PRL (2020)

Explosion dynamics are sensitive to nucleon effective mass

Schneider et al, PRC (2019); Yasin et al, PRL (2020)

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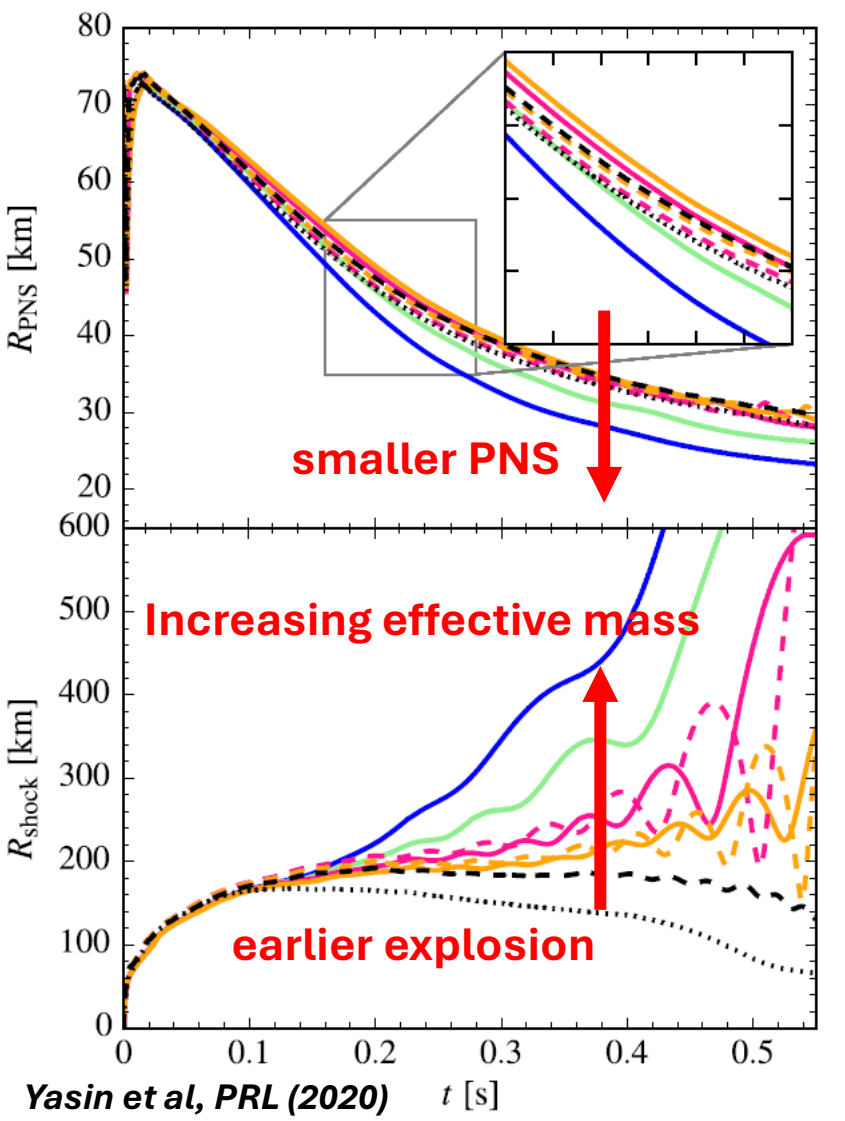

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(at fixed s)

- Faster PNS contraction
- Higher thermal ν energies
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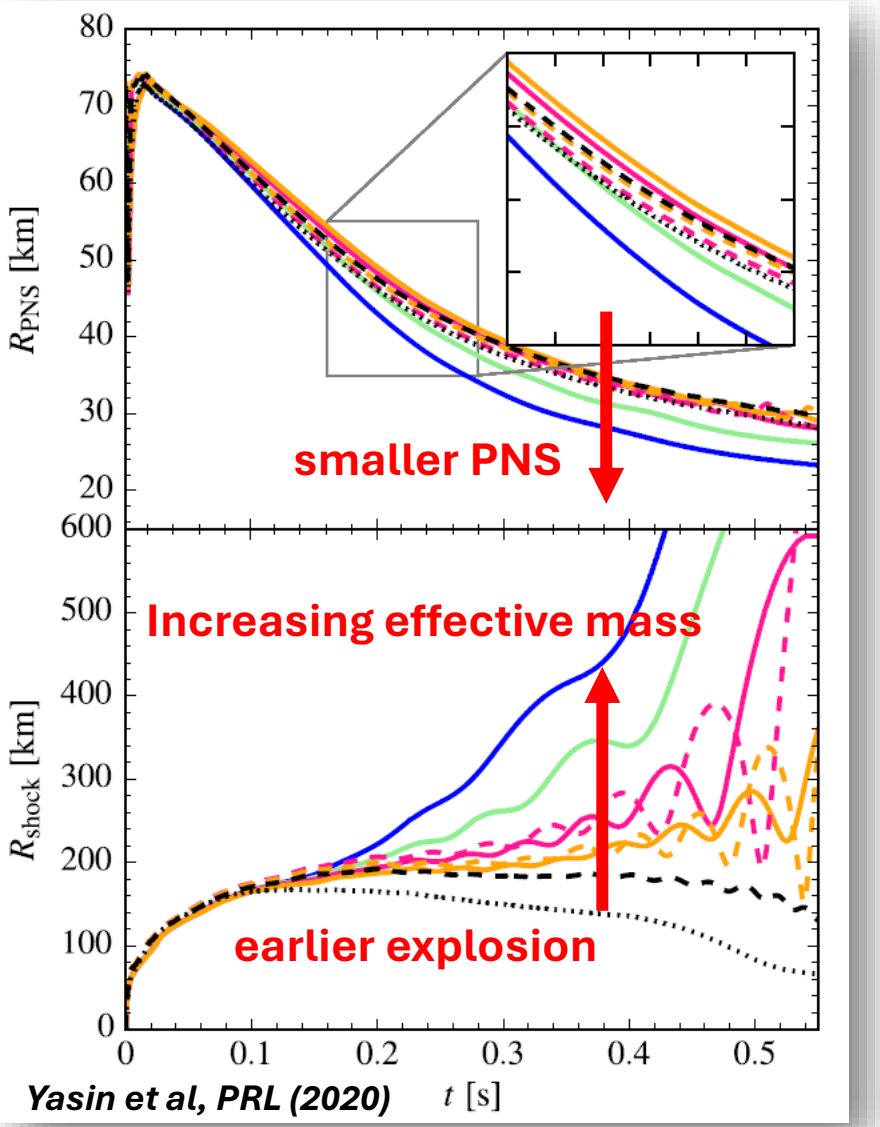

Explosion dynamics are sensitive to nucleon effective mass

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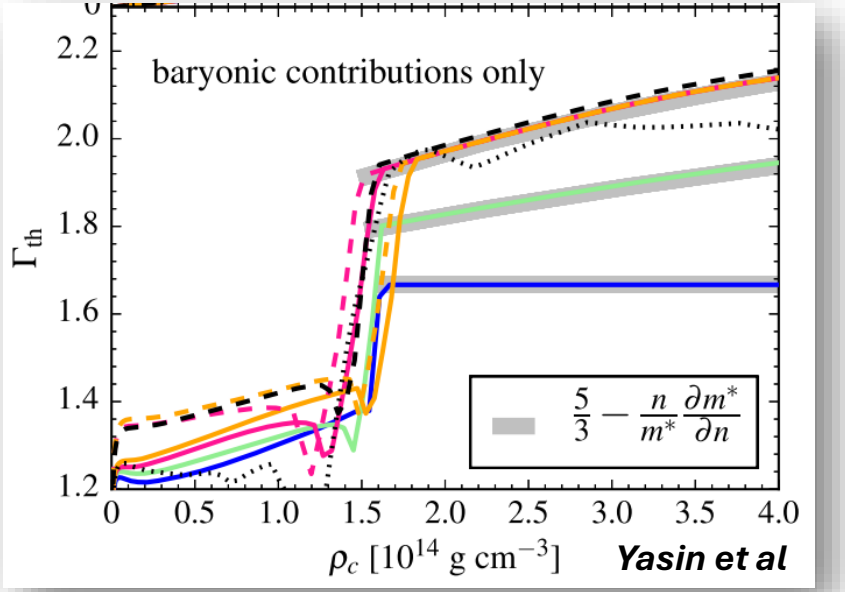
$$\Gamma_{\rm th} = 1 + \frac{P_{\rm th}}{\varepsilon_{\rm th}}$$

thermal index

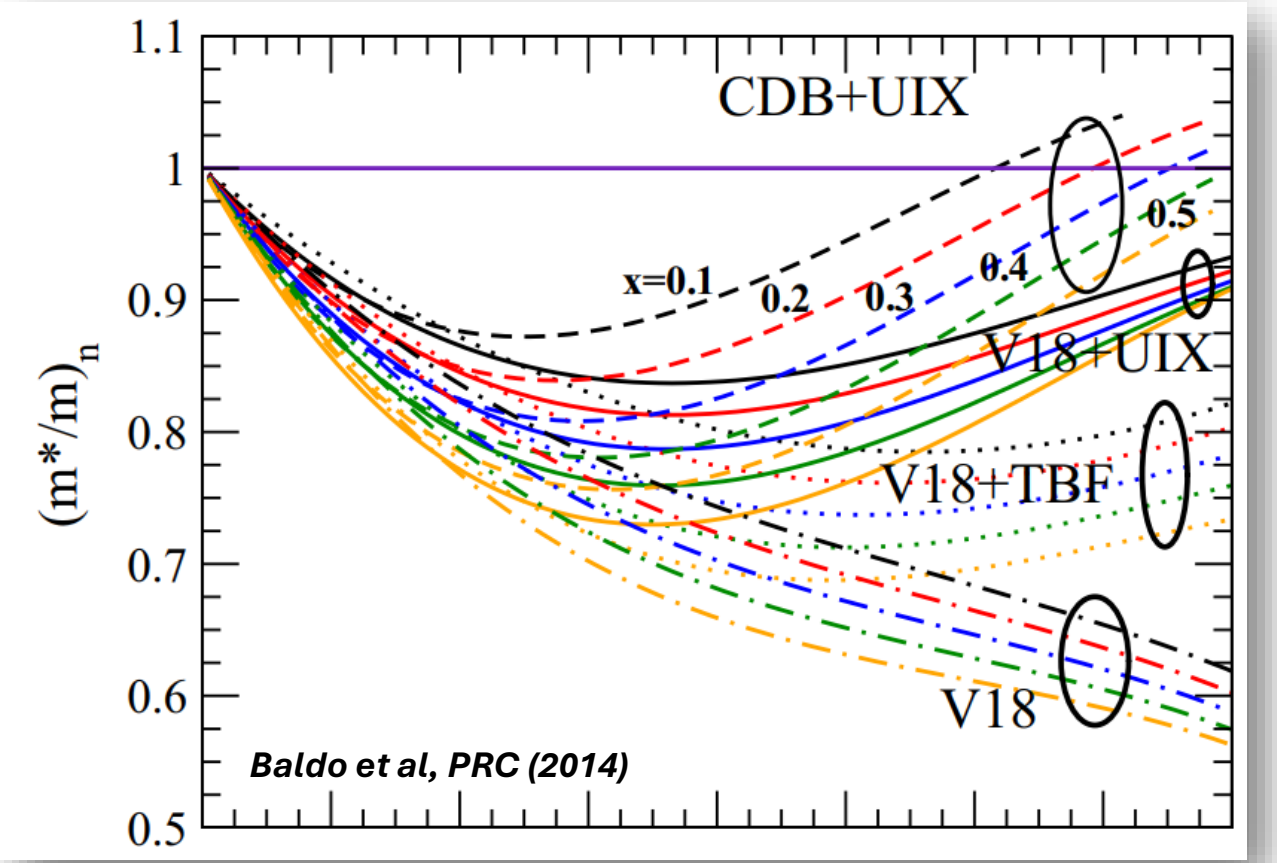


Mean-field level

Nucleon Effective Mass: Mean-Field

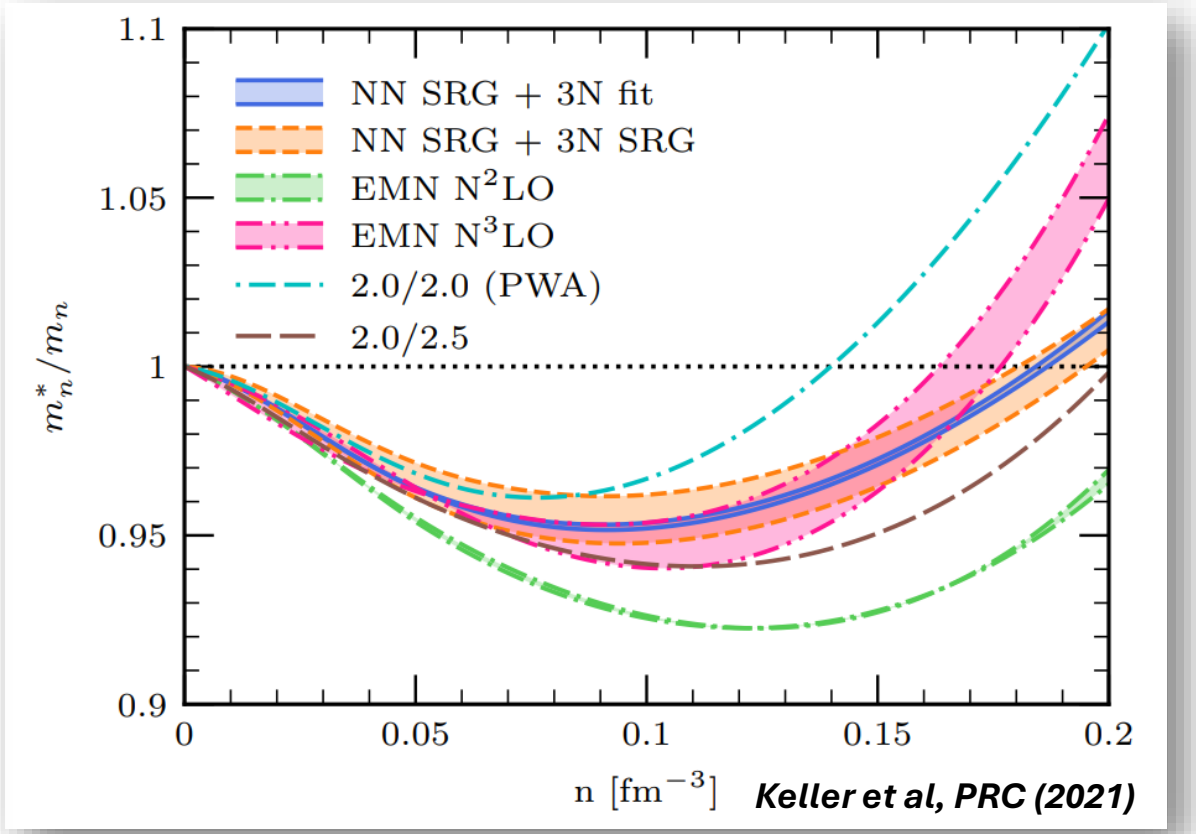
Brueckner-Hartree-Fock

direct: $$\frac{m^*(k)}{m} = \frac{k}{m}\left[\frac{de(k)}{dk}\right]^{-1}$$

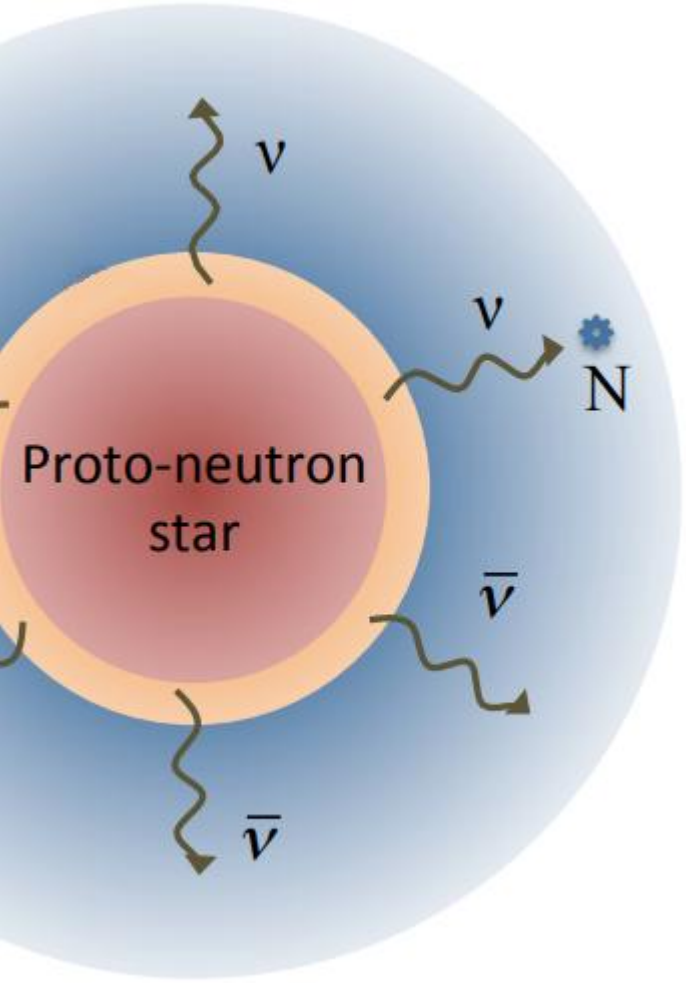


Chiral EFT + HF-MBPT

indirect: $$\Gamma^*_{\rm th}(n) = \frac{5}{3} - \frac{n}{m^*_n}\frac{\partial m^*_n}{\partial n}\,.$$



Supernova Neutrinos



Neutrinosphere:

$T = 4 - 8$ MeV

$\rho = 10^{11} - 10^{13}$ g/cm^3

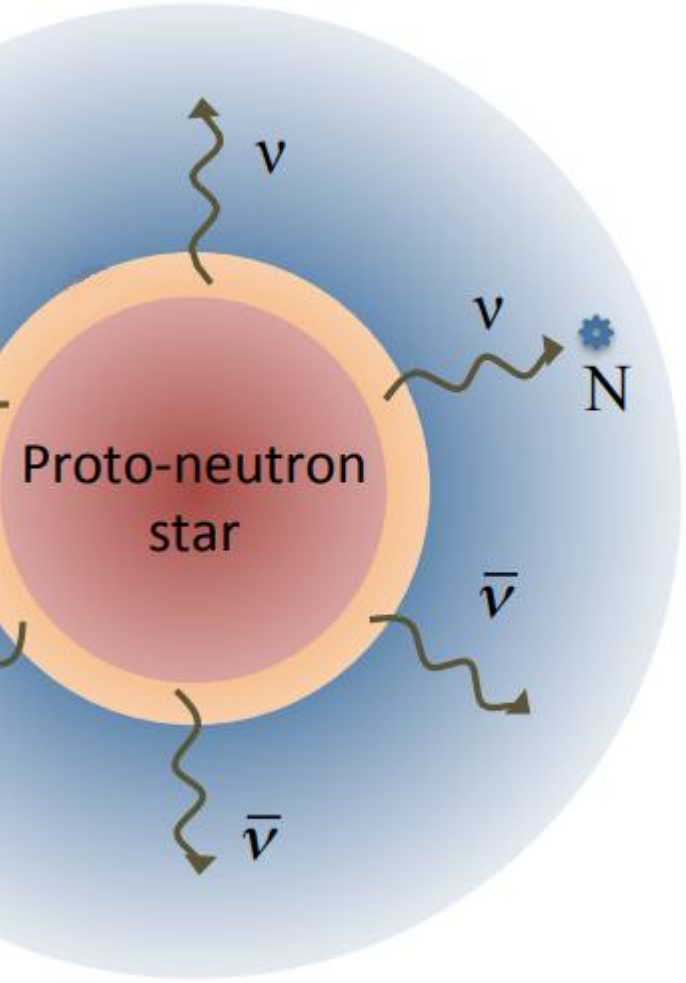
$Y_p = 0.05 - 0.10$

Neutrino Reactions in Supernovae

Beta processes:	• $e^- + p \rightleftharpoons n + \nu_e$ • $e^+ + n \rightleftharpoons p + \bar{\nu}_e$ • $e^- + A \rightleftharpoons \nu_e + A^*$
Neutrino scattering:	• $\nu + n, p \rightleftharpoons \nu + n, p$ • $\nu + A \rightleftharpoons \nu + A$ • $\nu + e^\pm \rightleftharpoons \nu + e^\pm$
Thermal pair processes:	• $N + N \rightleftharpoons N + N + \nu + \bar{\nu}$ • $e^+ + e^- \rightleftharpoons \nu + \bar{\nu}$
Neutrino-neutrino reactions:	• $\nu_x + \nu_e, \bar{\nu}_e \rightleftharpoons \nu_x + \nu_e, \bar{\nu}_e$ $(\nu_x = \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \text{ or } \bar{\nu}_\tau)$ • $\nu_e + \bar{\nu}_e \rightleftharpoons \nu_{\mu,\tau} + \bar{\nu}_{\mu,\tau}$

Martinez-Pinedo et al, PRL (2012)

Supernova Neutrinos



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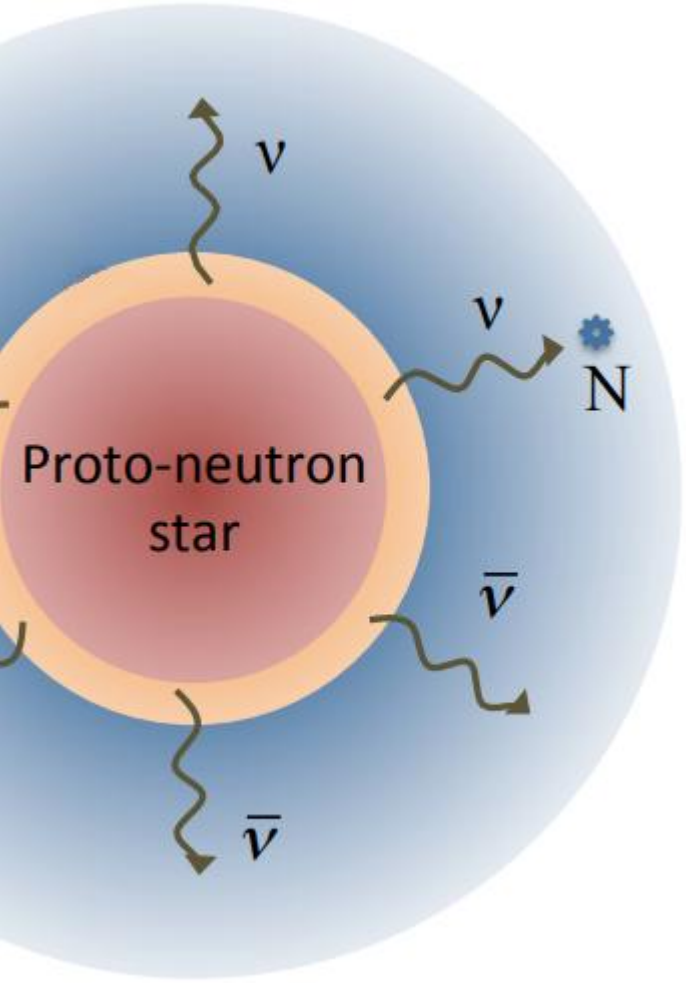

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Martinez-Pinedo et al, PRL (2012)

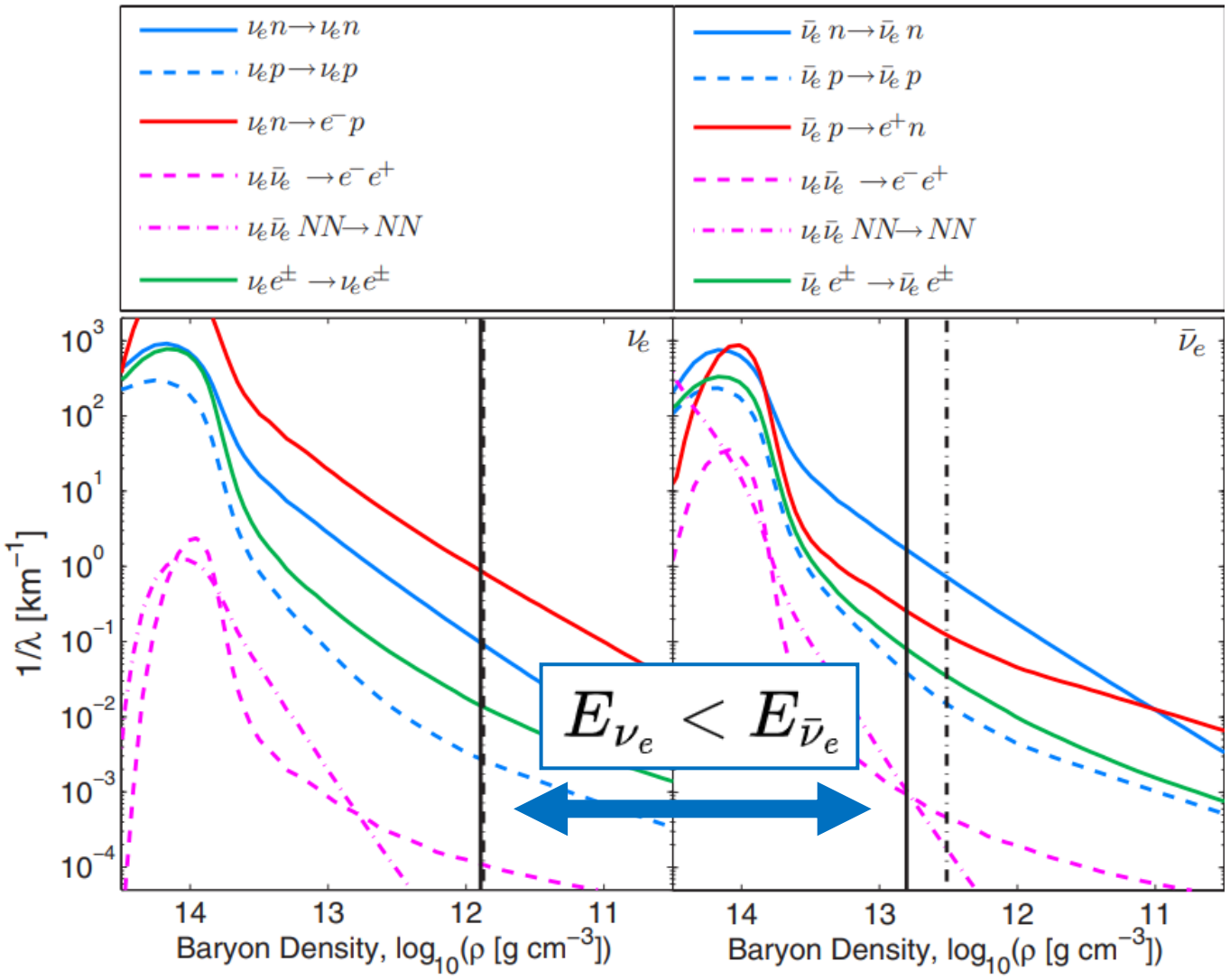
- Controls total luminosity and average energy - *explosion mechanism*

J. A. Pons et al, ApJ (1999)

- Governs spectral and temporal aspects of *detectable neutrino signal*

H.-T. Janka, et al, Ann. Rev. Nuc. (2016)

Neutrino opacity in the neutrinosphere



Martinez-Pinedo et al, J Phys G (2014)

$$\nu_e + n \longleftrightarrow e^- + p$$
$$\bar{\nu}_e + p \longleftrightarrow e^+ + n$$

charged-current reactions

Competing processes set *electron fraction* of ejected material

Neutrino-driven wind

Consequences for r-process nucleosynthesis

Martinez-Pinedo et al, PRL (2012); Roberts et al, PRC (2012)

Neutrino Absorption: Mean-Field and RPA

Electron phase space **Nucleon response**

$$\frac{1}{V}\frac{d^2\sigma}{d\cos\theta dE_e} = \frac{G_F^2\cos^2\theta_c}{4\pi^2}\; p_e E_e (1 - f_e(E_e))\; [(1+\cos\theta)S_{\tau\rho}(\omega,q) + g_A^2(3-\cos\theta)S_{\tau\sigma}(\omega,q)]$$

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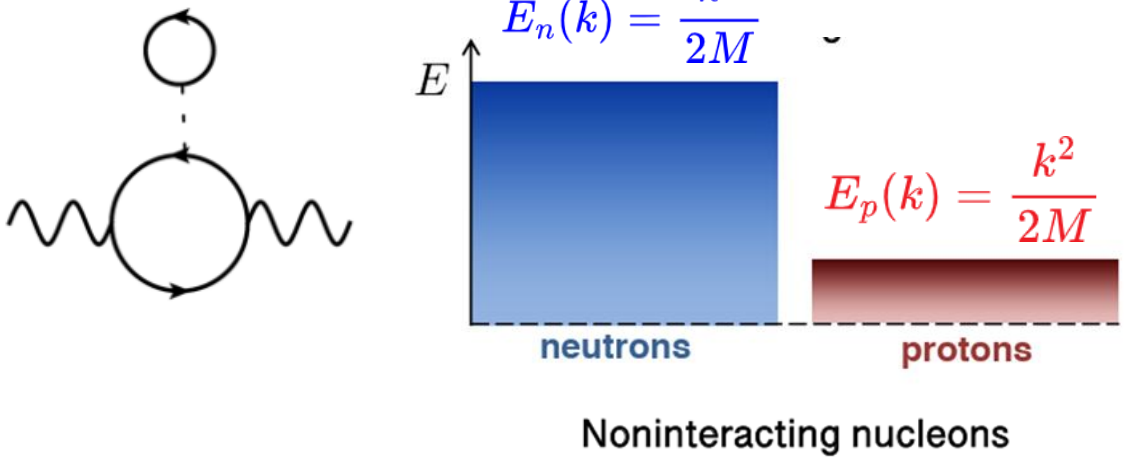
Nuclear Mean-Field Effects



Noninteracting nucleons

Neutrino Absorption: Mean-Field and RPA

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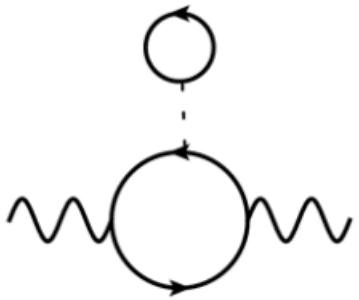

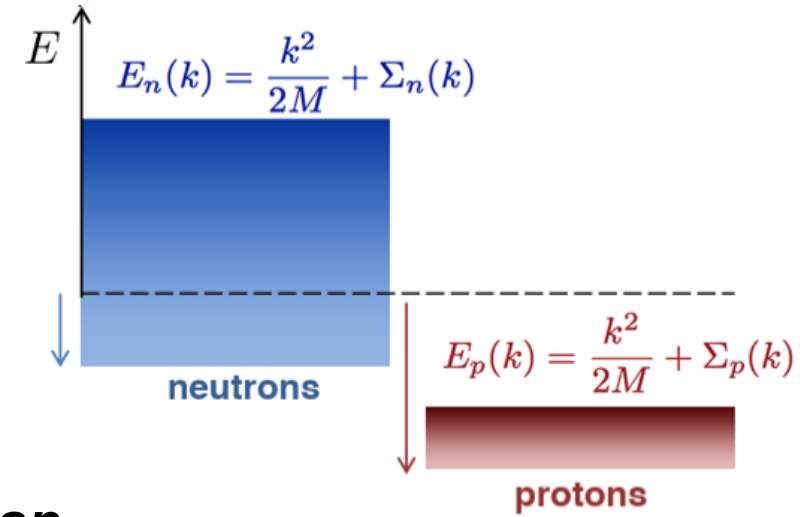

Nuclear mean fields

widen energy gap
between p, n

Neutrino Absorption: Mean-Field and RPA

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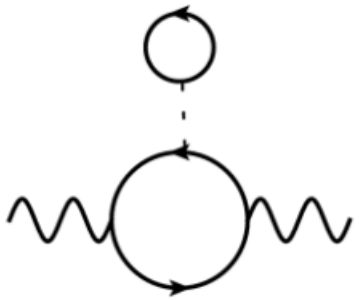

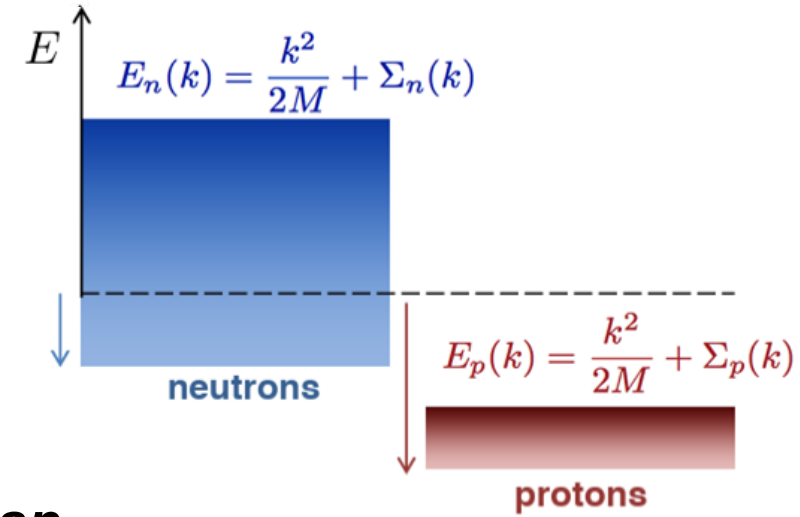

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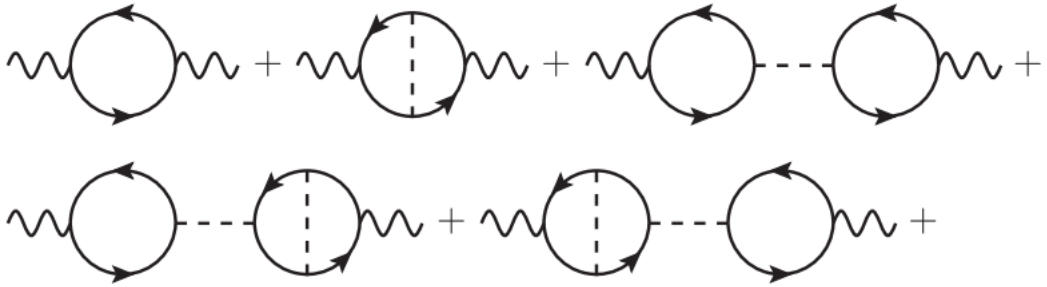
Random Phase Approximation (RPA)

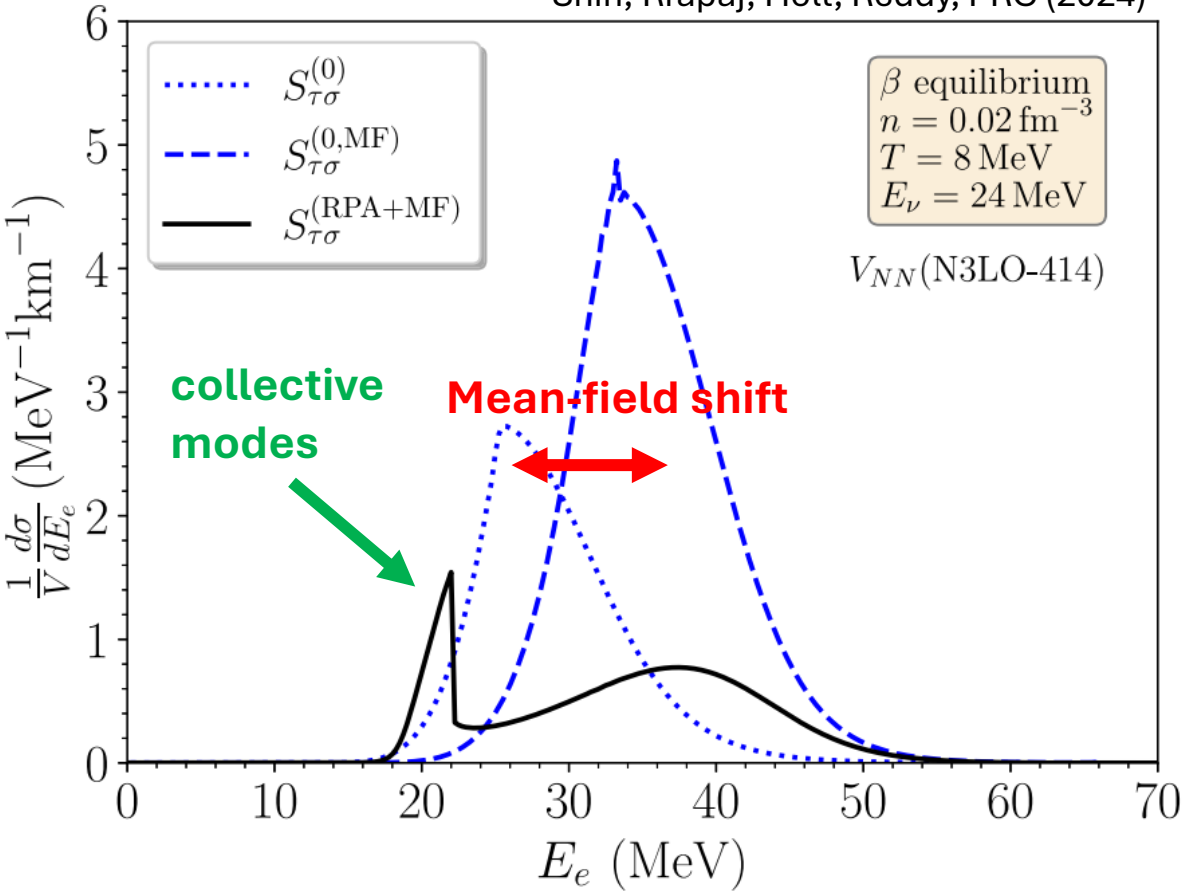


- Infinite summation over "bubble" diagrams
- Conserving approximations – *sum rules*
- Mean field approach – *collective excitations*

Neutrino Absorption: Mean-Field and RPA

Absorption cross section

Shin, Rrapaj, Holt, Reddy, PRC (2024)



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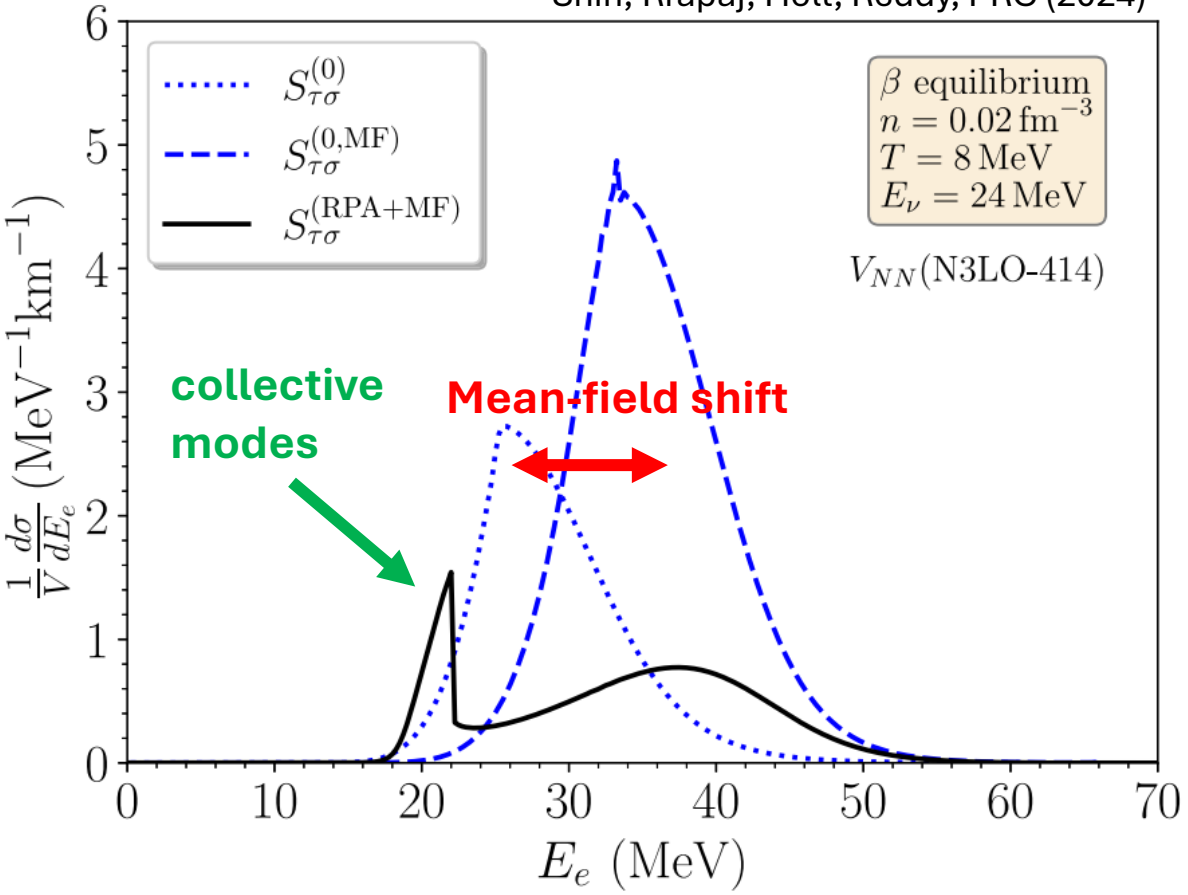
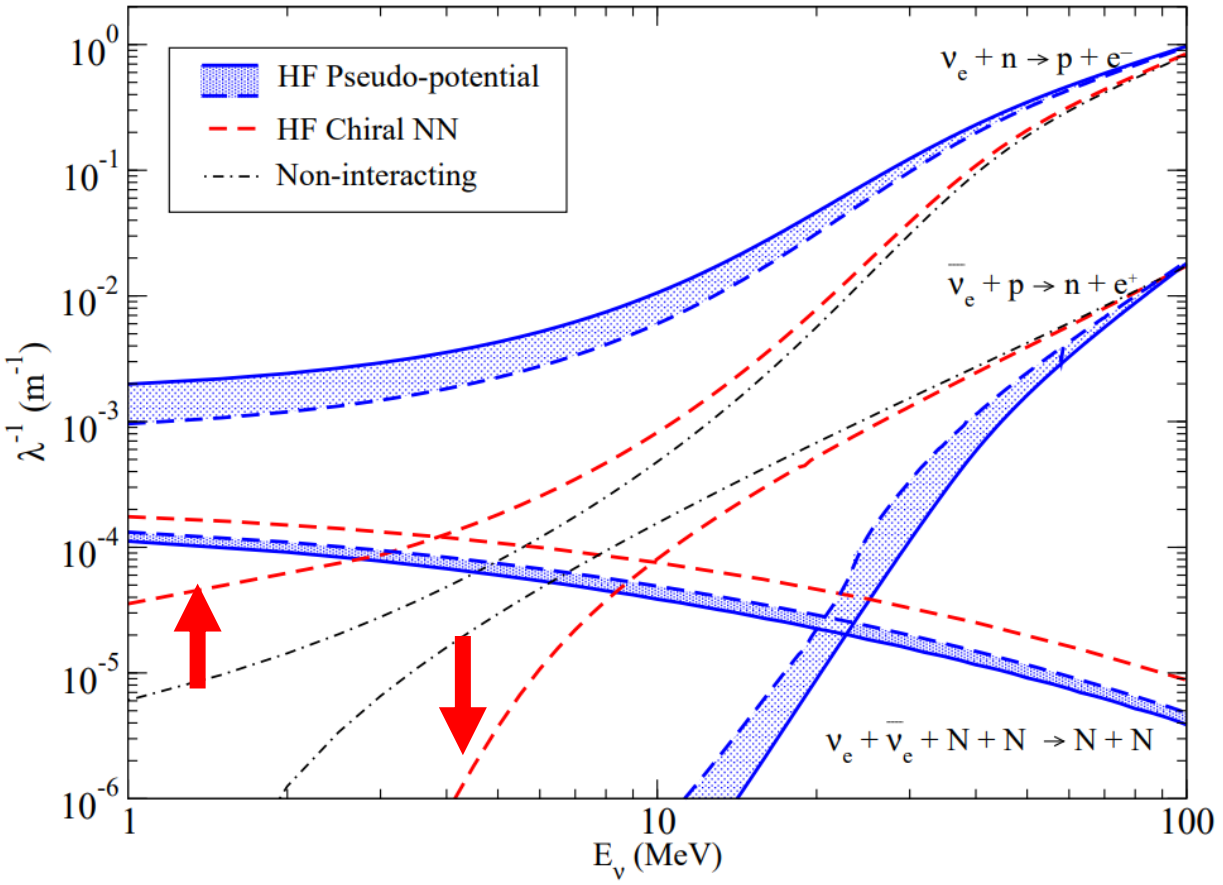


Neutrino opacity

Rrapaj, Holt, Bartl, Reddy, et al, PRC (2015)



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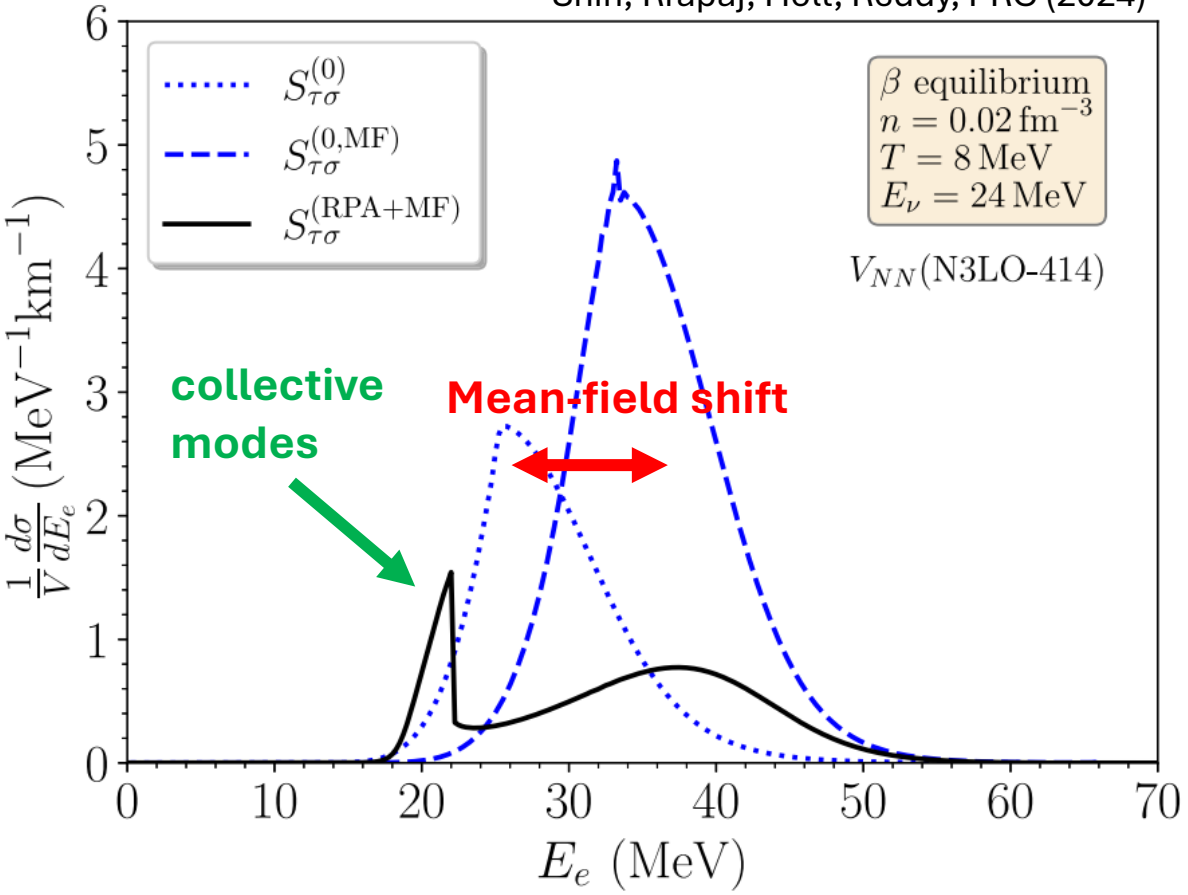

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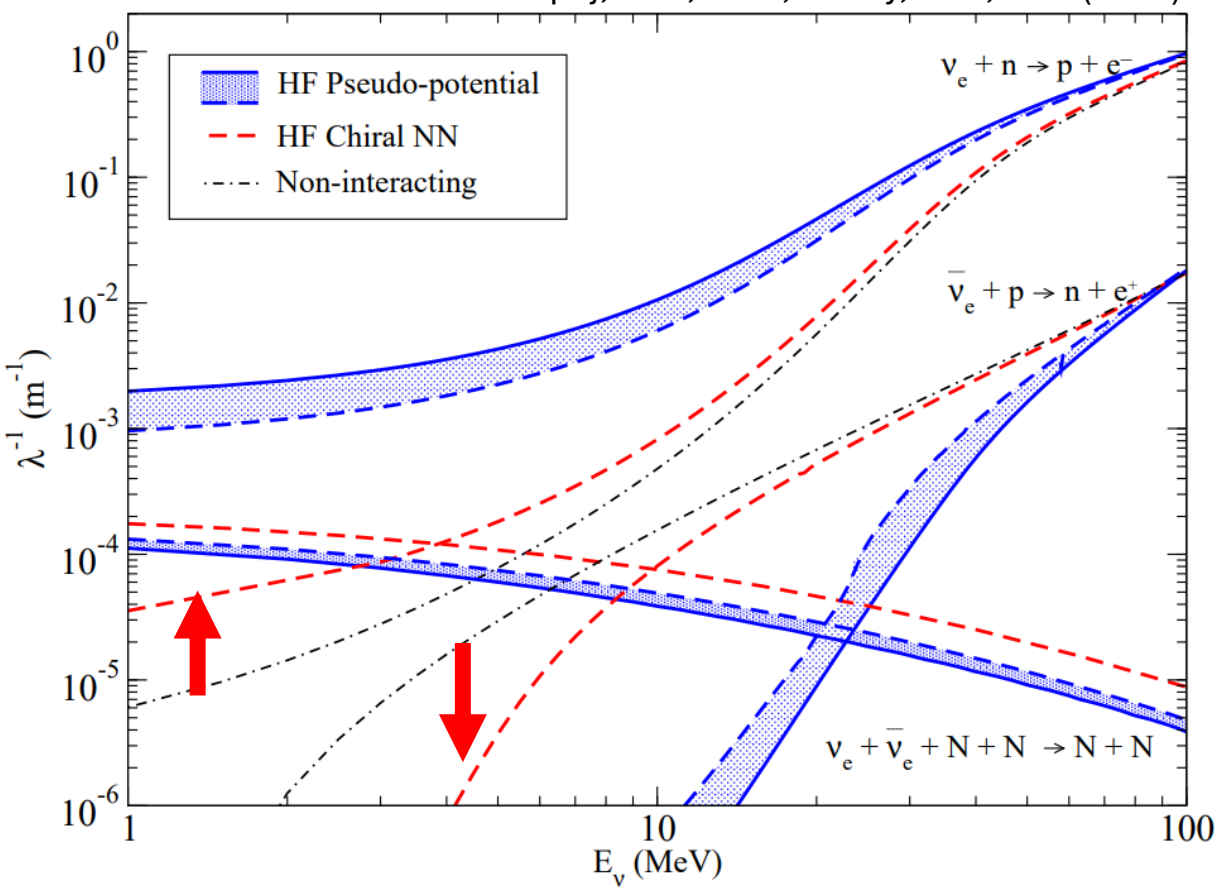


Takeaway: Nuclear mean-fields enhance neutrino absorption

Nuclear Forces from (Next-To) First Principles

Quark/gluon (high energy) dynamics

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \bar{q}_L i\gamma_\mu D^\mu q_L$$

$$+\bar{q}_R i\gamma_\mu D^\mu q_R - \bar{q}\mathcal{M}q$$

- Approximate chiral symmetry (left and right-handed quarks approximately decouple)

Nucleon/pion (low energy) dynamics

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(0)}_{NN} + \mathcal{L}^{(2)}_{NN} + \cdots$$

- Explicit and spontaneous **chiral symmetry breaking** → *pion mass*

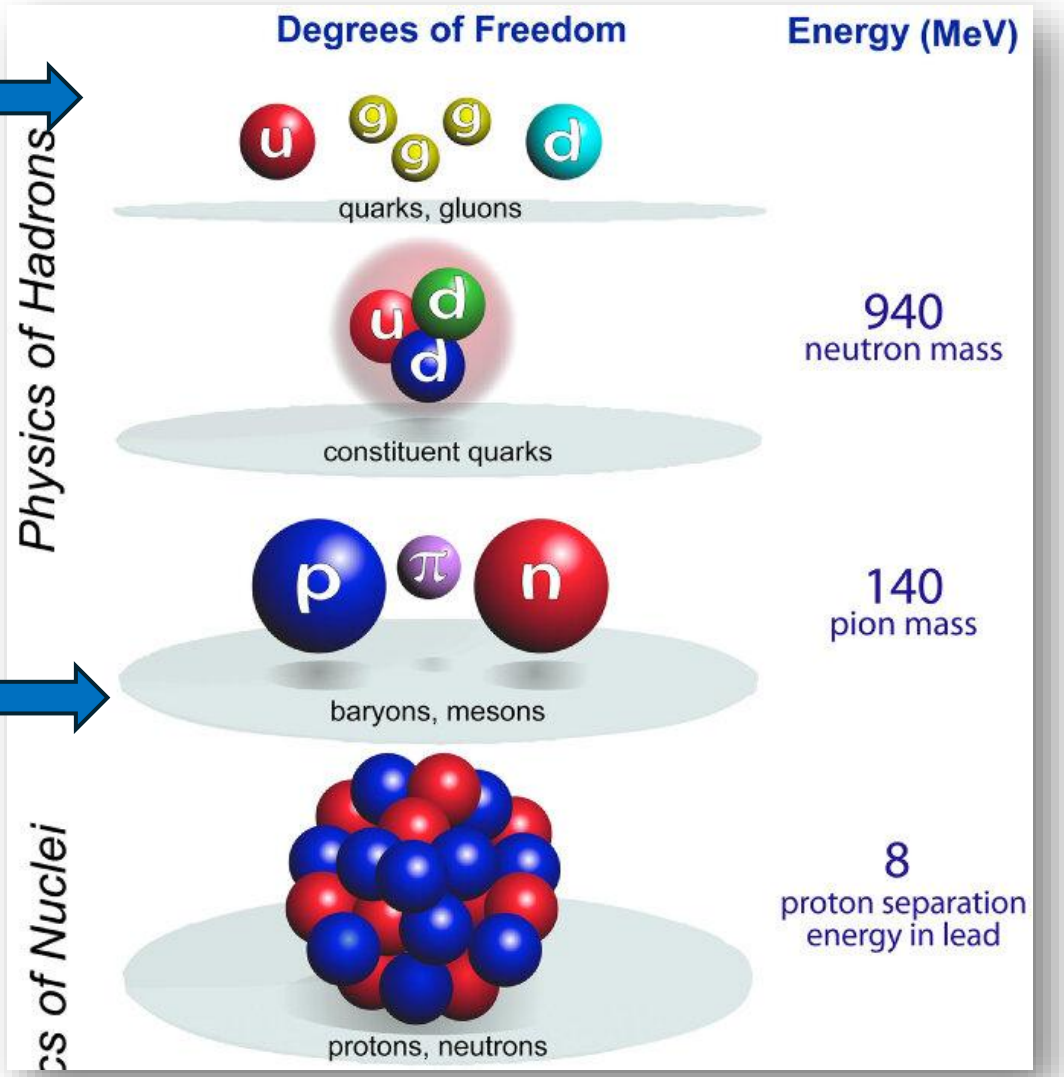
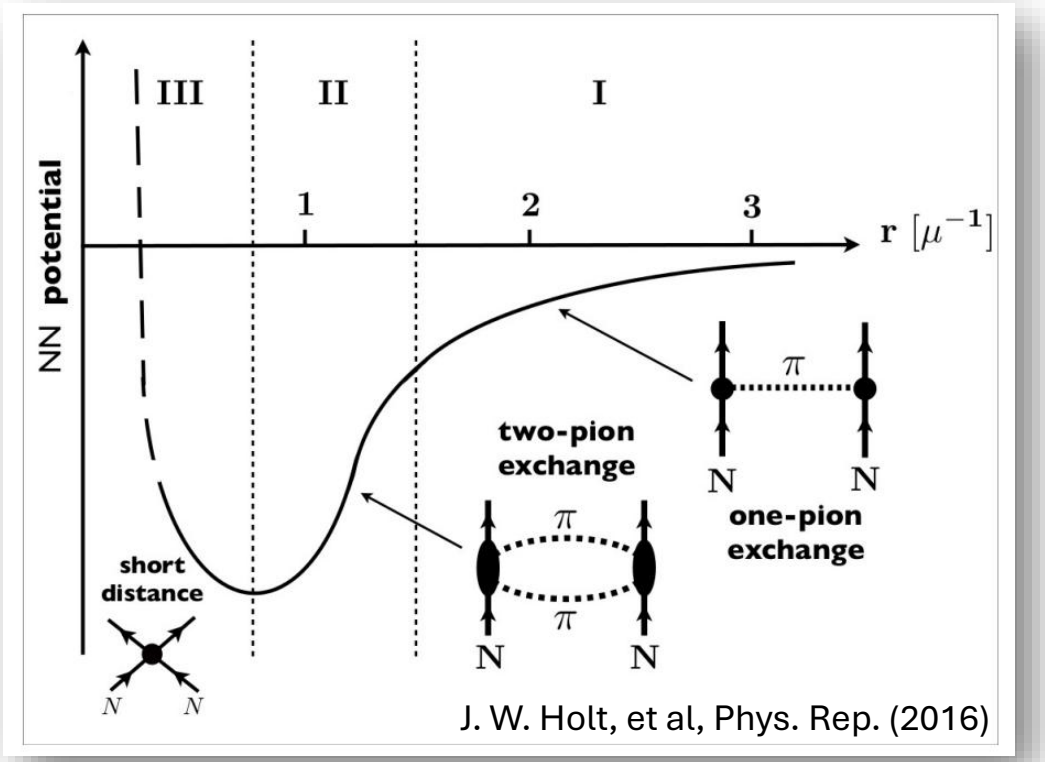


Image Credit: Oak Ridge Natl. Lab

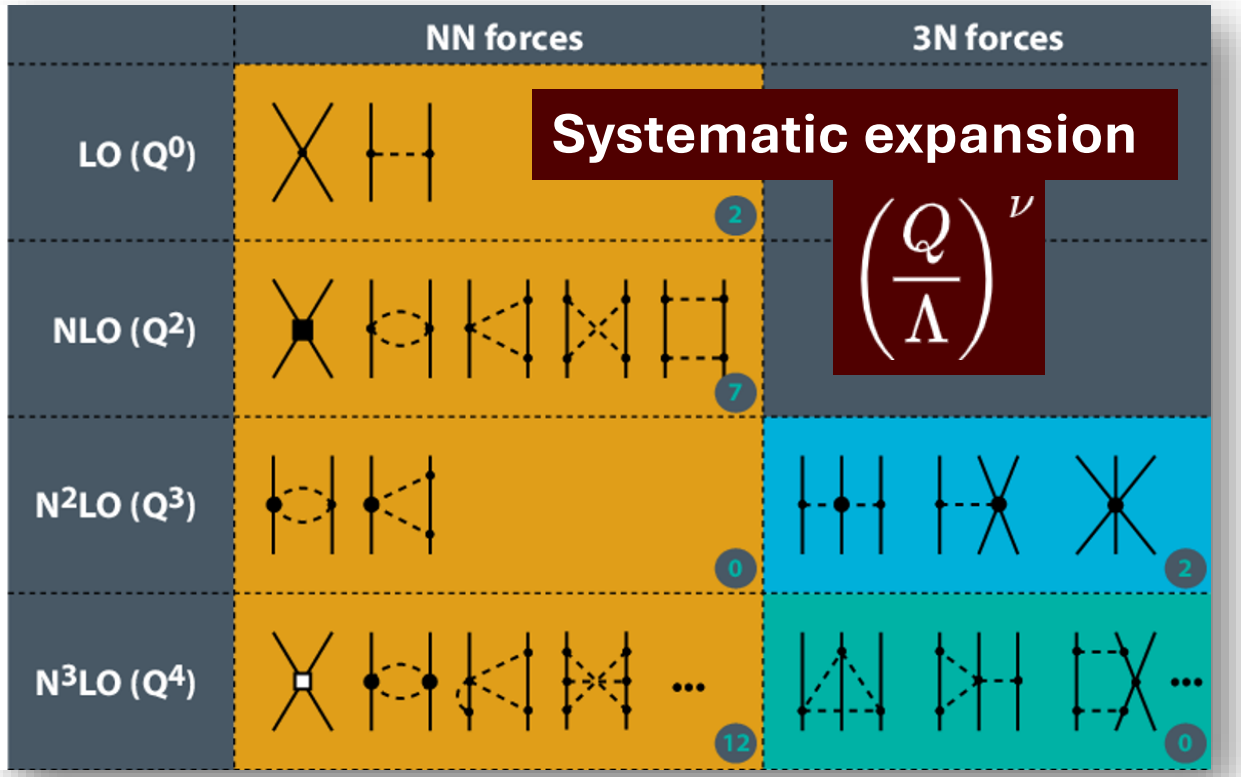
Chiral Effective Field Theory

Low energy theory of nucleons and pions

pion exchange (long range) and nucleon-nucleon contact (short range)



J. W. Holt, et al, Phys. Rep. (2016)



C. Drischler, J. W. Holt, et al, Annu. Rev. Nucl. Part. Sci. (2021)

- Fit low-energy constants (πN and NN vertices) to NN scattering data and bound states (H, He, etc.)

- Naturally incorporates 3-nucleon interactions (**nuclear saturation**)

Microscopic modeling of nuclear matter

- **Many-Body Perturbation Theory** (MBPT): Perturbation series in the *grand canonical potential*

$$n_\tau = 2 \int \frac{d^3\mathbf{k}}{(2\pi)^3}\, n_{k,\tau}\left(\mu_\tau, T\right)$$

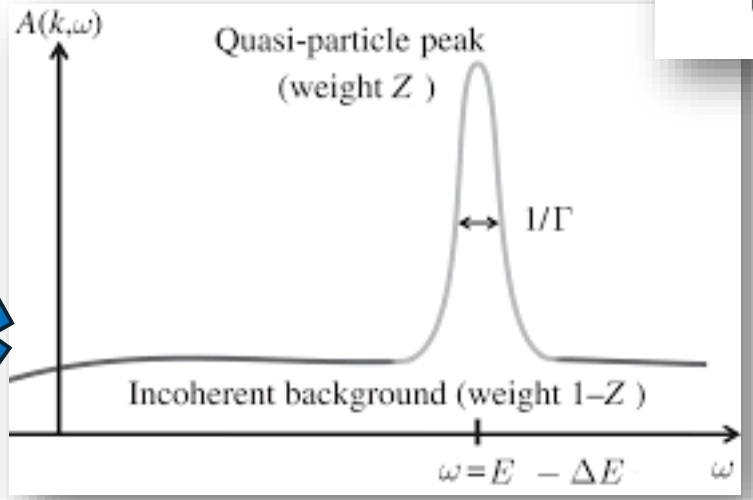
- **Dyson Equation**: nucleon Green's functions

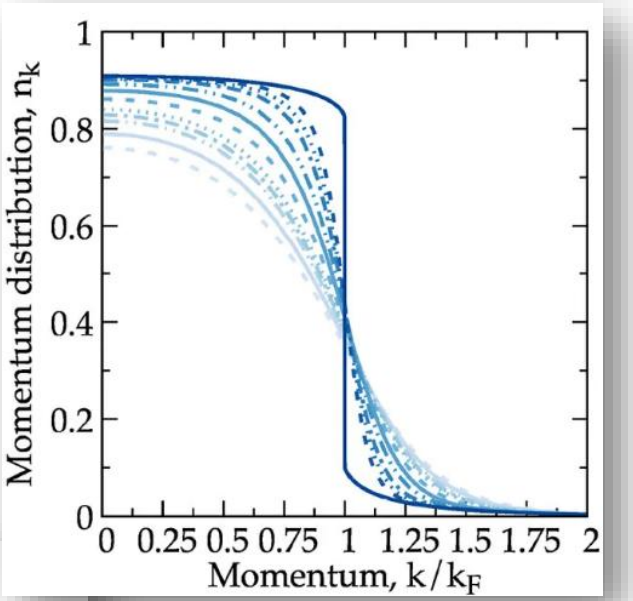
$$G(\mathbf{k},\omega) = G^0(\mathbf{k},\omega) + G^0(\mathbf{k},\omega)\Sigma(\mathbf{k},\omega)G(\mathbf{k},\omega)$$

Nucleon self-energy "dresses" the particles with medium interactions – ***quasiparticle approach***

$$\varepsilon_\tau(k) = \frac{k^2}{2M_N} + \mathrm{Re}\,\Sigma_\tau(k, \varepsilon_\tau(k))$$

$$W_\tau(k) = -2\,\mathrm{Im}\,\Sigma_\tau(k, \varepsilon_\tau(k))$$



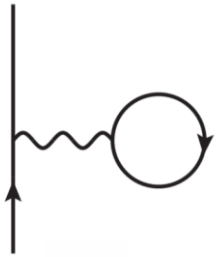


Rios, Front. Phys. (2020)

Nucleon Self-Energy in MBPT

- **Hartree-Fock contribution** (real, energy-independent)

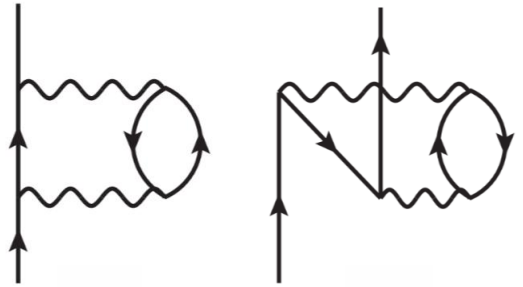
$$\Sigma_\tau^{(1)}(k;\mu_p,\mu_n,T) = \sum_1 \langle \mathbf{k}\,\mathbf{h}_1; s\,s_1; \tau\,\tau_1 | \overline{V}_{NN} | \mathbf{k}\,\mathbf{h}_1; s\,s_1; \tau\,\tau_1 \rangle\, n_1$$



- **Second Order perturbative contributions** (complex, energy-dependent)

$$\Sigma_\tau^{(2a)}(k,\omega;\mu_p,\mu_n,T) = \frac{1}{2}\sum_{123} \frac{\left| \langle \mathbf{p}_1\,\mathbf{p}_2; s_1\,s_2; \tau_1\,\tau_2 | \overline{V}_{NN} | \mathbf{k}\,\mathbf{h}_3; s\,s_3; \tau\,\tau_3 \rangle \right|^2}{\omega + \varepsilon_3 - \varepsilon_1 - \varepsilon_2 + i\eta}$$

$$\bar{n}_1 \bar{n}_2 n_3 \,(2\pi)^3\, \delta^3(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{h}_3 - \mathbf{k})$$

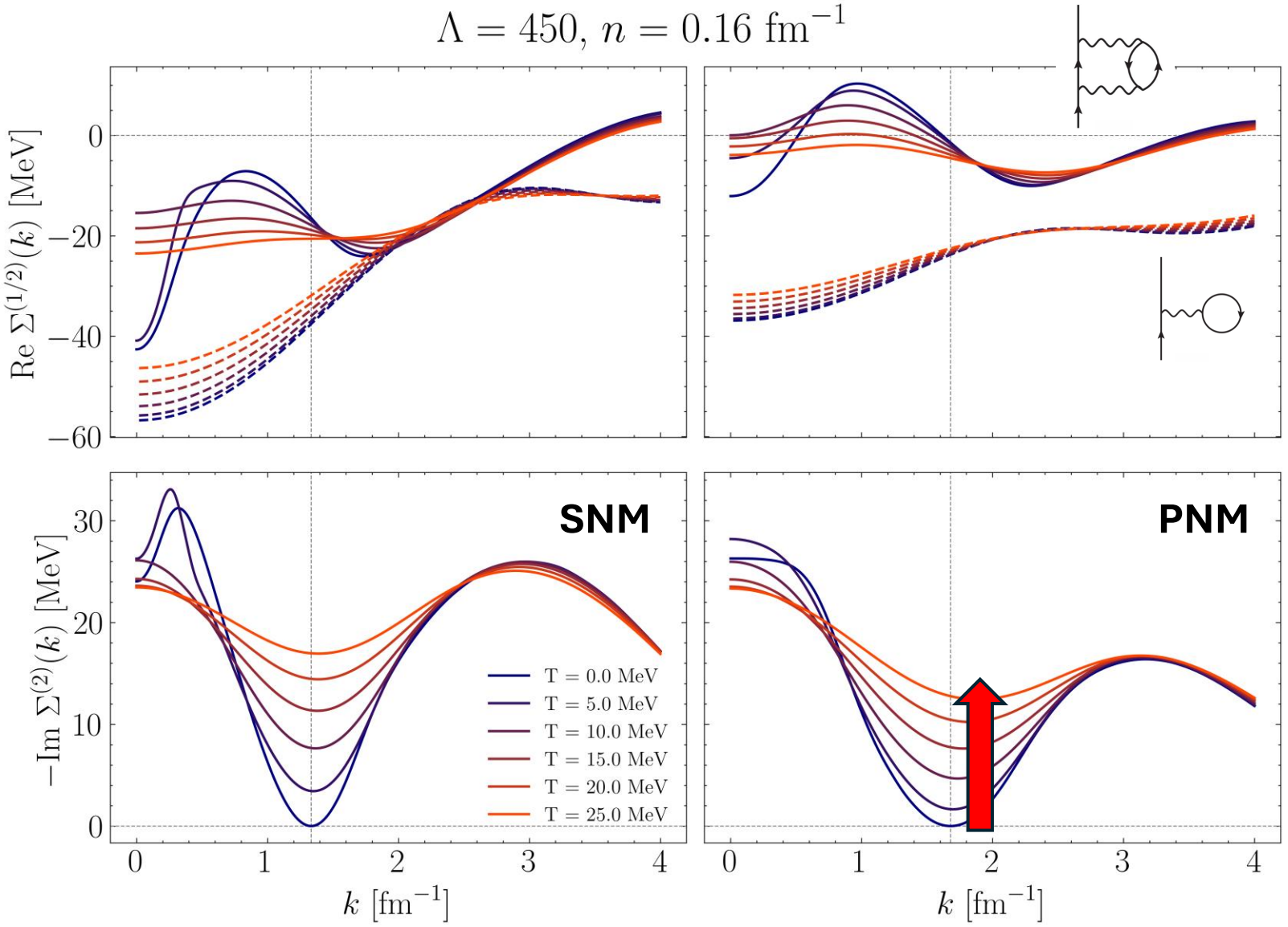


- **Self-Consistent Approach**

  (quasiparticle approximation)

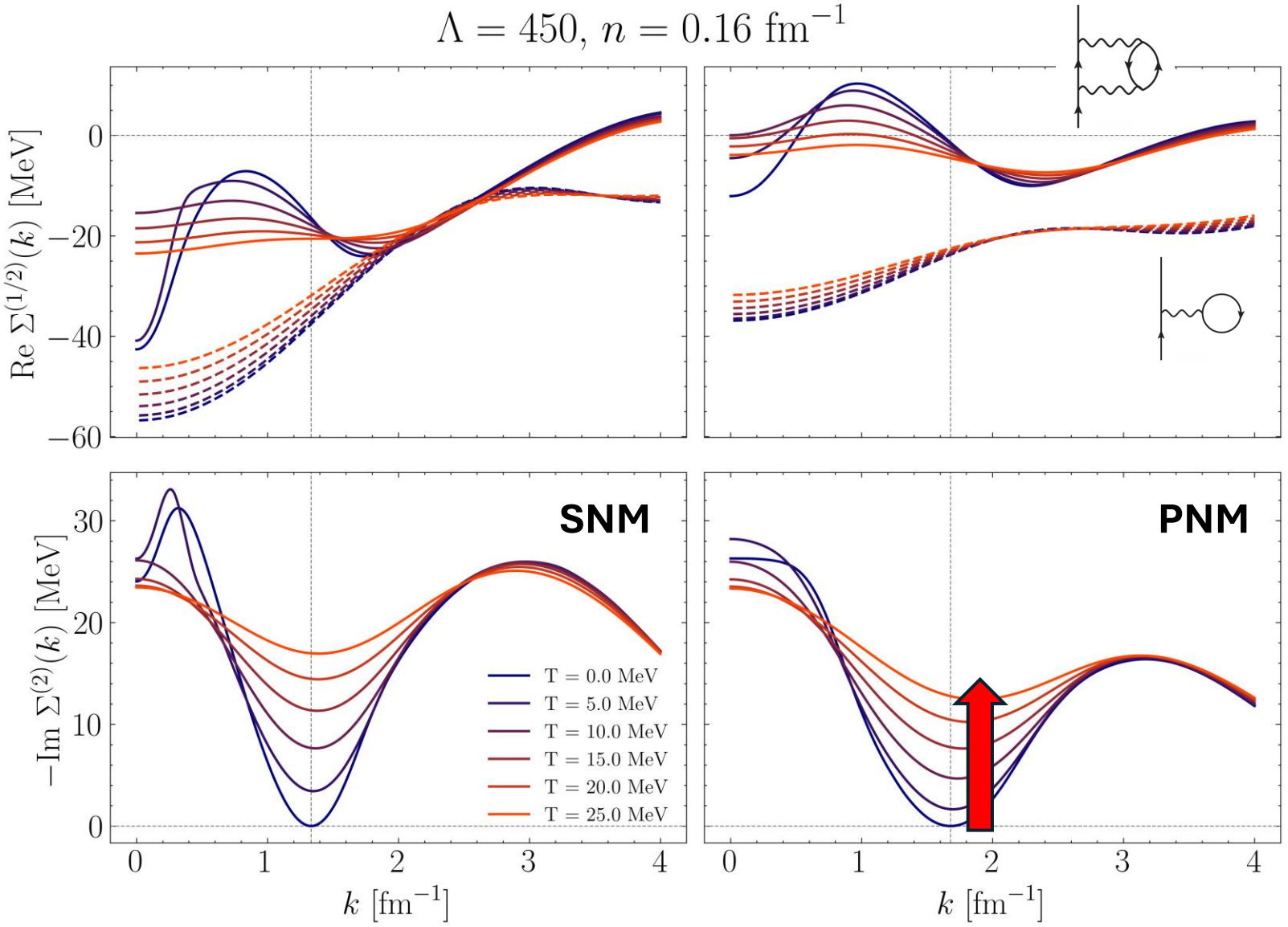
$$\varepsilon_\tau(k) = \frac{k^2}{2M_N} + \mathrm{Re}\,\Sigma_\tau(k, \varepsilon_\tau(k))$$

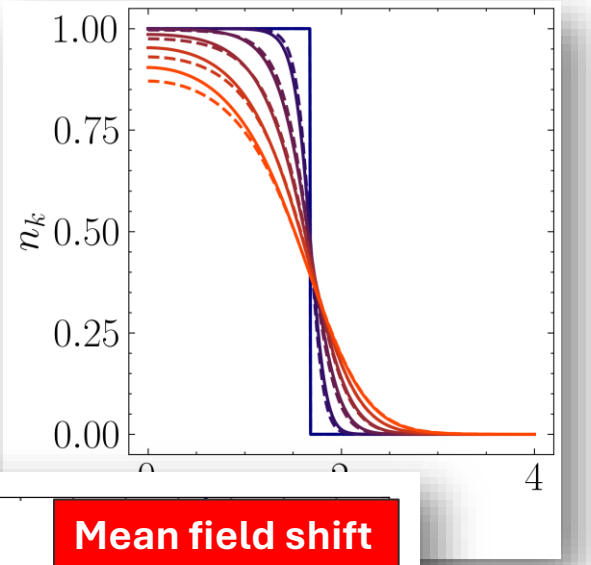
Nucleon Self-Energy: Results

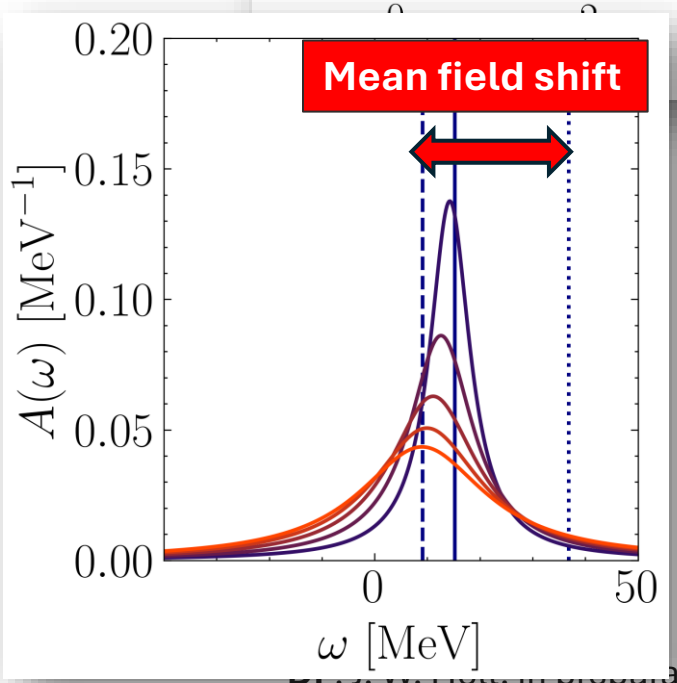


DF, J. W. Holt, in preparation (2026)

Nucleon Self-Energy: Results



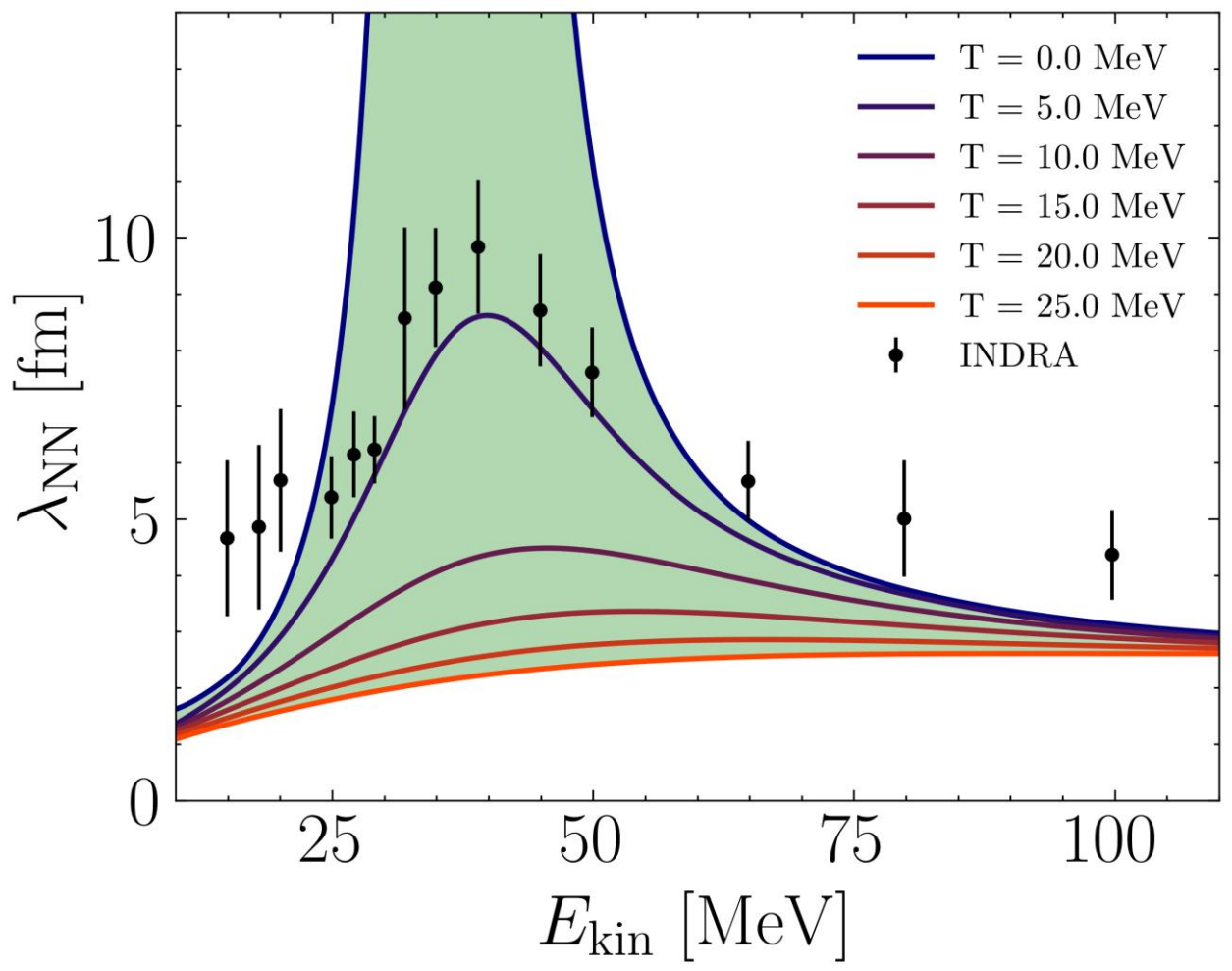




[1] D. Friedenberg, J. W. Holt, in preparation (2026)

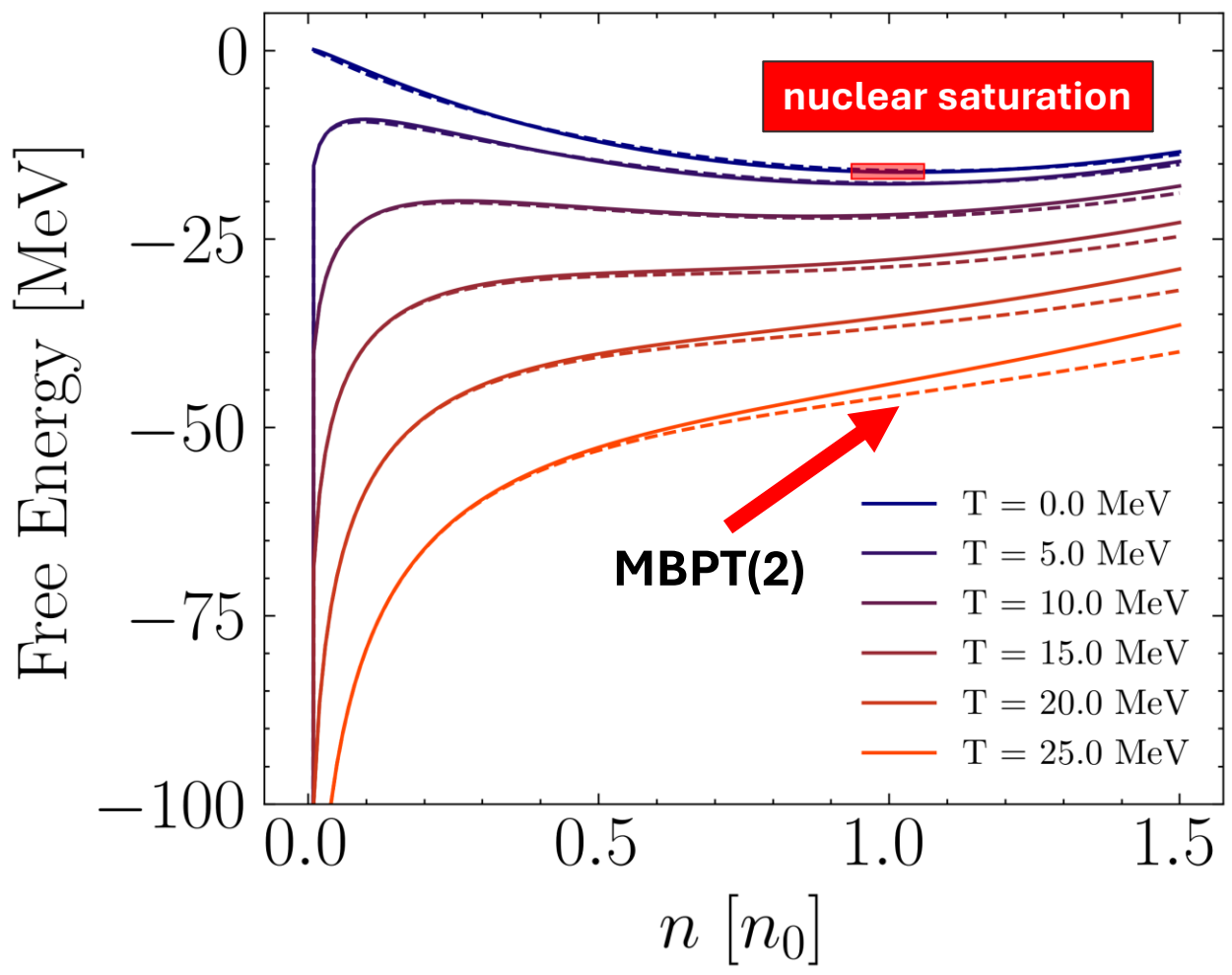
Nucleon Self-Energy: Benchmarks

Nucleon Mean-Free-Path $\lambda_{\mathrm{NN}} = v_F / W$



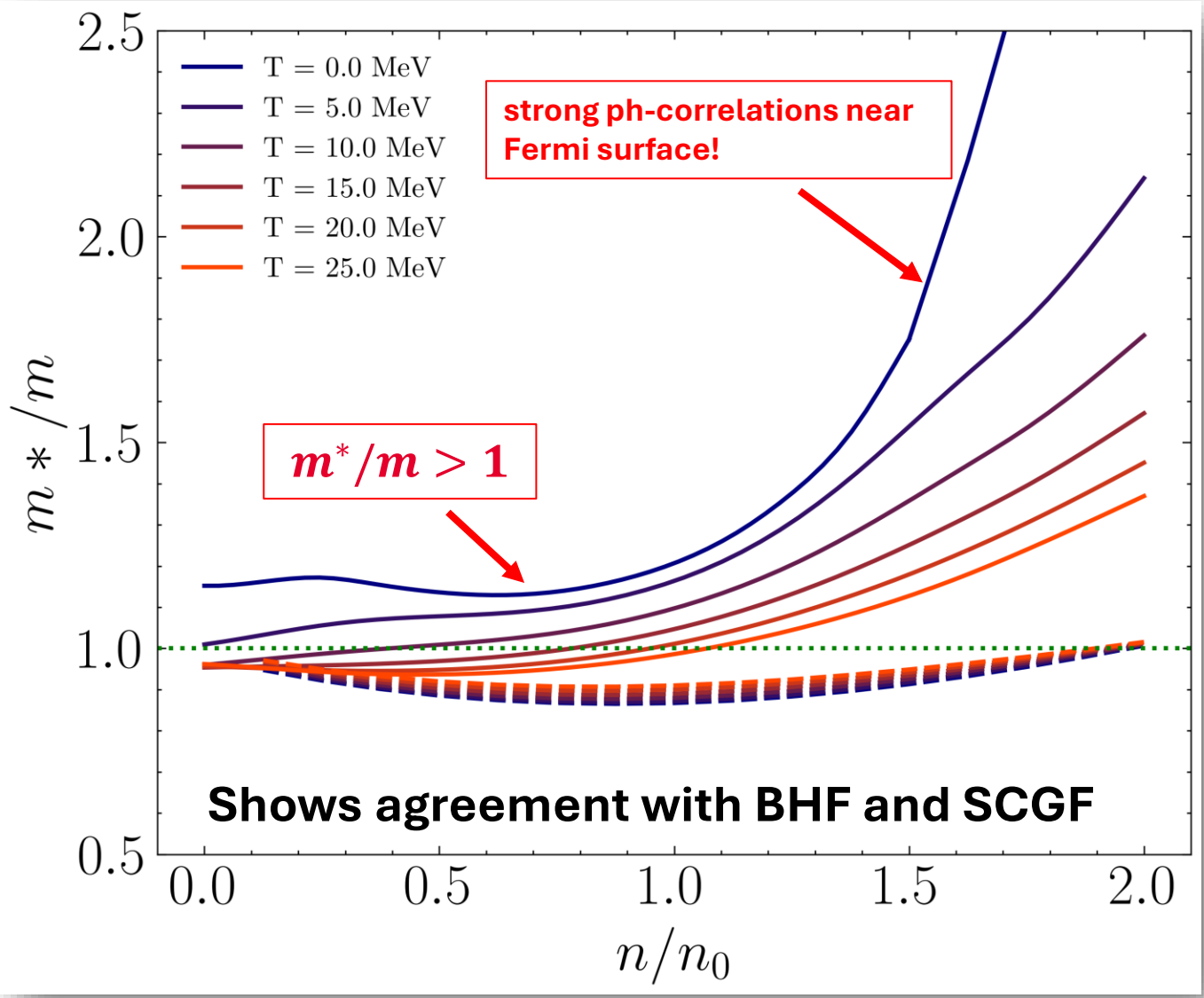
INDRA Collaboration, Lopez, Durand, Lehaut, EPJ (2015)

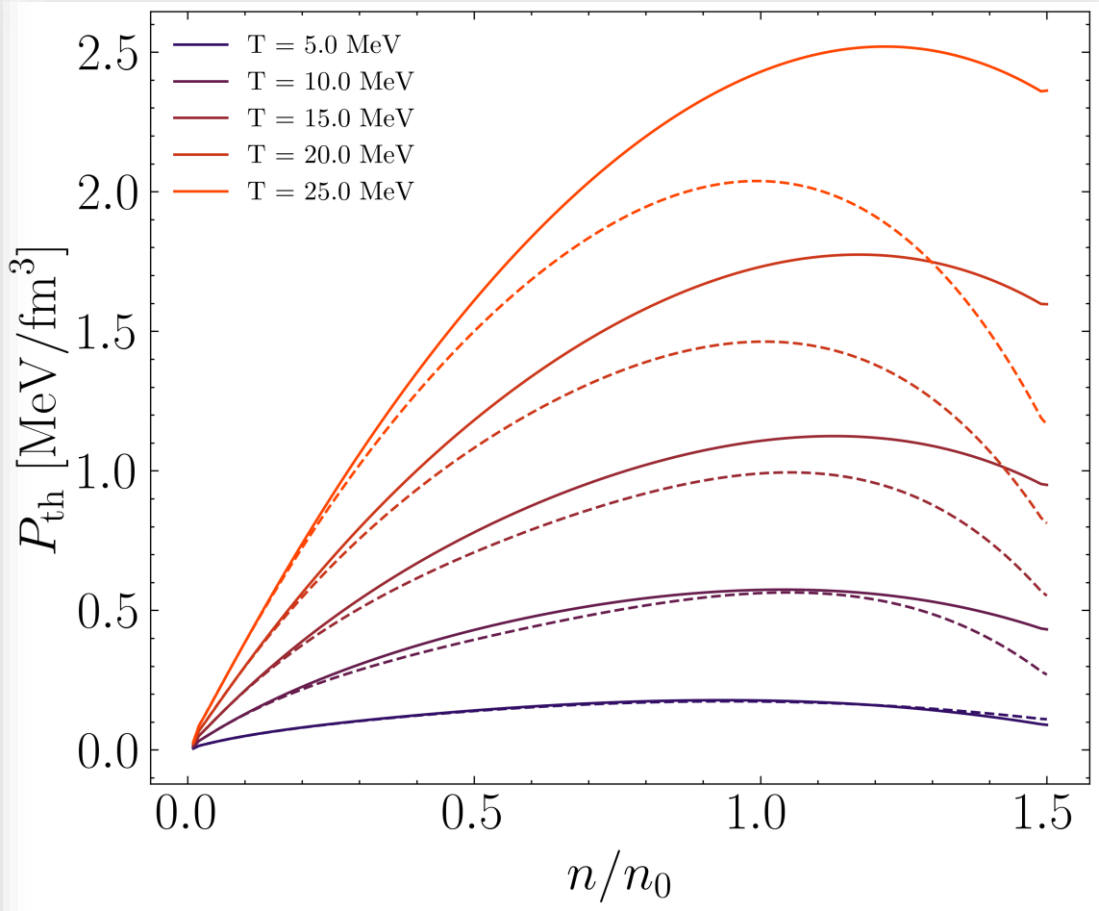
Nuclear Matter EOS $x_n \mu_n + x_p \mu_p = \dfrac{\partial F}{\partial n_B}$



DF, J. W. Holt, in preparation (2026)

Nucleon Effective Mass



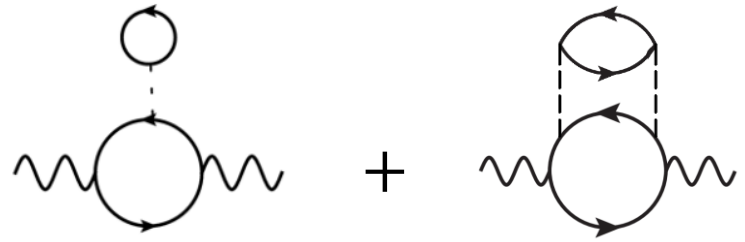


Increased thermal pressure

DF, J. W. Holt, in preparation (2026)

Neutrino Absorption: Beyond Mean Field

- Further shift in quasiparticle energy
- Stronger correlations – *two-body currents*
- **Collisional broadening**: reduced Pauli blocking



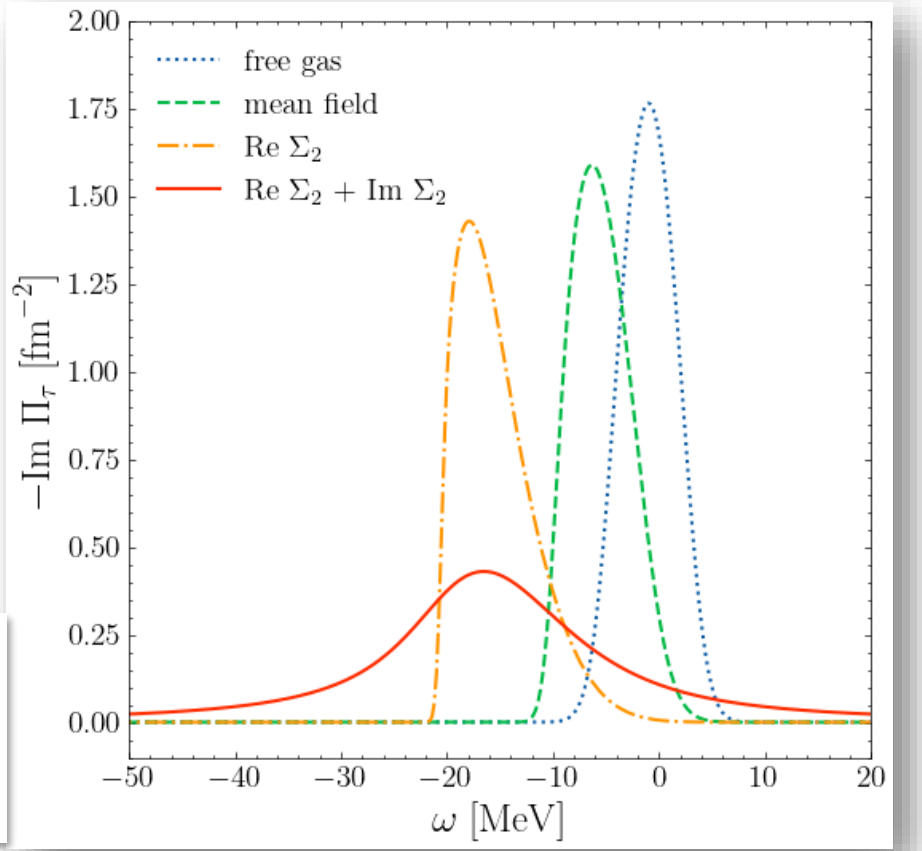
$$S_{\mathrm{F}}(q_0, q) = \frac{1}{2\pi^2} \int d^3 p_2 \delta(q_0 + E_2 - E_4) f_2 (1 - f_4),$$

$$L(\Gamma) = \frac{1}{\pi} \frac{\Gamma}{(\tilde{q}_0 - \Delta\epsilon_{p+q})^2 + \Gamma^2},$$

Roberts, Reddy, Shen, PRC (2012)

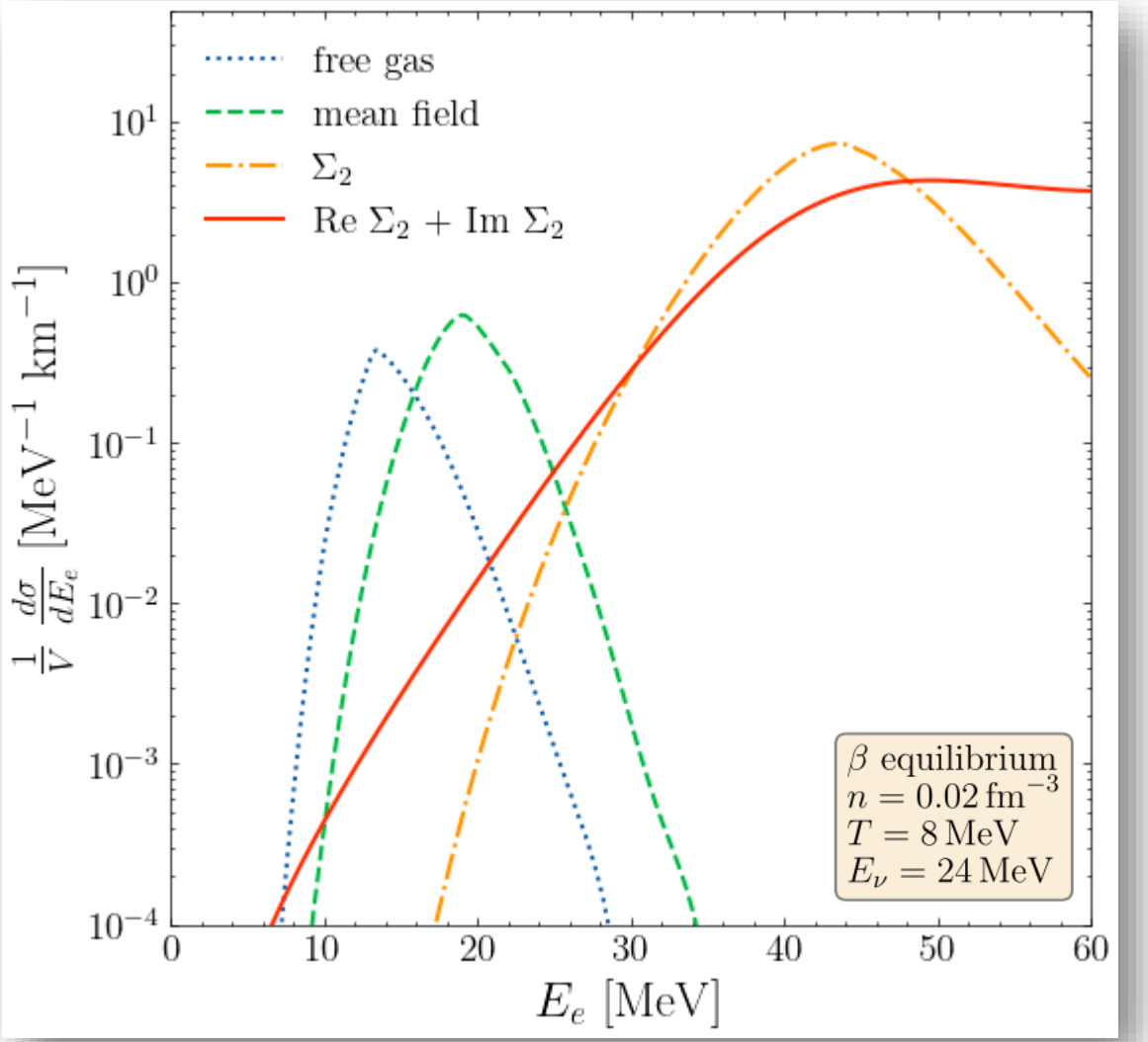
Nucleon width

β equilibrium
$n = 0.02\,\mathrm{fm}^{-3}$
$T = 8\,\mathrm{MeV}$
$q = 24\,\mathrm{MeV}$

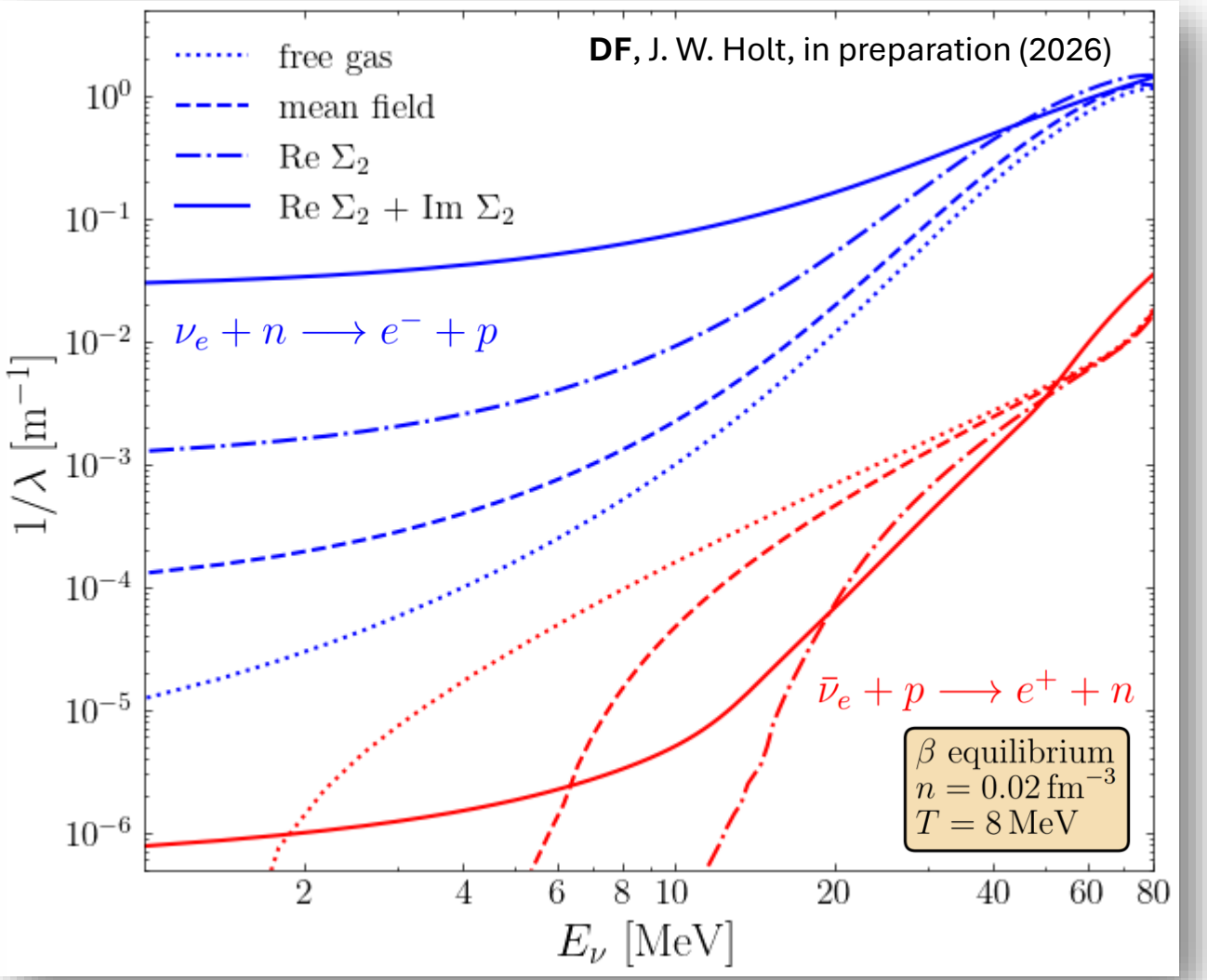


Neutrino Opacities: Beyond Mean Field

Absorption cross section



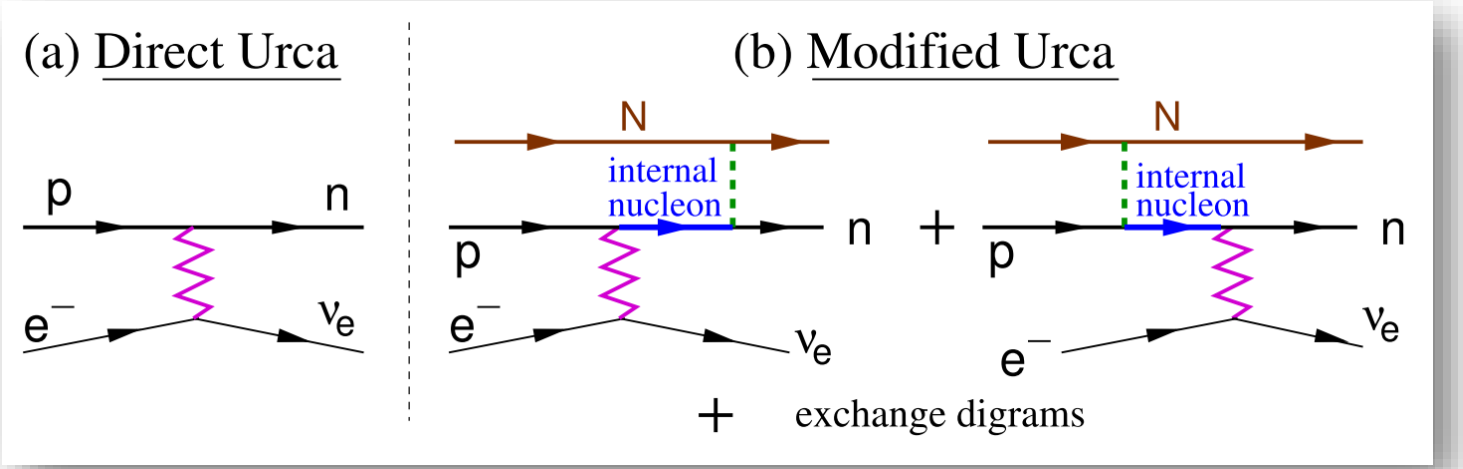
Neutrino opacity



Future Outlook: Urca Cooling

Relevant for:

- *Beta Equilibrium* – relaxation of proton fraction
- *Bulk viscosity* – damping density oscillations in NS mergers



Alford et al, PRC (2024)

Direct Urca:

- Strongly degenerate matter: dominated by Fermi surface
- Momentum conservation on Fermi surface: *heavily suppressed at low densities*

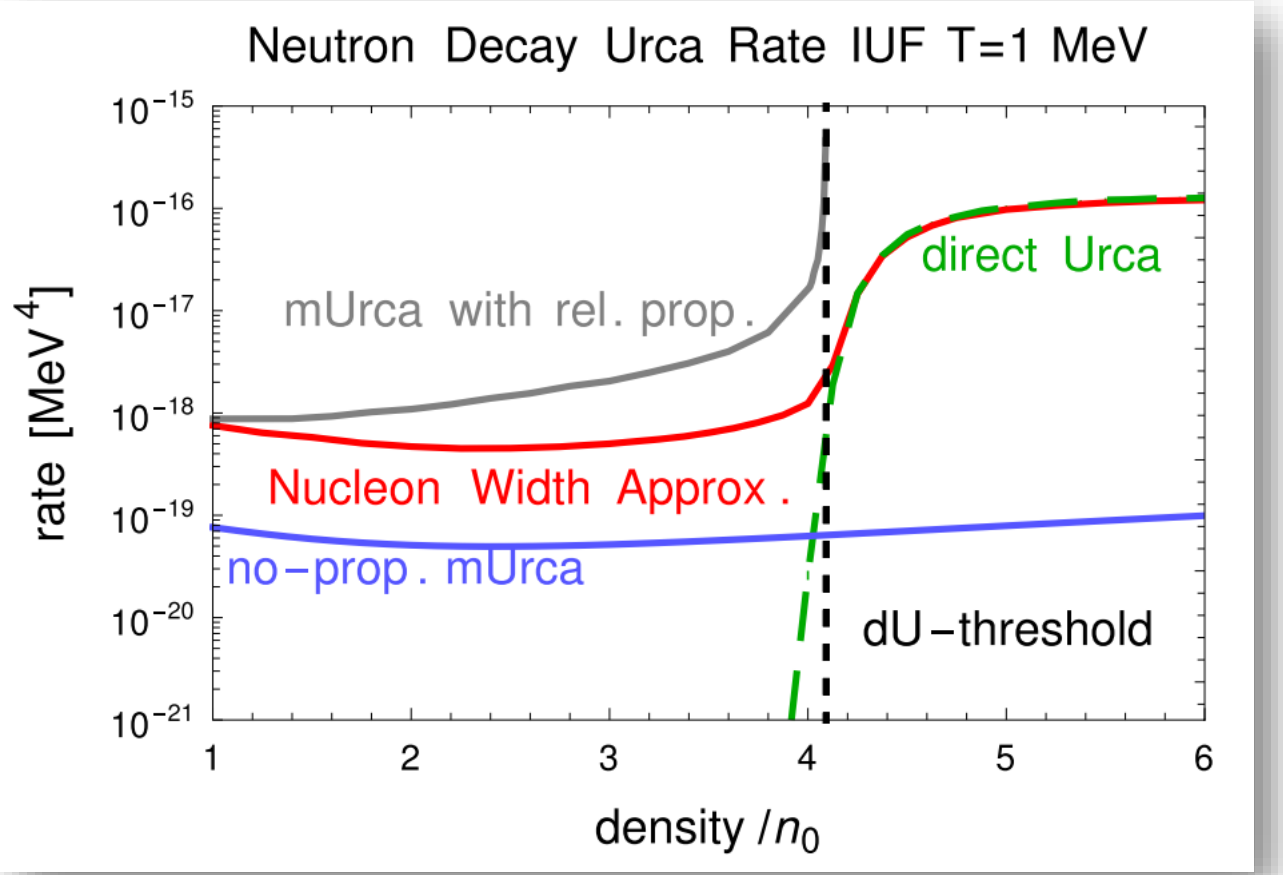
Modified Urca:

- Spectator nucleon extends kinematics at Fermi surface
- Typically approximated with constant mean-field effects (*no propagator*) or unrealistic NN interactions (F&M)

Future Outlook: Nucleon Width Approximation

$$\Gamma^{\rm NWA} = \int_{-\infty}^{\infty} dm_{\rm n} dm_{\rm p} \Gamma^{\rm dUrca}(m_{\rm n}, m_{\rm p}) R_{\rm n}(m_{\rm n}) R_{\rm p}(m_{\rm p}) \, .$$

$$R_a(m) \equiv \frac{1}{\pi} \frac{W_a/2}{(m - M_a^*)^2 + W_a^2/4} \, .$$

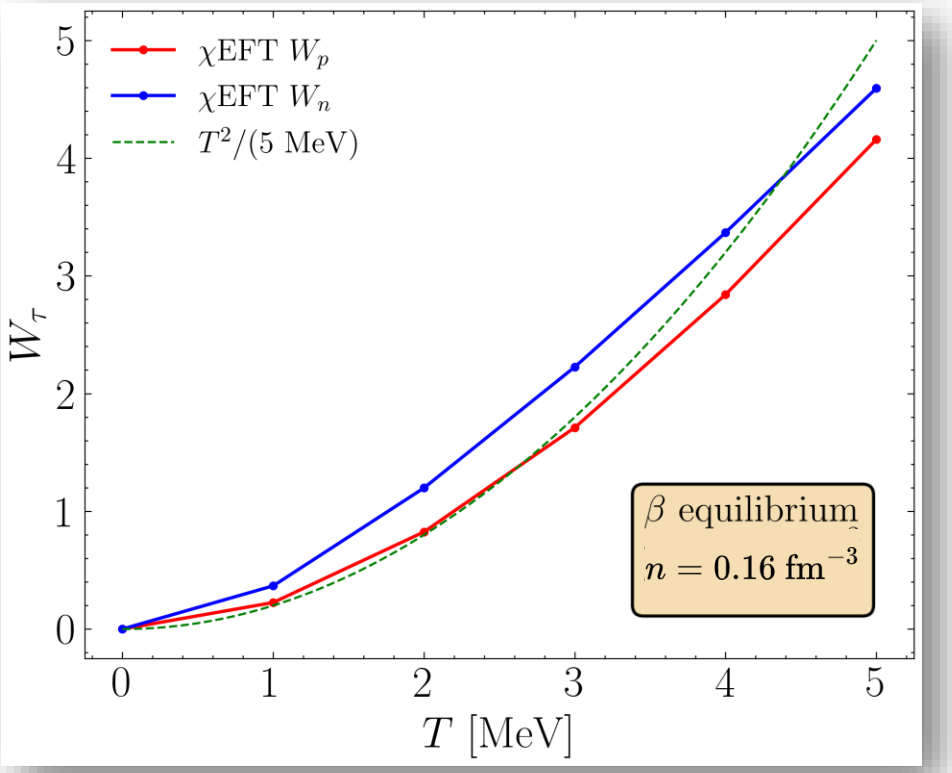


- Agreement above and below dUrca threshold
- Generalizes to higher temperatures, nonzero magnetic field, etc.
- Phenomenological width: $\Gamma_{n,p} = T^2/T_0$

Future Outlook: Nucleon Width Approximation

In progress: Use widths from Chiral EFT + MBPT2

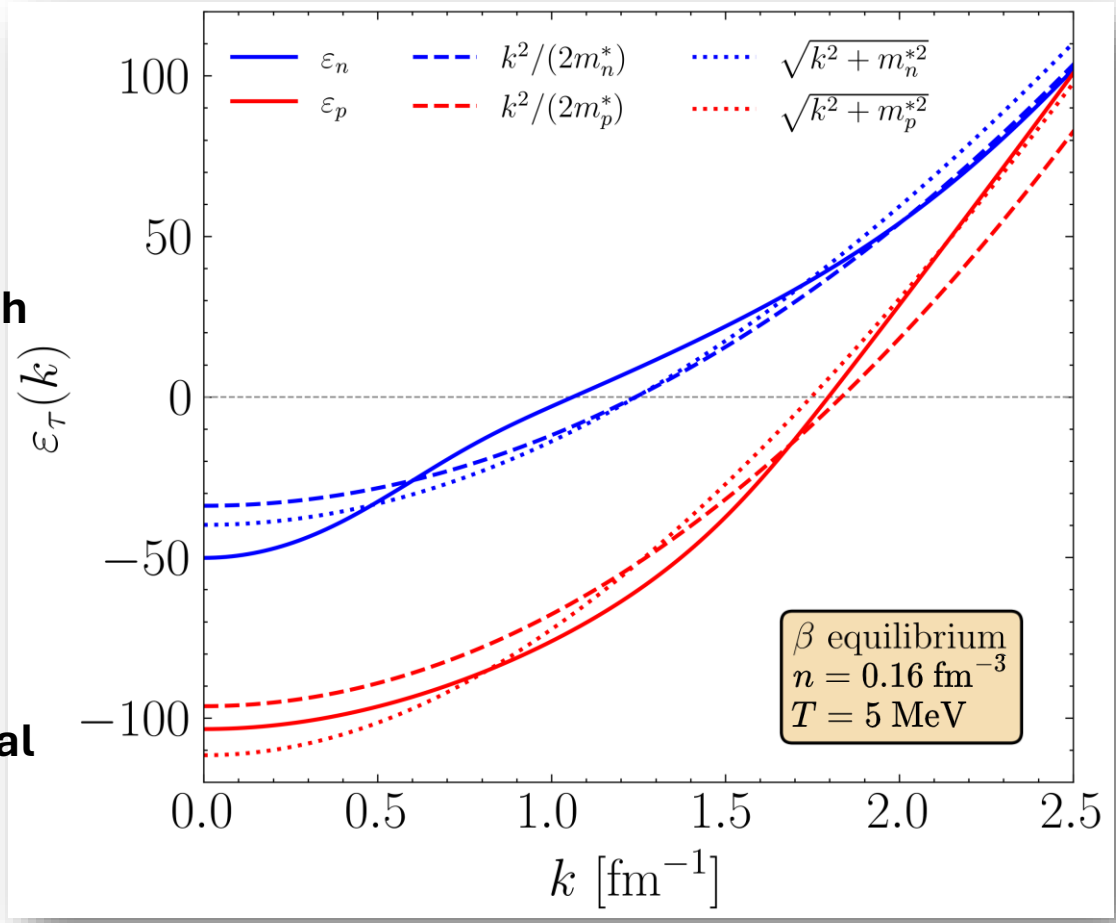
$$G_a^{\mathrm{NWA}}(k) = \frac{1}{(k_0 - U_a)\gamma^0 + k_i\gamma^i + M_a^* + iW_a/2} \,.$$



Consistent effective mass and nucleon width

Corrections to phenomenological approach

Relativistic vs. non-relativistic single-particle spectrum



Conclusions

- Effective mass of nucleons dominates thermal effects in Equation of State
- **Beyond mean field:** Larger effective masses due to correlations & 3NF ($m^*/m > 1$)!
- Charged-current reactions dominate neutrino absorption rates/opacities
- **Beyond mean field:** Wider energy shift and collisional broadening lead to larger difference in neutrino-antineutrino rates/opacities, implications for r-process in core-collapse supernovae

- **<u>Future Outlook:</u>** Urca cooling, flavor relaxation, uncertainty quantification, supernova simulations

Thank You!

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