Exploring sub-barrier fusion reactions

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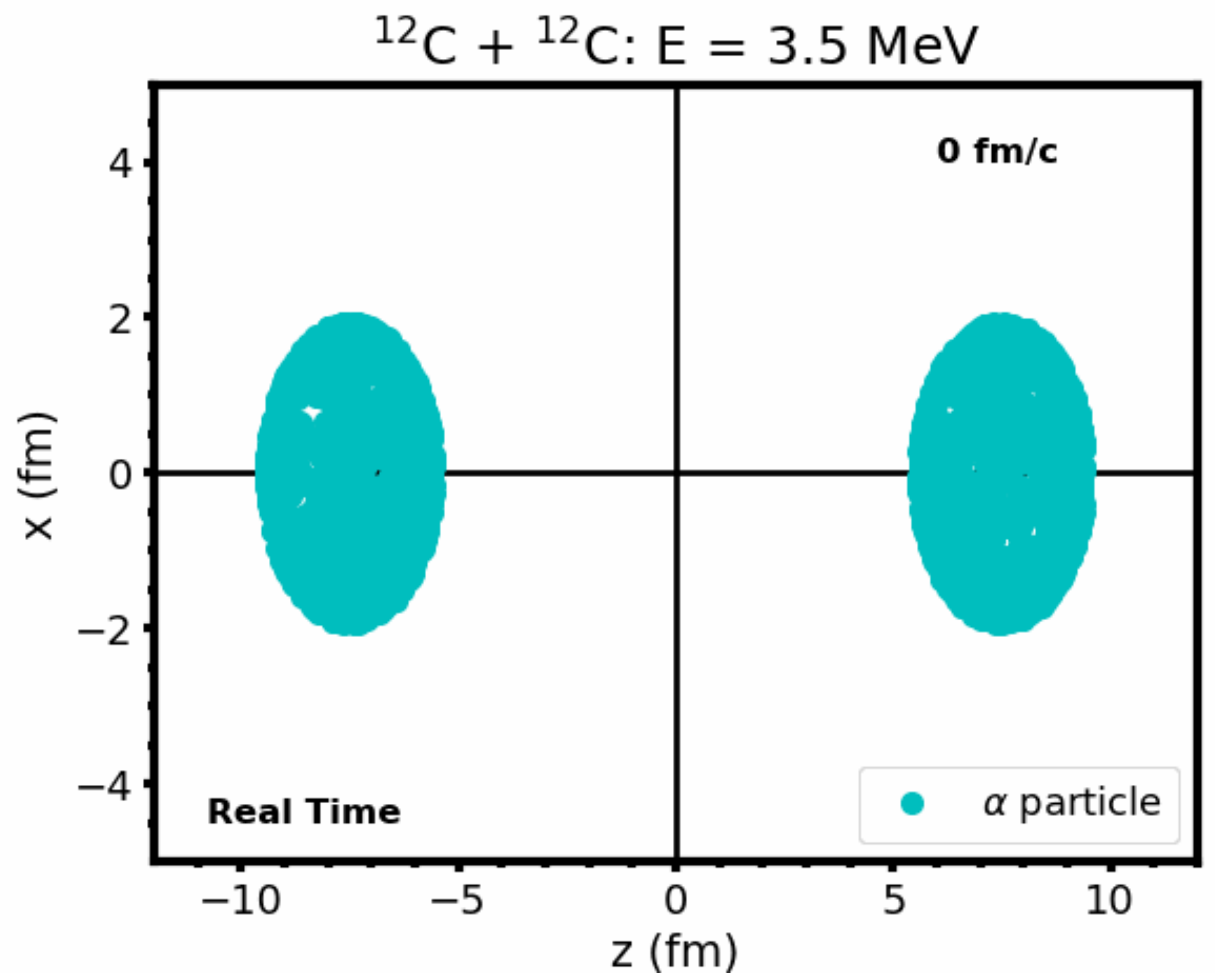
Introduction



Microscopic calculations: Vlasov dynamics in imaginary times.
Future directions-Constrained Molecular Dynamics (CoMD)
Hybrid Alpha Cluster model (T.Depastas).

Macroscopic calculations: Neck model in imaginary times (S. Sun).

Experimental challenges: pair production in sub barrier fusion reactions (T. Settlemyre)

Conclusions

Exploring sub-barrier fusion reactions

A.B. and J.B.Natowitz

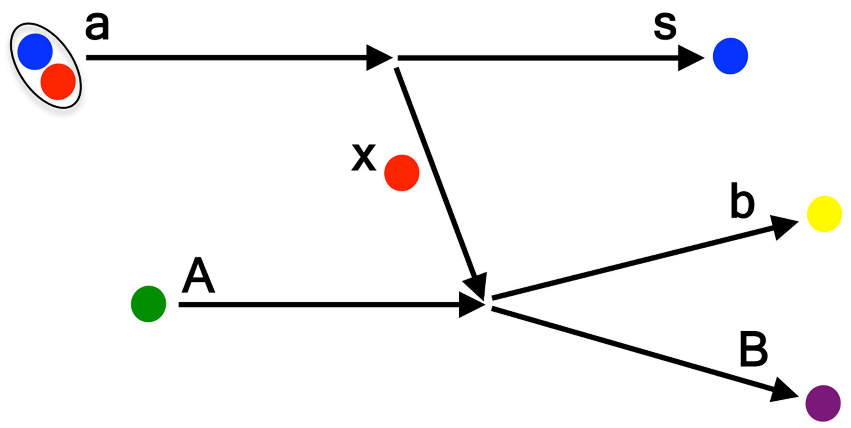
PHYSICAL REVIEW C **102**, 061602(R) (2020)

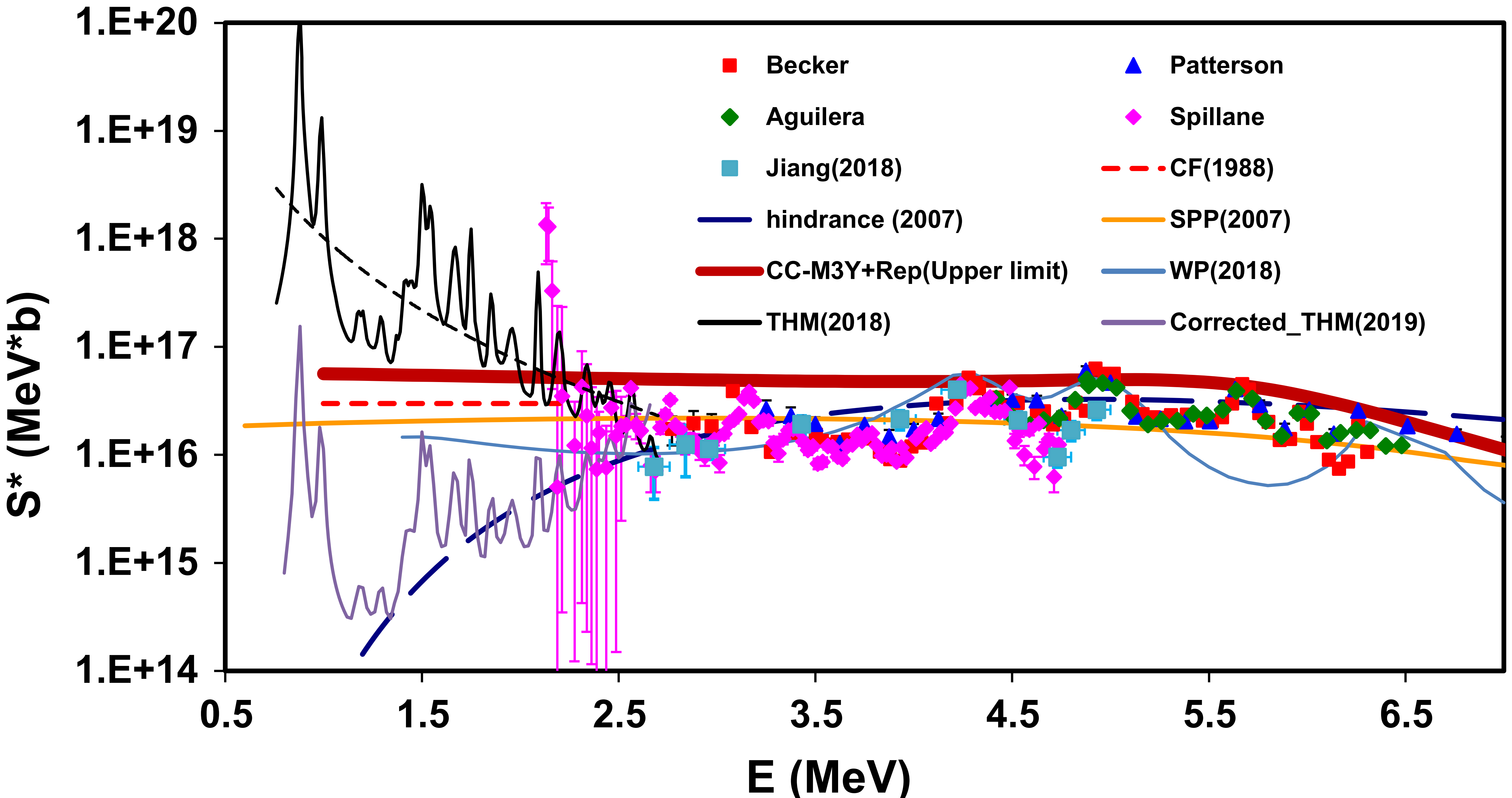
INTRODUCTION&MOTIVATION

Letter 688| Nature | VOL557 | 31MAY2018

An increase in the $^{12}C + ^{12}C$ fusion rate from resonances at astrophysical energies

A. Tumino[1,2*], C. Spitaleri[2,3], M. La Cognata[2], S. Cherubini[2,3], G. L. Guardo[2,4], M. Gulino[1,2], S. Hayakawa[2,5], I. Indelicato[2], L. Lamia[2,3], H. Petrascu[4], r. G. Pizzone[2], S. M. r. Puglia[2], G. G. rapisarda[2], S. romano[2,3], M. L. Sergi[2], r. Spartá[2] & L. Trache[4]





Status on $^{12}C + ^{12}C$ fusion at deep subbarrier energies: impact of resonances on astrophysical S^* factors

C. Beck[1,a], A. M. Mukhamedzhanov[2,b], X. Tang[3,4,c]

Eur. Phys. J. A (2020) 56:87

$$S^*(E_{c.m.}) = E_{c.m.}\sigma(E_{c.m.})\exp\left(87.12E_{c.m.}^{-1/2} + 0.46E_{c.m.}\right)$$
$$= S(E_{c.m.})\exp(0.46E_{c.m.}) \quad (1)$$

Feynman path integration in phase space

Physics Letters B 339 (1994) 207–210

Phys.Rev.Lett.78(1997)187

Aldo Bonasera, Vladimir N. Kondratyev [1]

Laboratorio Nazionale del Sud, INFN, v. S. Sofia 44, 95125 Catania, Italy

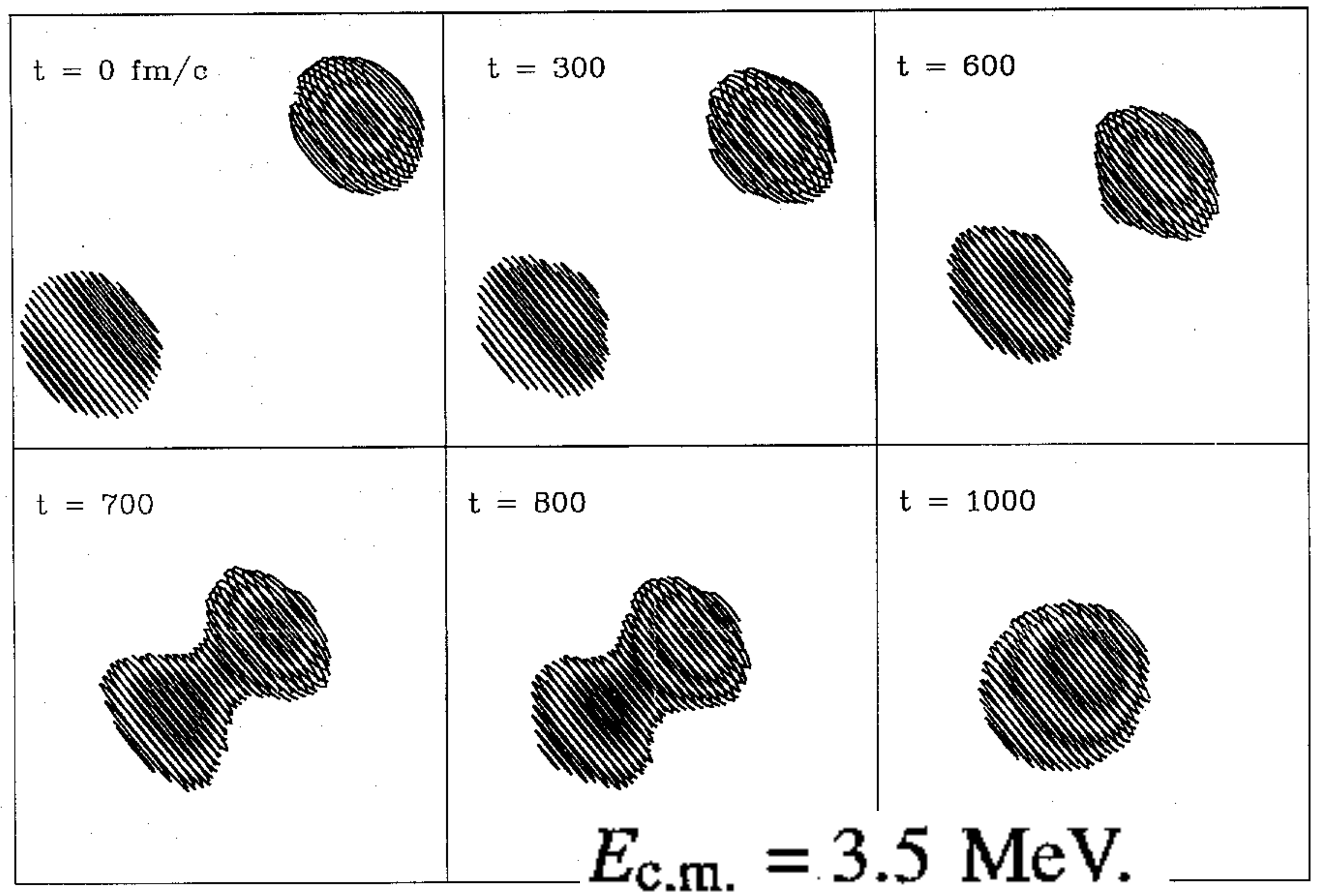
Solve the Vlasov equation in imaginary time. Define collective variables R&P

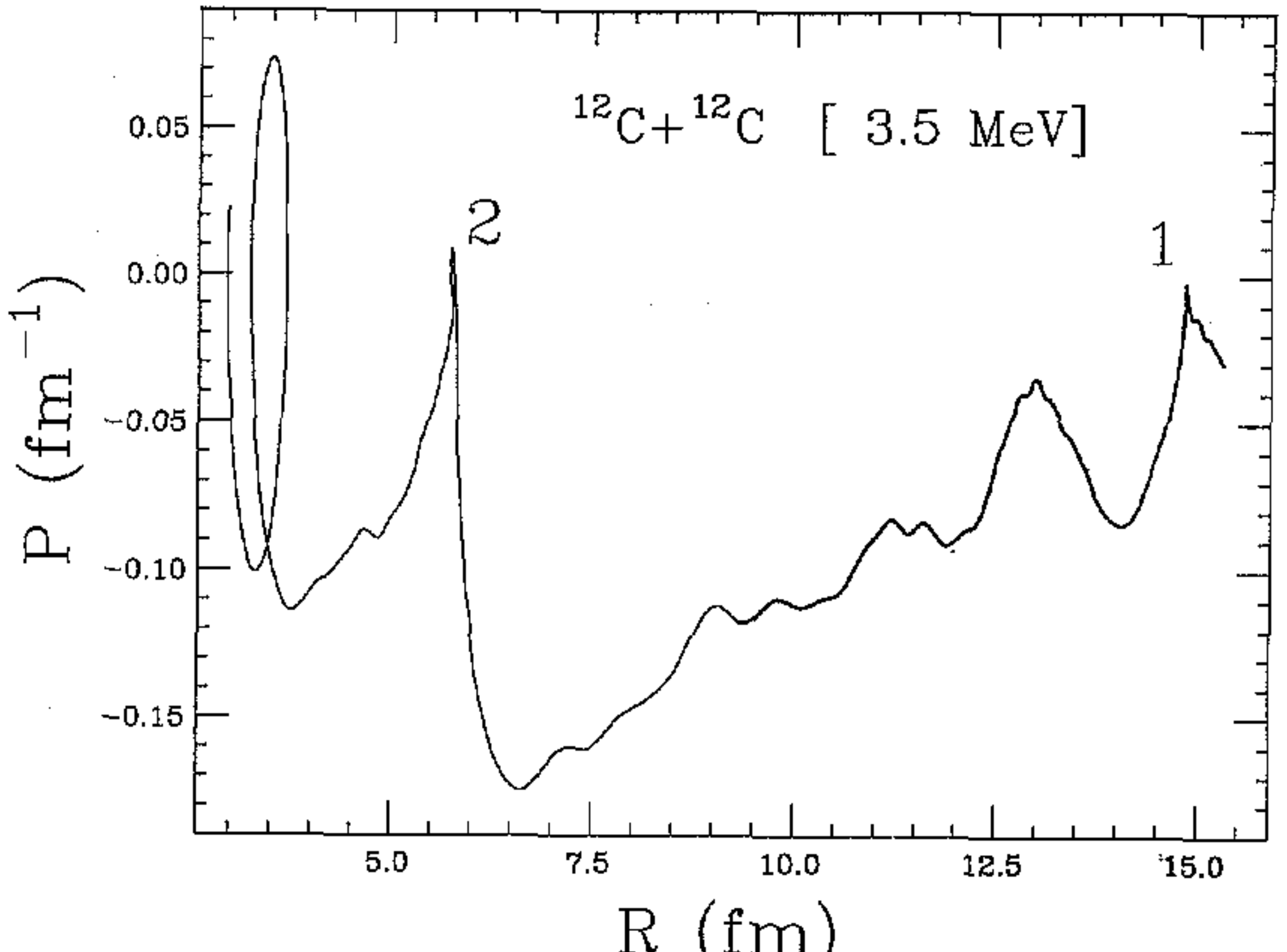
$$\begin{Bmatrix} \boldsymbol{R} \\ \boldsymbol{P} \end{Bmatrix} = \int_A d\boldsymbol{r}d\boldsymbol{p} \begin{Bmatrix} \boldsymbol{r} \\ \boldsymbol{p} \end{Bmatrix} f(\boldsymbol{r},\boldsymbol{p};\, t) - \int_B d\boldsymbol{r}d\boldsymbol{p} \begin{Bmatrix} \boldsymbol{r} \\ \boldsymbol{p} \end{Bmatrix} f(\boldsymbol{r},\boldsymbol{p};\, t)$$

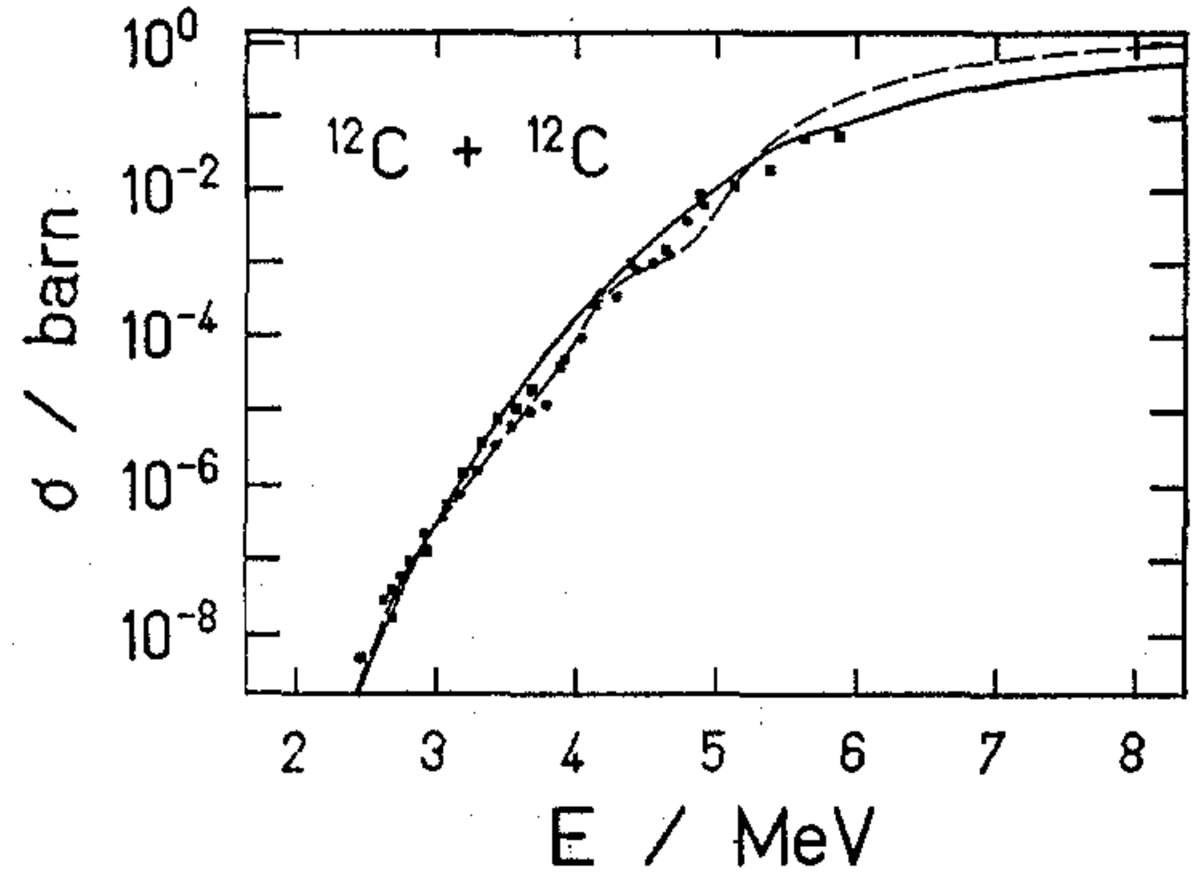
$$\frac{d\boldsymbol{R}_{A(B)}}{dt} = \frac{\boldsymbol{P}_{A(B)}}{m}\,; \qquad \frac{d\boldsymbol{P}_{A(B)}}{dt} = \boldsymbol{F}_{A(B)}$$

in imaginary time t—>it

$$\frac{d\boldsymbol{R}^i_{A(B)}}{dt} = \frac{\boldsymbol{P}^i_{A(B)}}{m}\,; \qquad \frac{d\boldsymbol{P}^i_{A(B)}}{dt} = -\boldsymbol{F}_{A(B)}$$







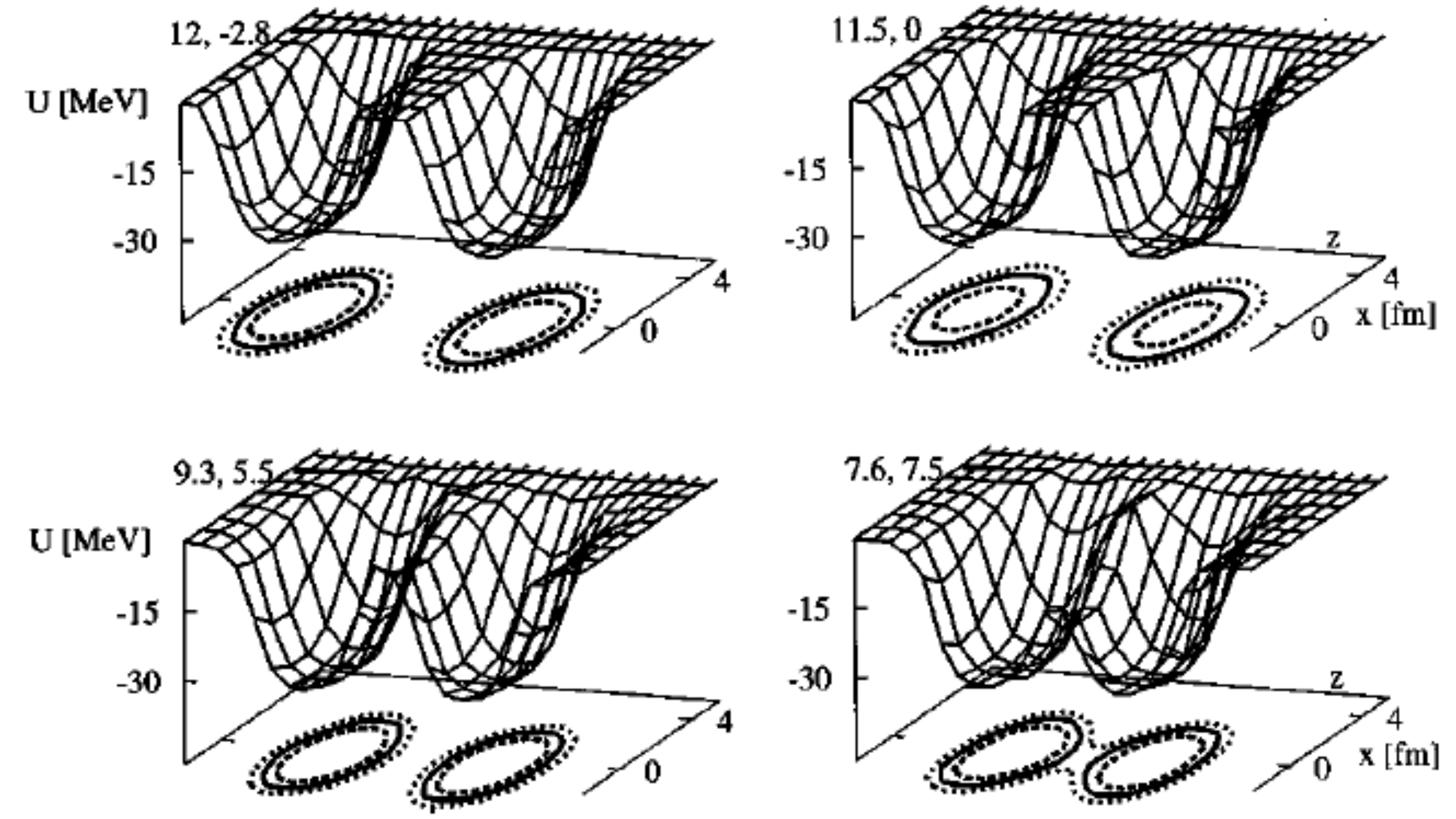


FIG. 1. Snapshots from the mean-field simulation of the fusion reaction for a head-on $^{16}O+^{16}O$ collision at the energy 8 MeV. The

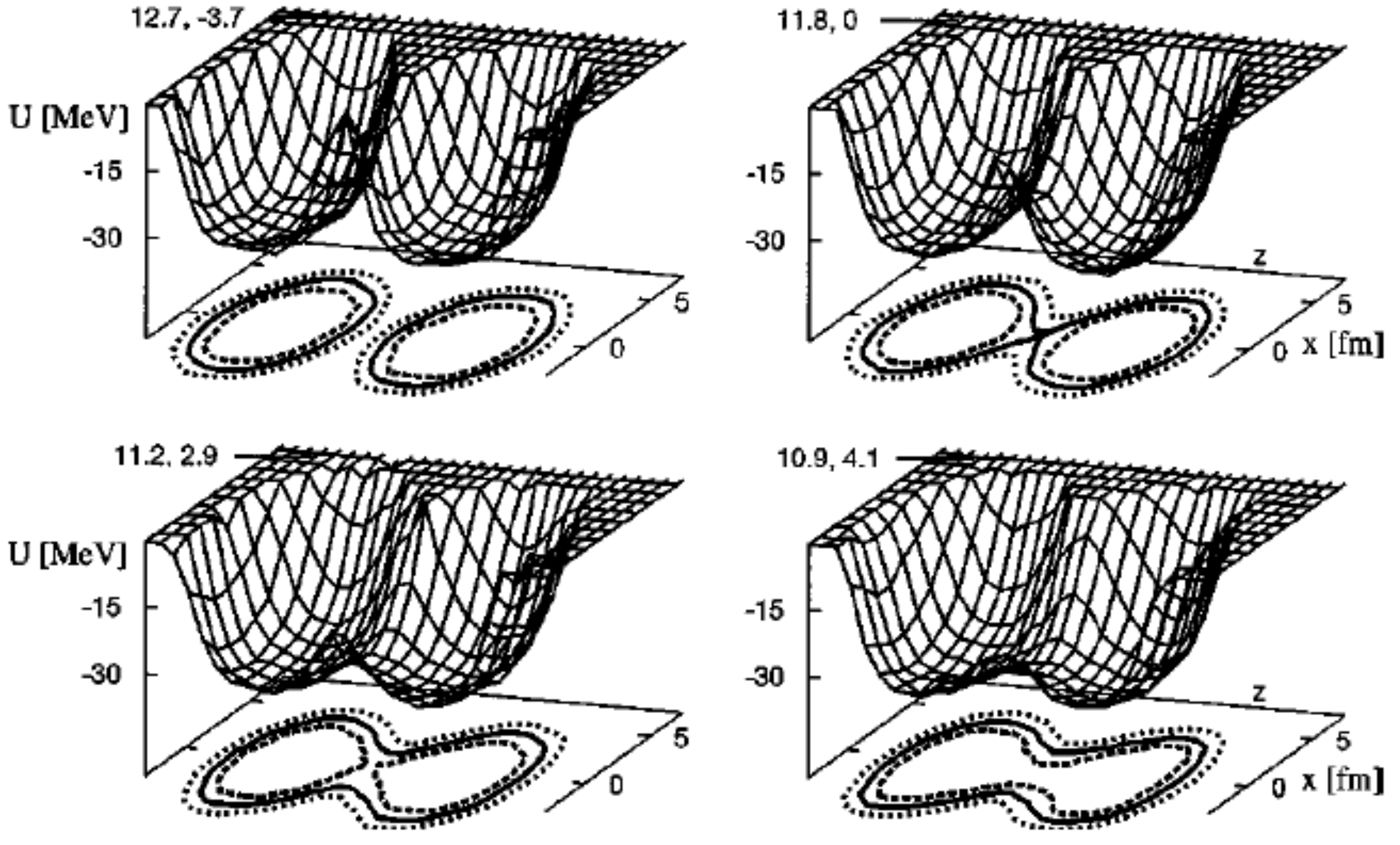


FIG. 2. The same as Fig. 1 for a $^{58}Ni+^{58}Ni$ collision at the energy 93 MeV.

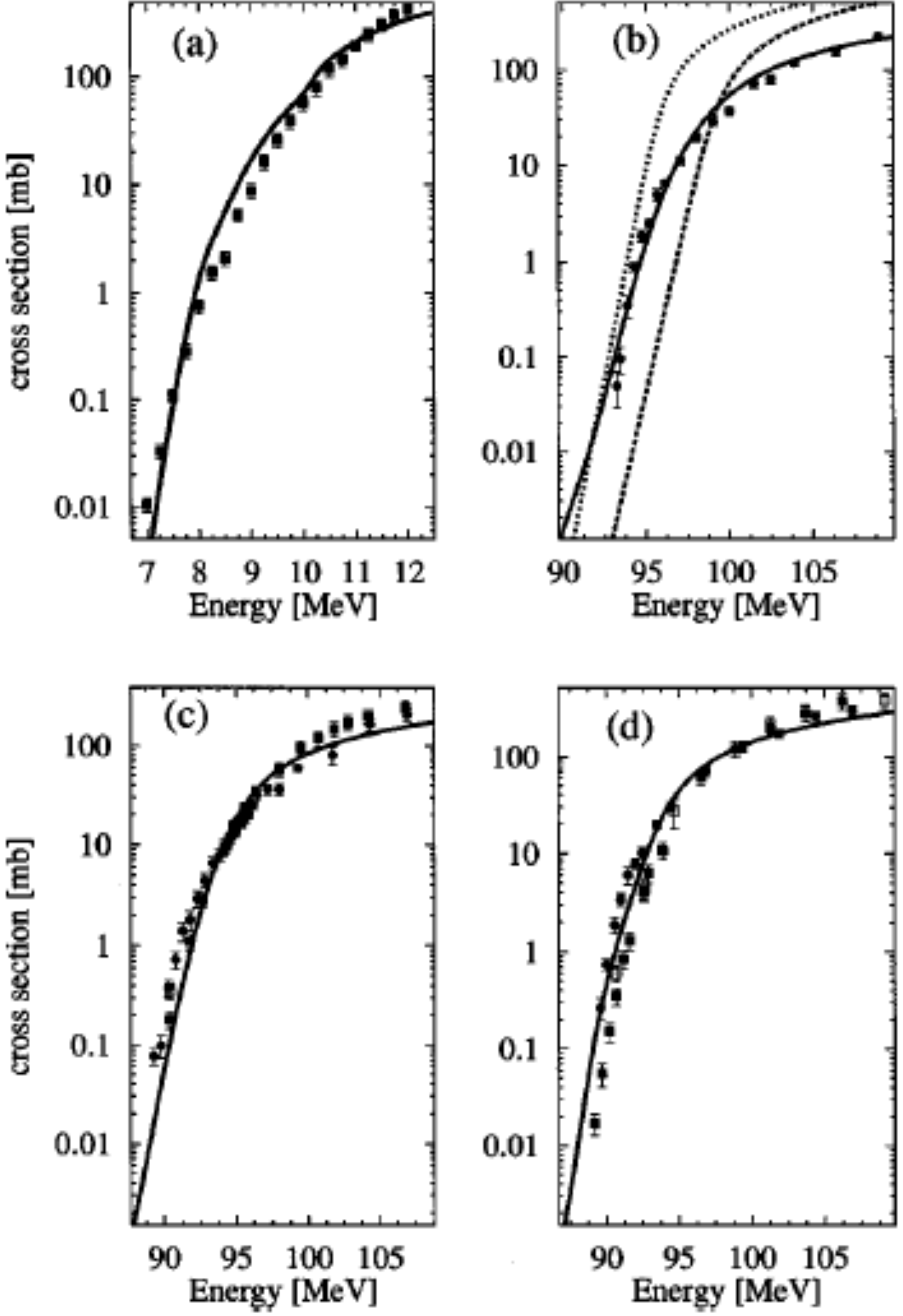
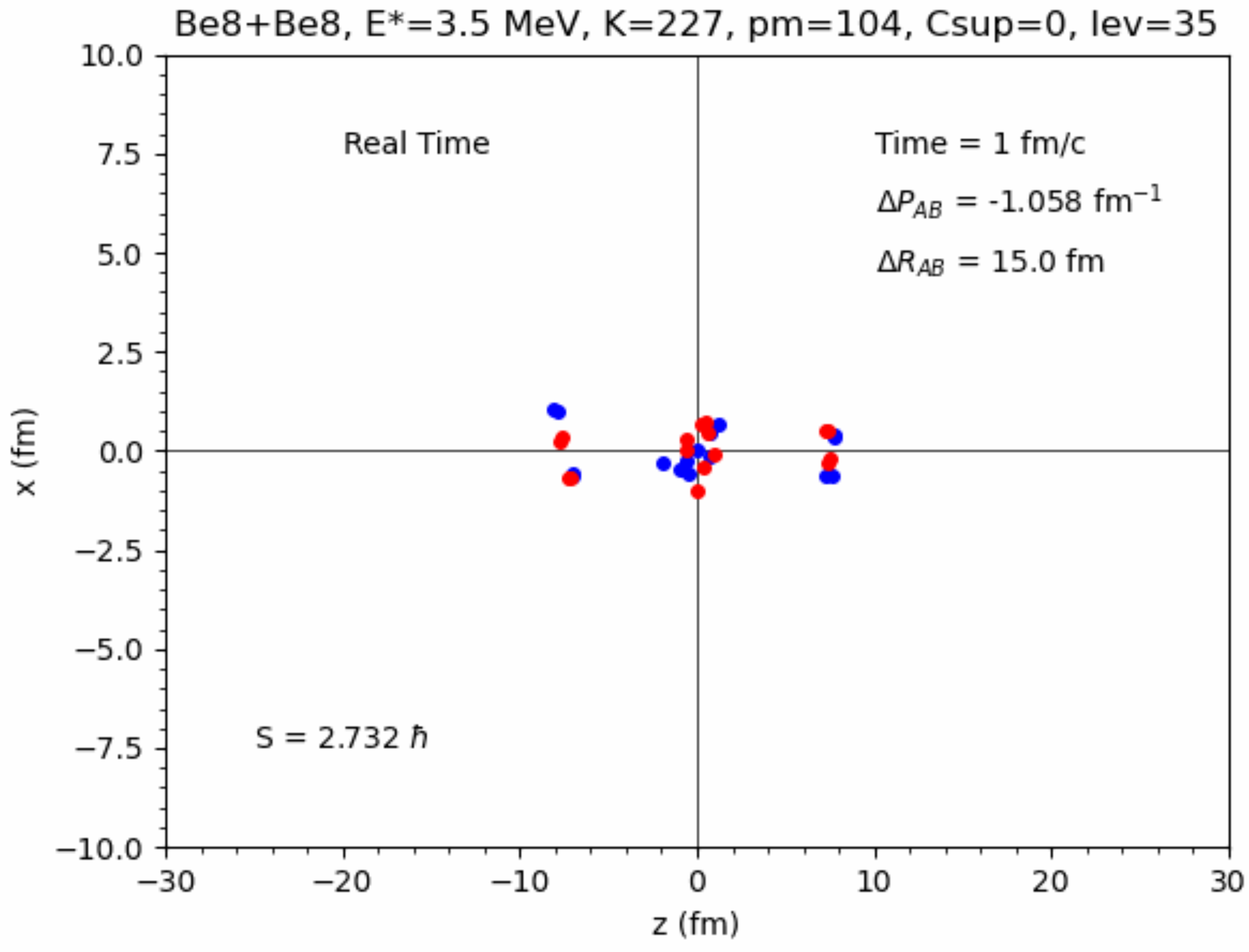
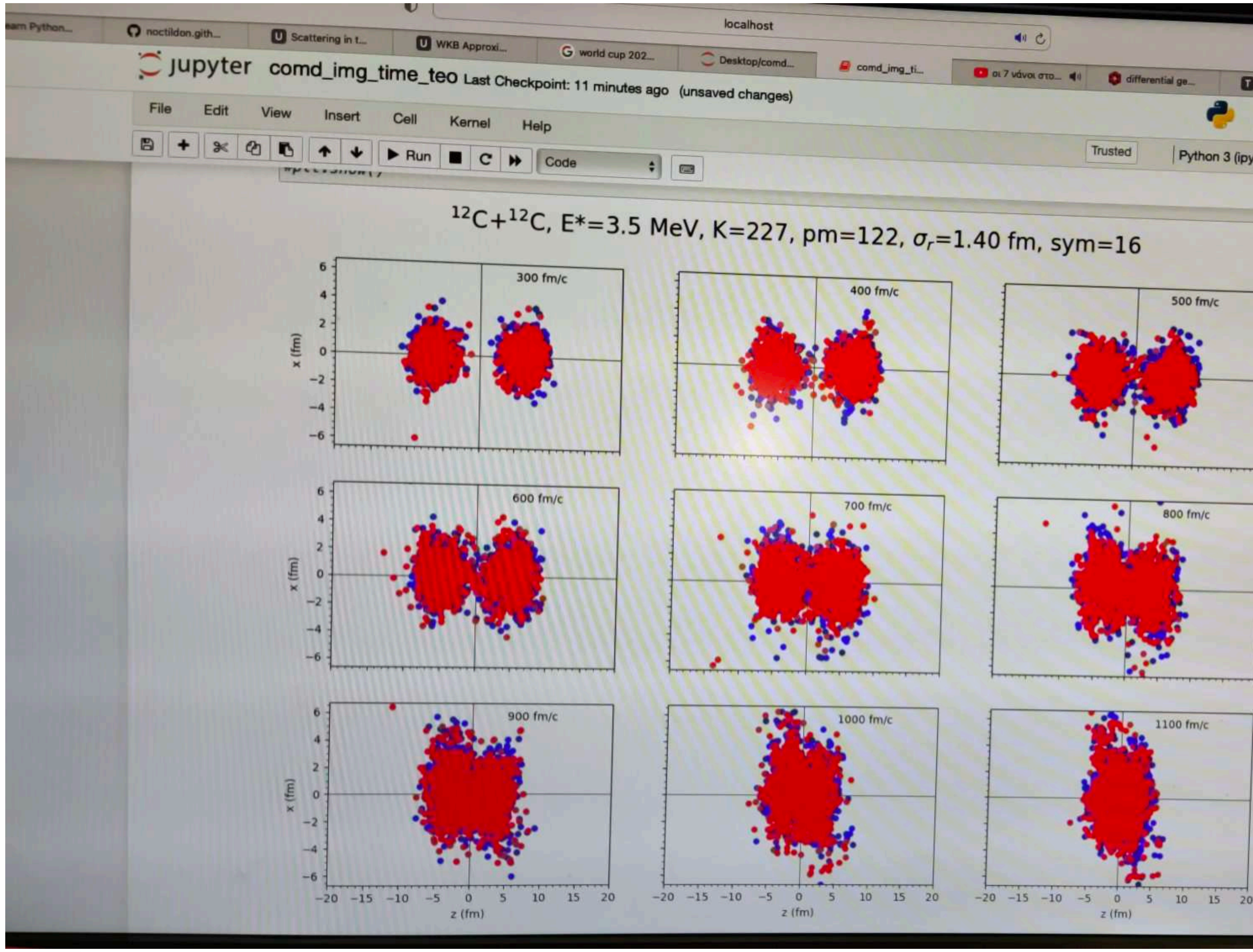


FIG. 5. The fusion excitation functions. Part (a) represents the $^{16}O+^{16}O$ system with the experimental data from Ref. [47]. Parts (b)–(d) display the results for the isotope pairs: (b) $^{58}Ni+^{58}Ni$; (c) $^{58}Ni+^{64}Ni$; and (d) $^{64}Ni+^{64}Ni$. The circles show the experimental

Microscopic calculations in progress- T. Depastas

Preliminary attempts using CoMD



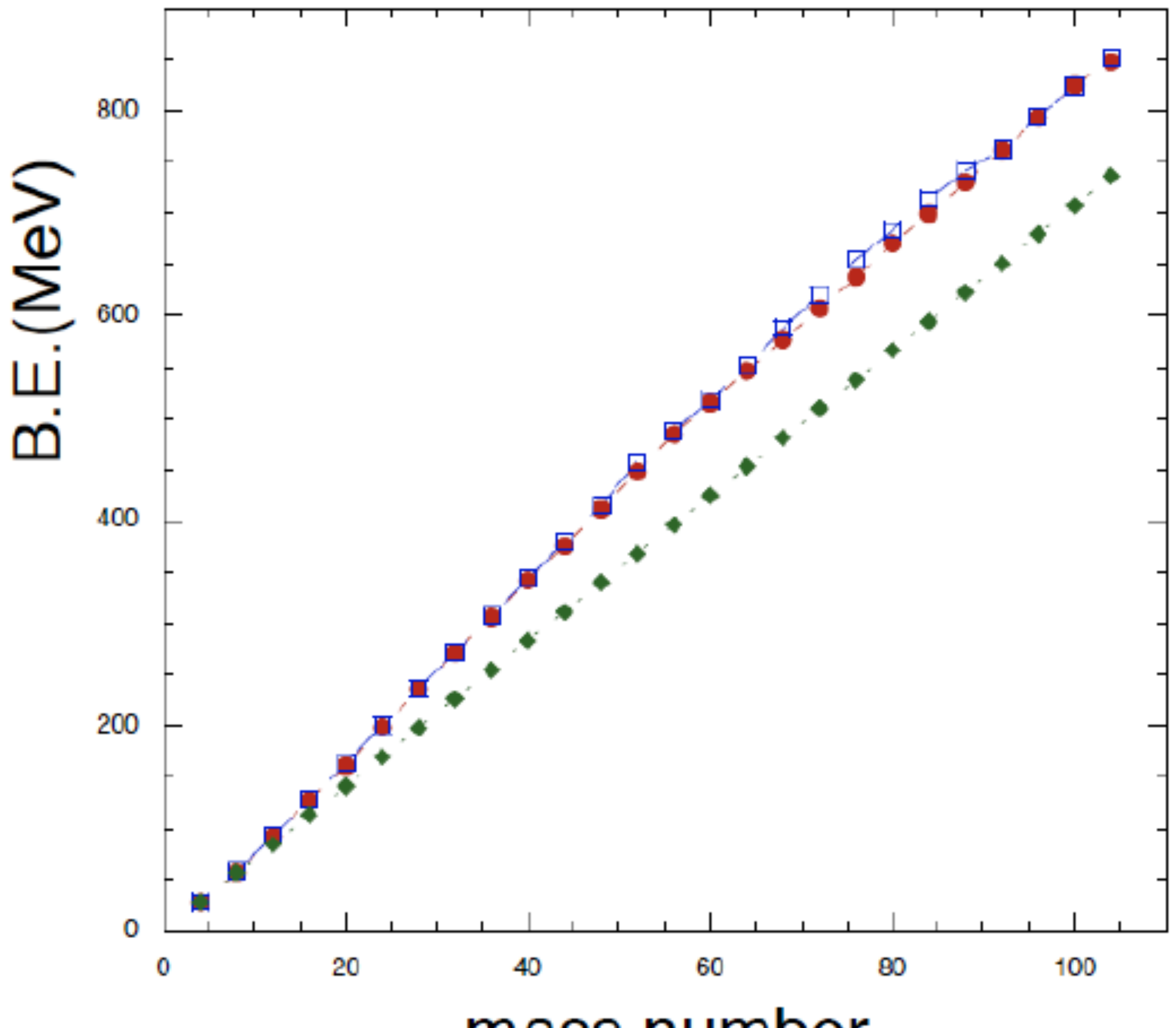


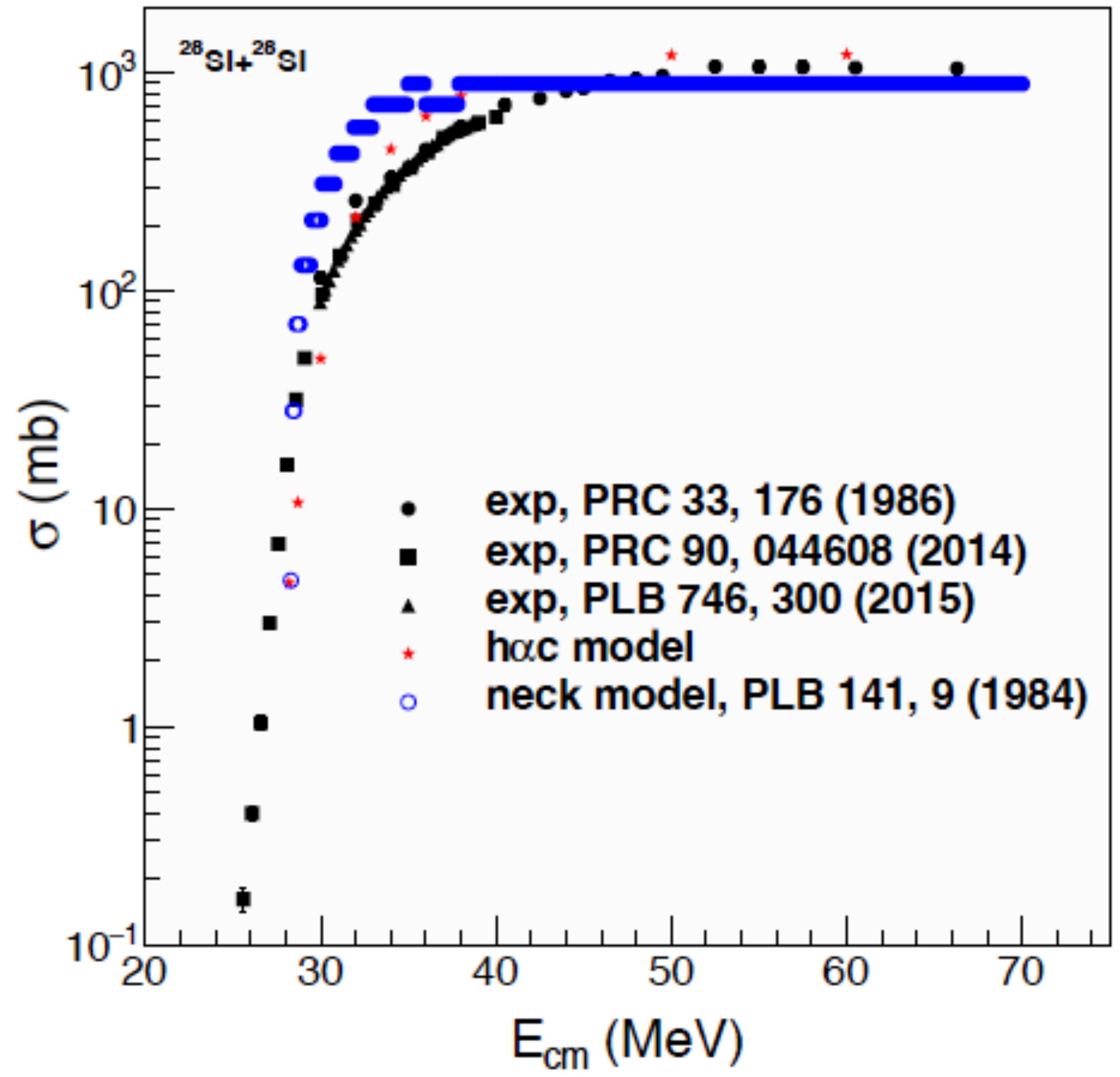
Hybrid Alpha Cluster model (HAC)

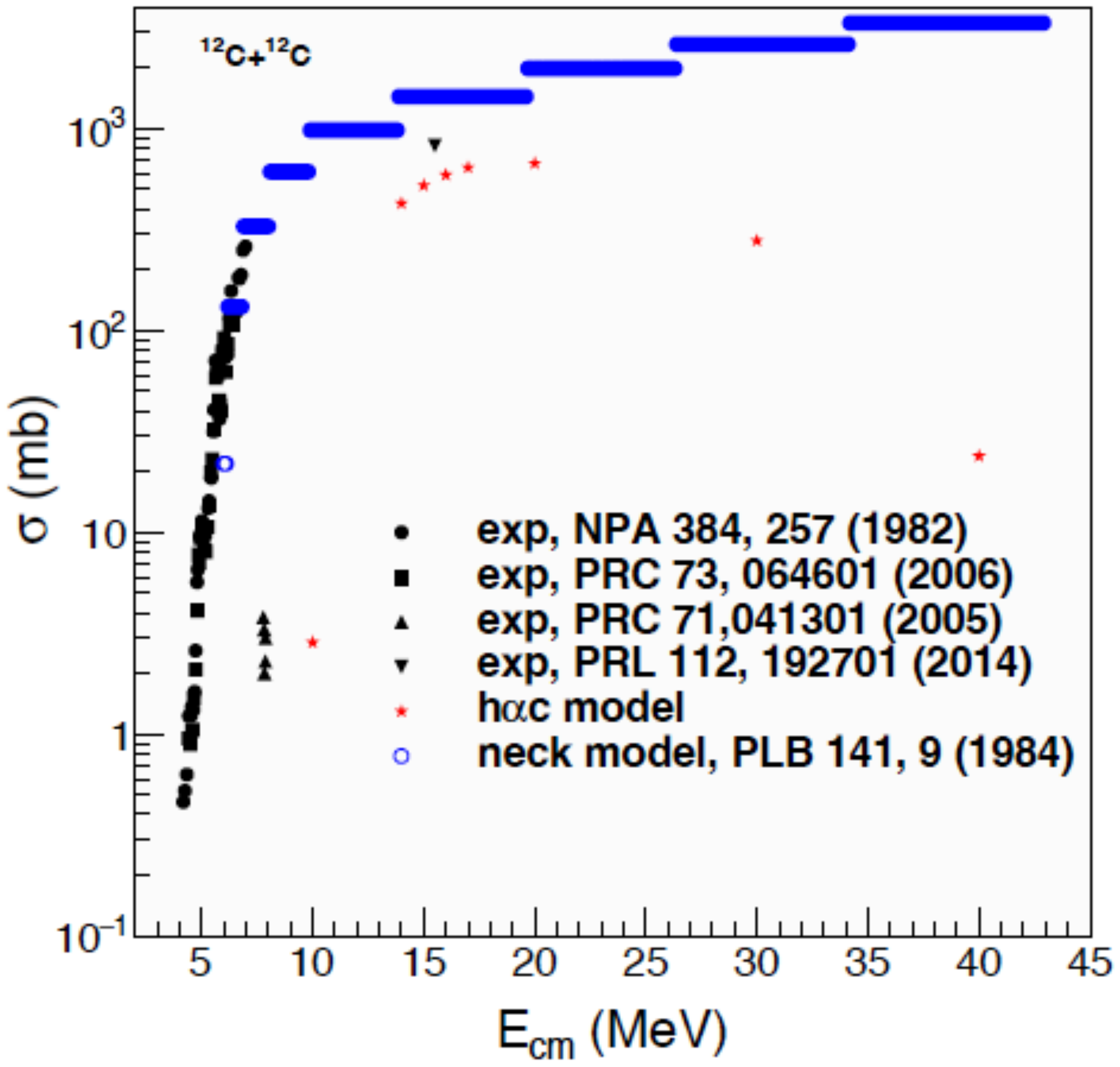
Zheng, H.; Bonasera, A. Spin Quantization in Heavy Ion Collision. Symmetry 2021, 13, 1777. https://doi.org/10.3390/sym13101777

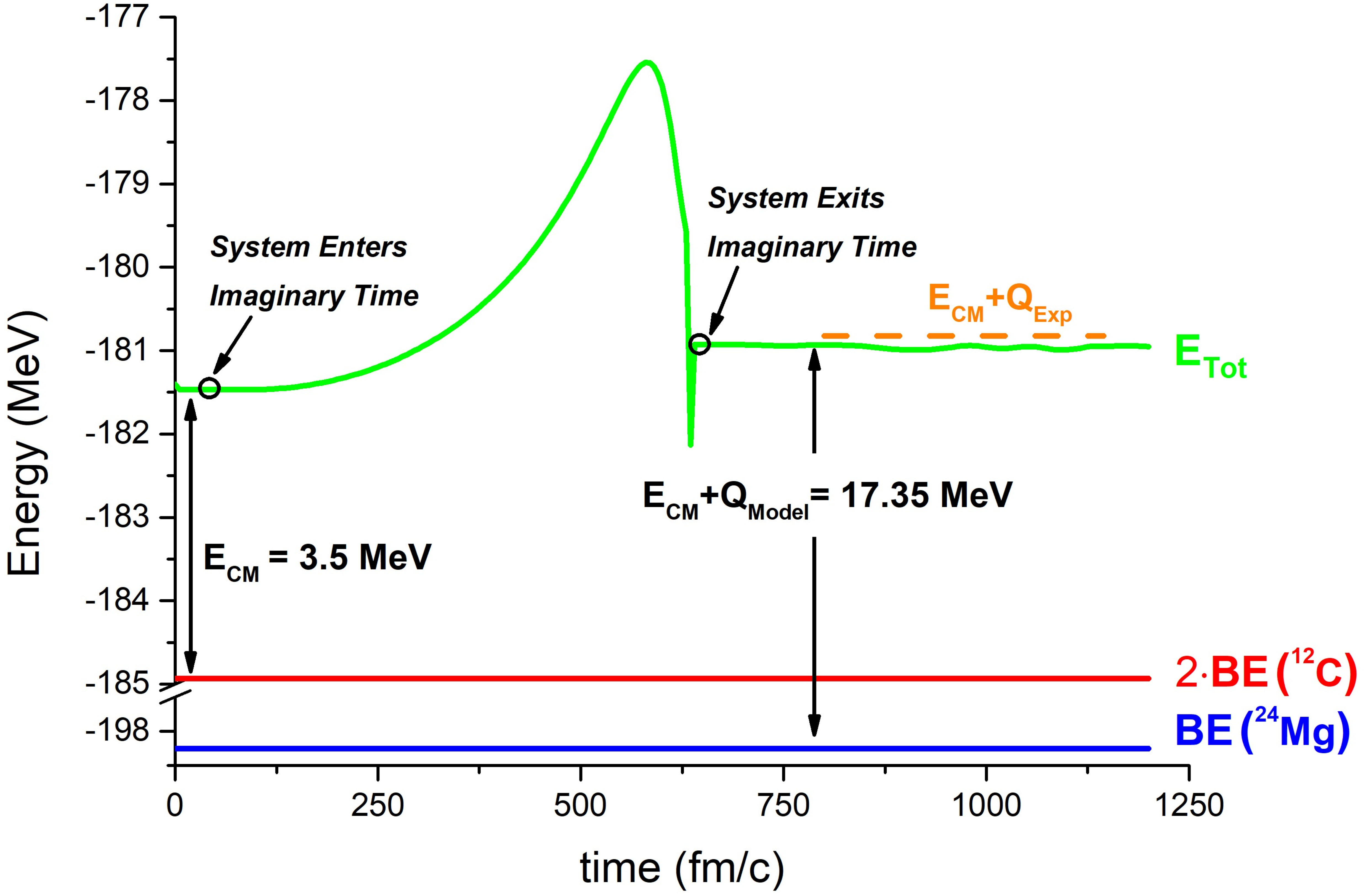
In our model a-degrees of freedom are treated explicitly while nucleon (protons and neutrons) degrees of freedom are treated implicitly hence the hac acronym. The interaction between the a-particles is given by the Coulomb repulsion (in the monopole–monopole approximation for simplicity) and the nuclear attraction. The latter is approximated as $V_{aa} = V_{Bass}(A = 4)$, i.e., the Bass potential for mass A = 4 nuclei [30]. Coulomb repulsion is not sufficient to prevent a strong overlap among a-particles. Overlapping nuclei increase the repulsion due to the combined action of the Pauli principle and Heisenberg uncertainty principle, in particular the Fermi energy (per a-particle) is given by:

$$\frac{E_F}{N_\alpha} = 4 \cdot x_F \cdot \overline{\varepsilon_F} \cdot \overline{\rho}^{2/3}.$$

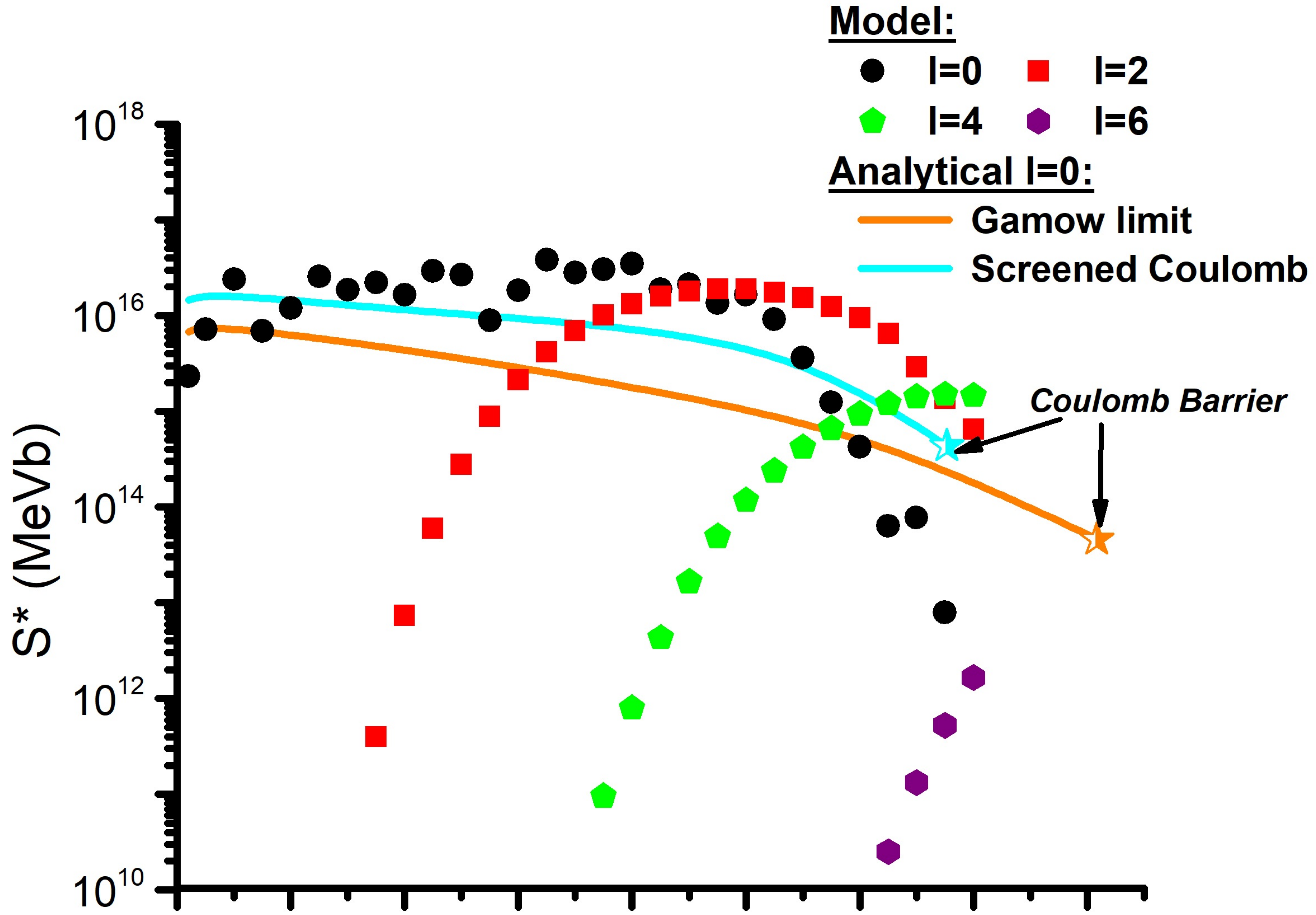




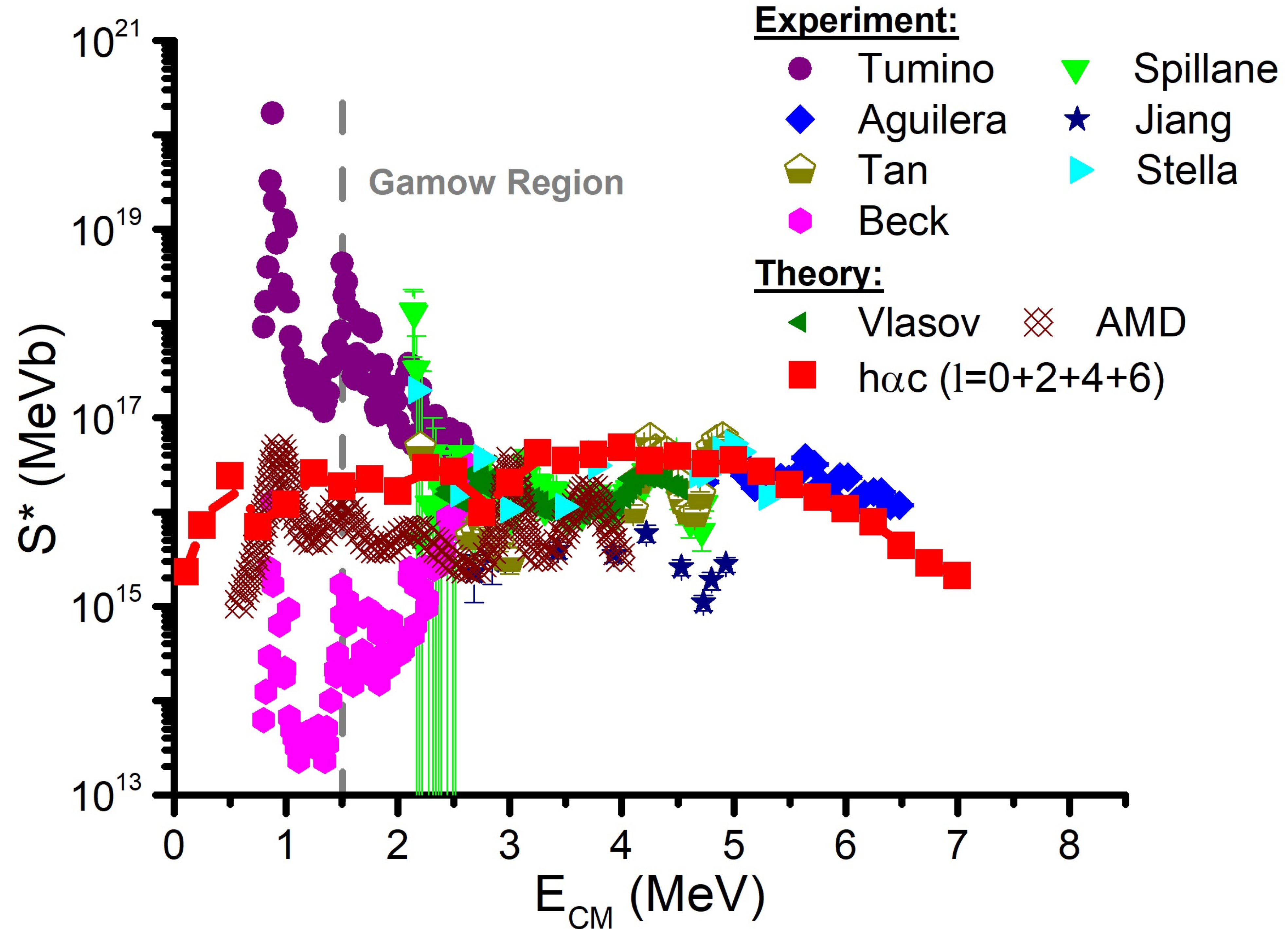




Energy (MeV)
time (fm/c)
System Enters
Imaginary Time
System Exits
Imaginary Time
$E_{CM}+Q_{Exp}$
E_{Tot}
$E_{CM}+Q_{Model}$ = 17.35 MeV
E_{CM} = 3.5 MeV
2·BE (^{12}C)
BE (^{24}Mg)
-177
-178
-179
-180
-181
-182
-183
-184
-185
-198
0
250
500
750
1000
1250



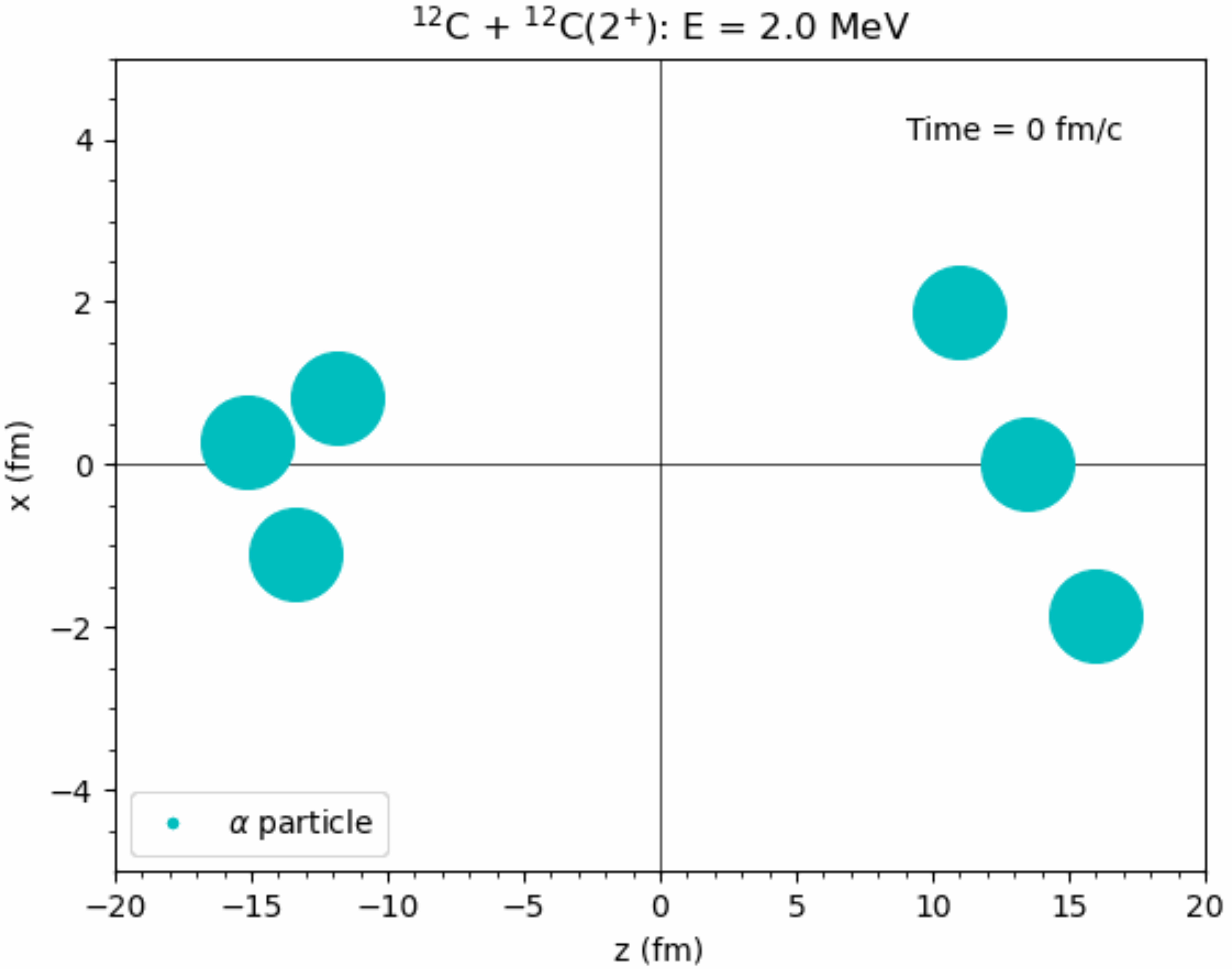
Model:
l=0
l=2
l=4
l=6
Analytical l=0:
Gamow limit
Screened Coulomb
Coulomb Barrier
S* (MeVb)
10^{18}
10^{16}
10^{14}
10^{12}
10^{10}



Experiment:
Tumino
Spillane
Aguilera
Jiang
Tan
Stella
Beck
Theory:
Vlasov
AMD
hαc (l=0+2+4+6)
Gamow Region
S* (MeVb)
E_{CM} (MeV)
10^{21}
10^{19}
10^{17}
10^{15}
10^{13}
0
1
2
3
4
5
6
7
8

(T.Depastas)

Quare id faciam fortasse requiris! 12C E*=4.44MeV l=2



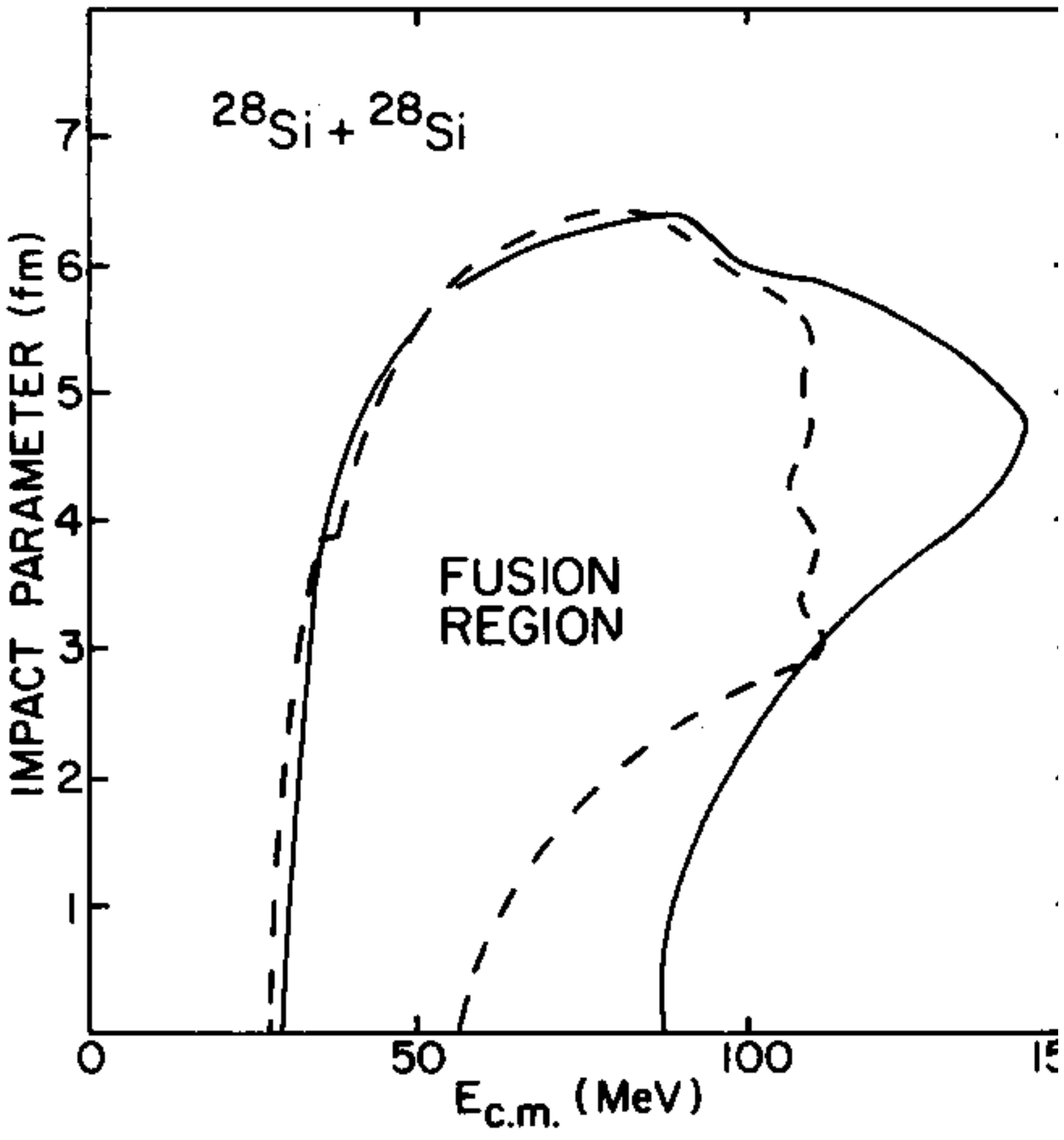
NEWTONIAN DYNAMICS OF TIME-DEPENDENT MEAN FIELD THEORY

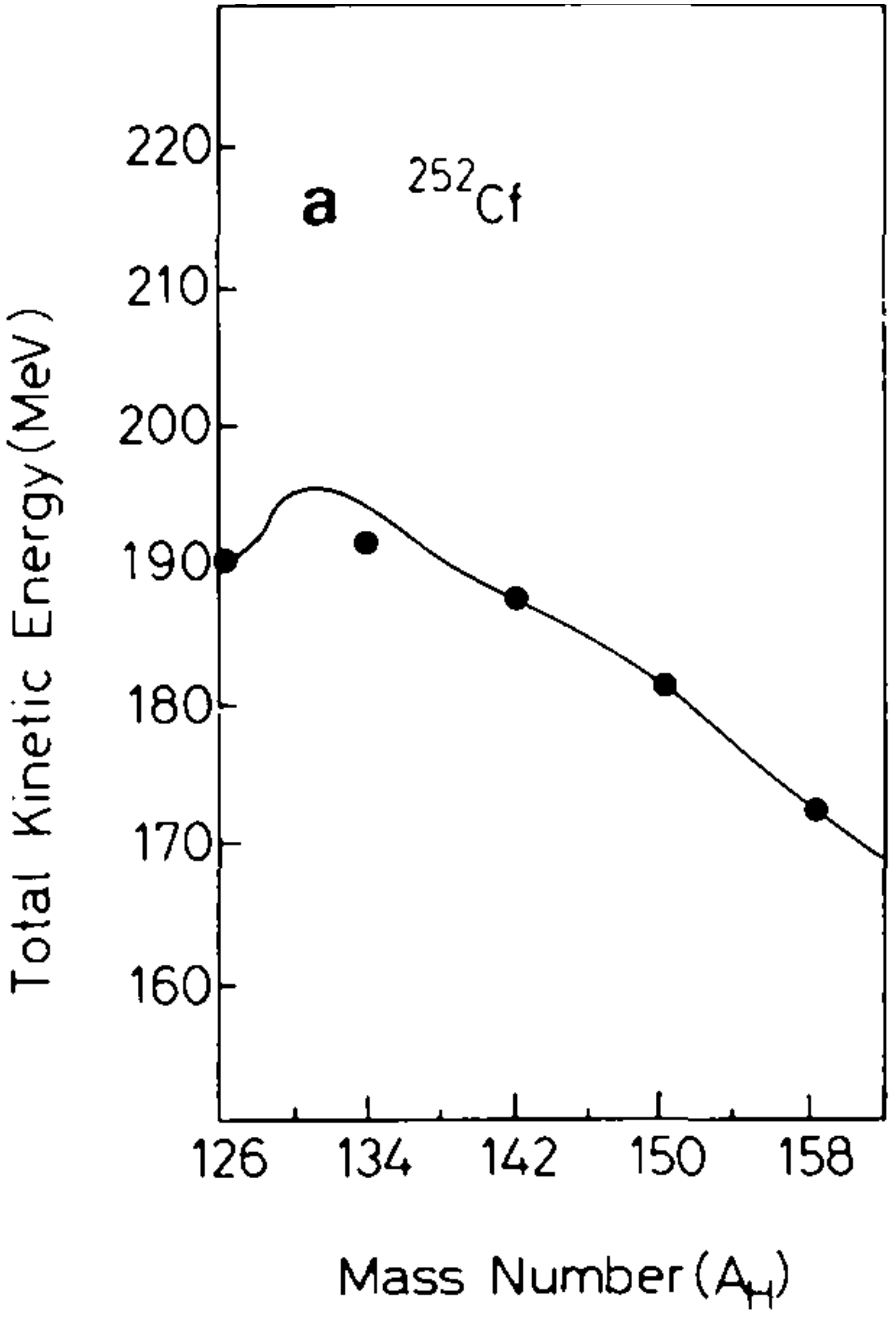
Phys.Lett.B141(1984)9; 168B(1986)35.

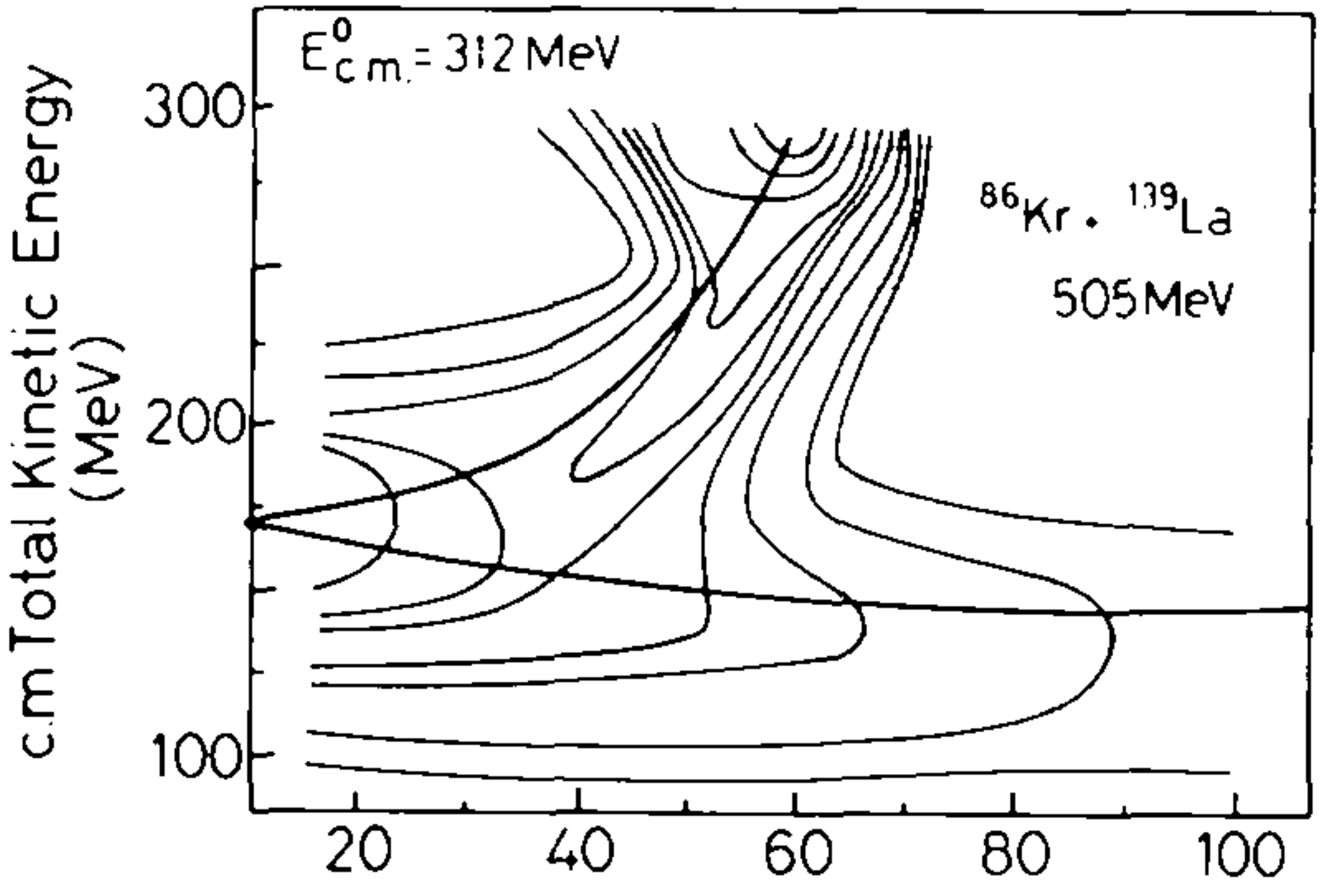
Nuclear Physics **A439** (1985) 353–370

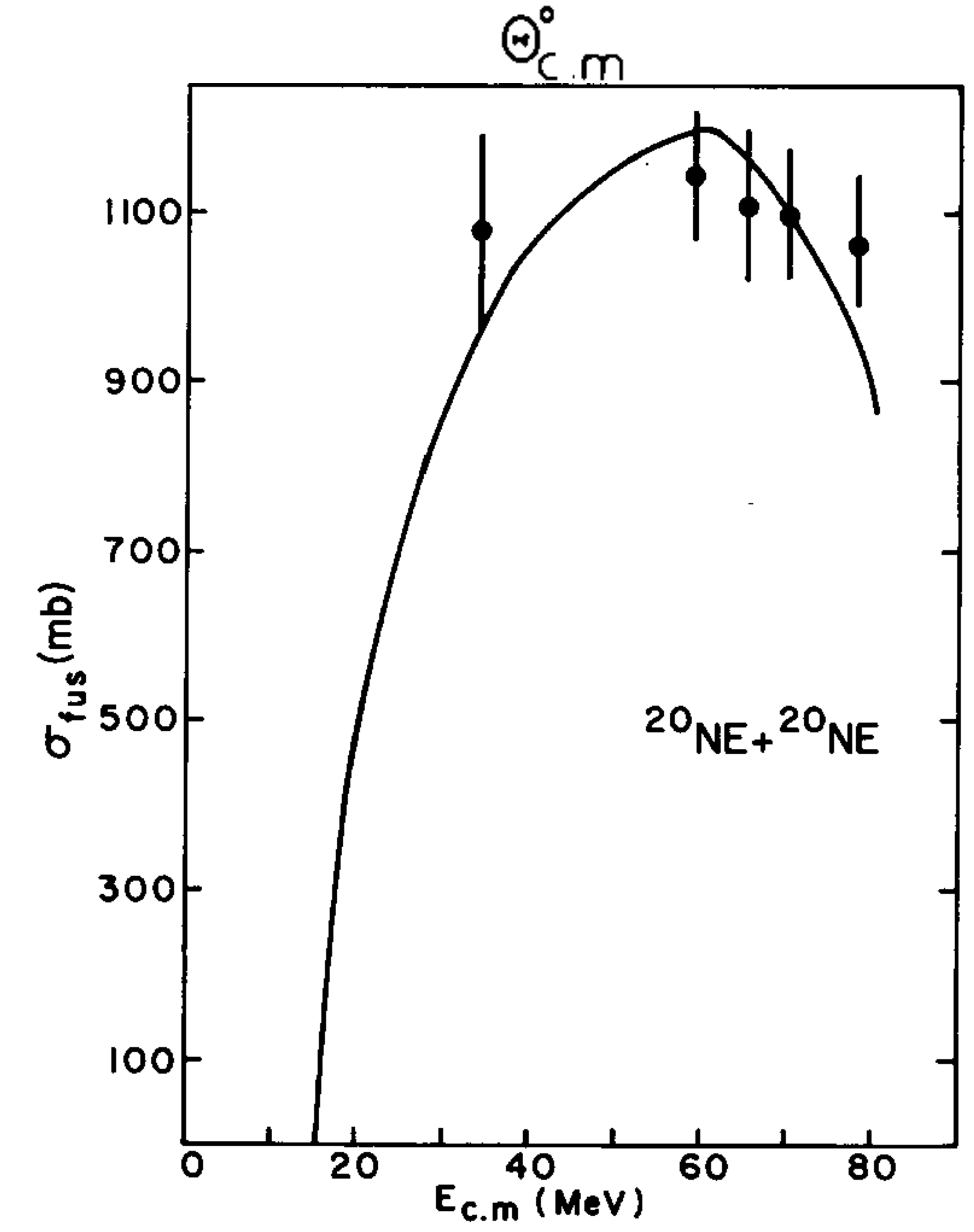
A. BONASERA, G.F. BERTSCH and E.N. EL-SAYED

Cyclotron Laboratory, Michigan State University, East Lansing, MI 48824, USA





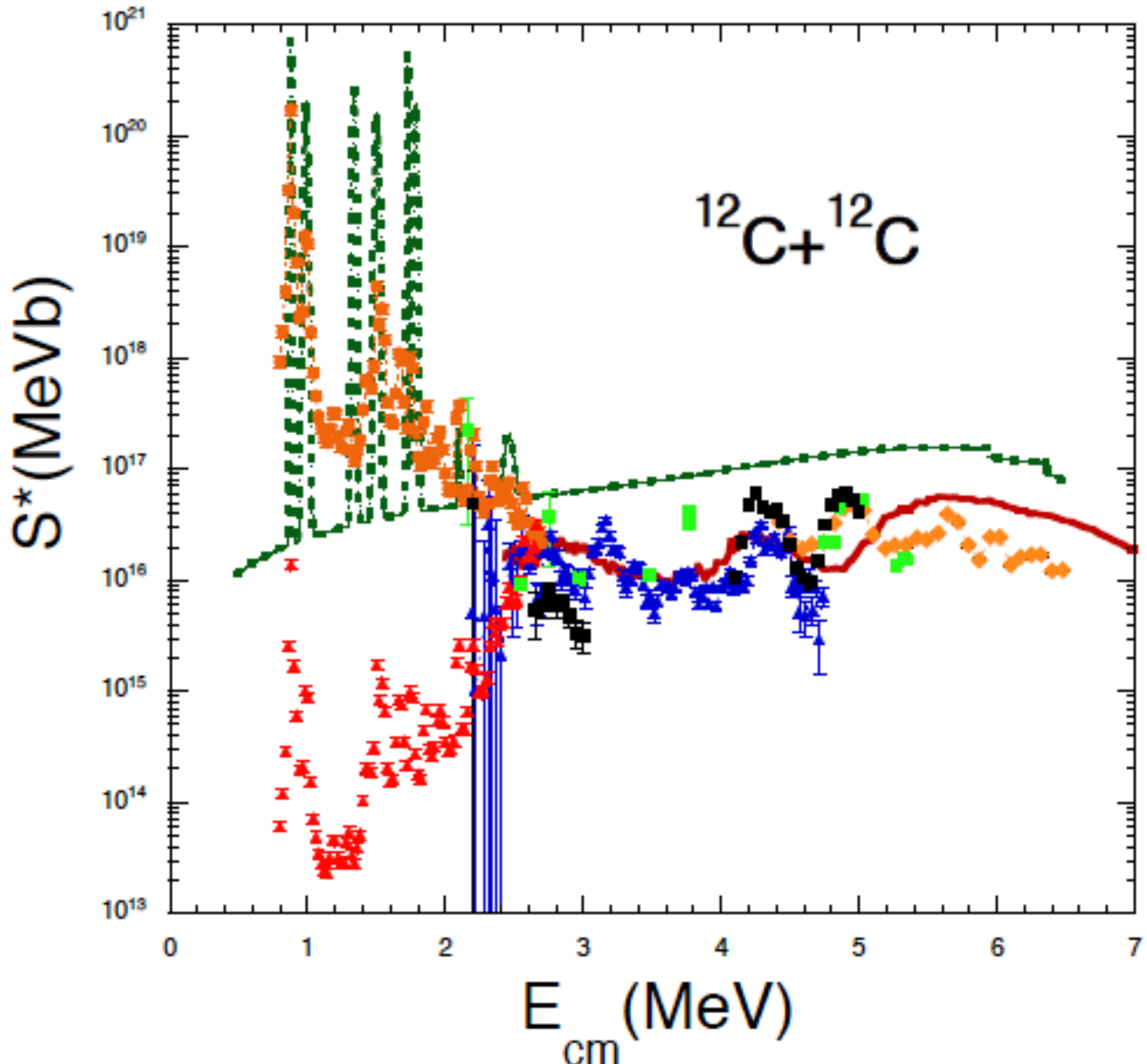




Neck model in imaginary times

PHYSICAL REVIEW C **102**, 061602(R) (2020)

The probability of fusion for the *l* th-partial wave is given by $T_l = 1/(1 + \exp\{2A\})$, $A = \int_1^2 P \, dR$.



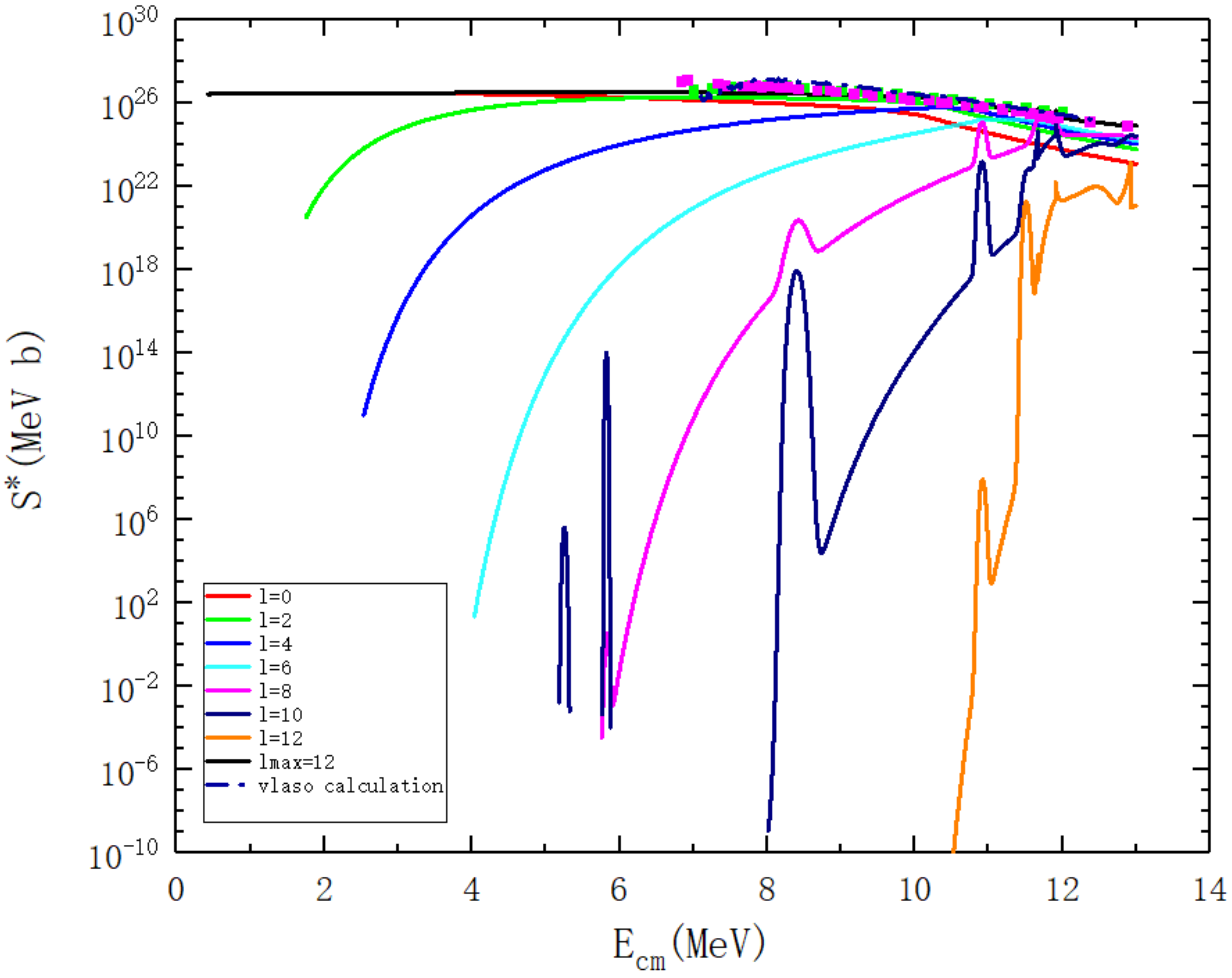
To take into account resonances modify the Bass potential as:

$$V_B \to V_B[1 + g(x, \gamma, \sigma)],$$

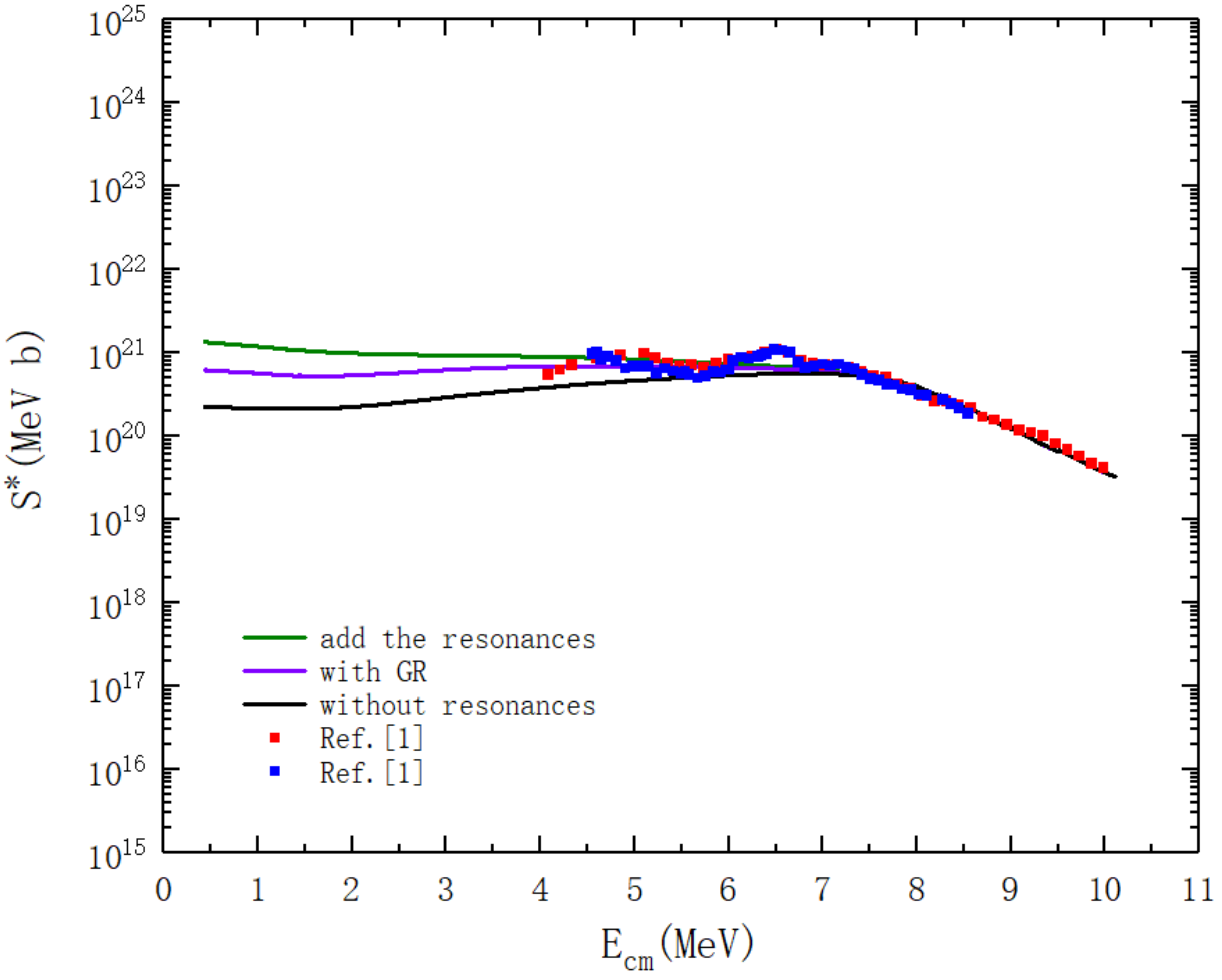
Calculation of the O16+O16 sub_barrier fusion cross section

Shuting Sun

supervisors: H. Zheng and A. Bonasera   School of Physics & Information Technology , Shaanxi Normal University

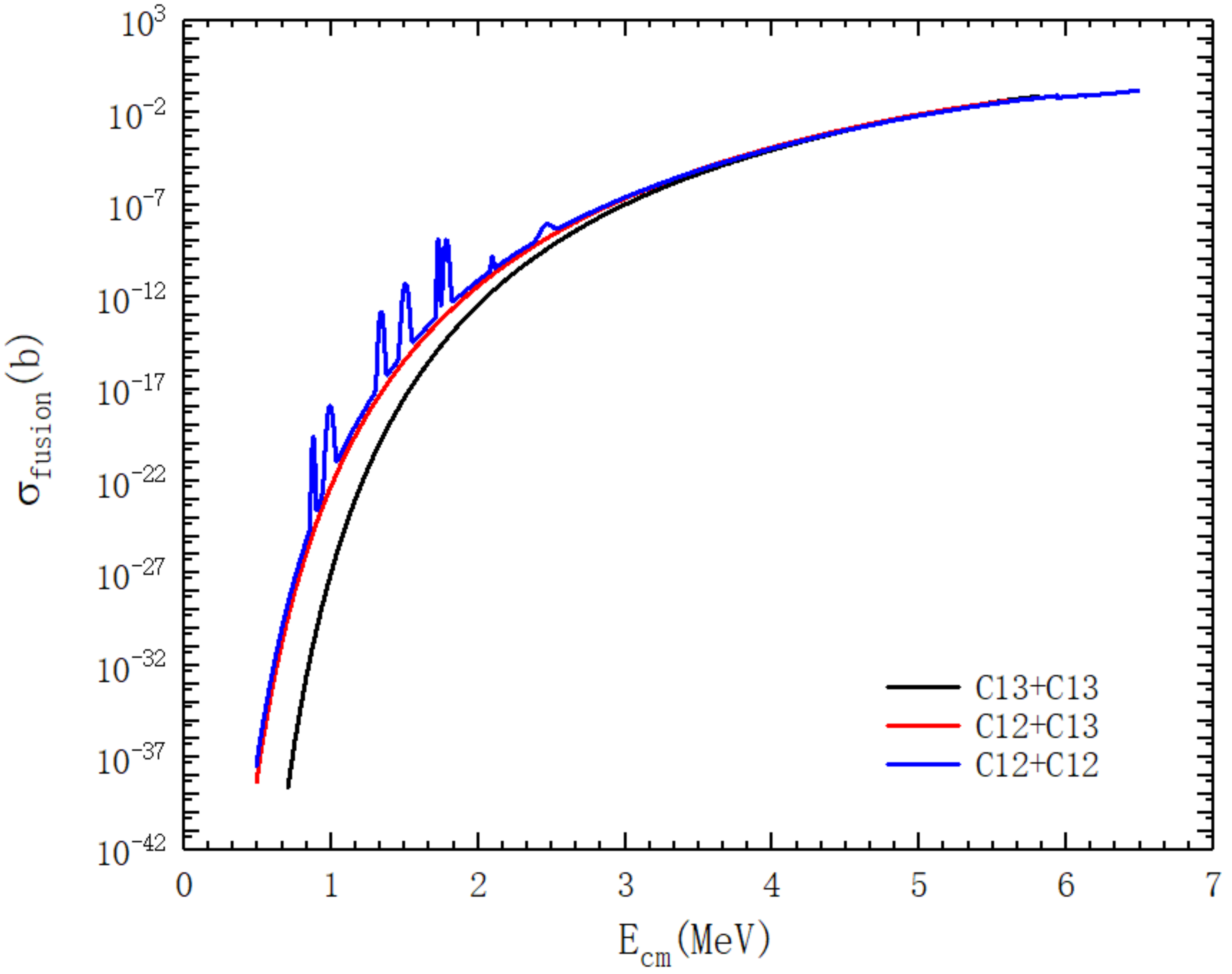


12C+16O



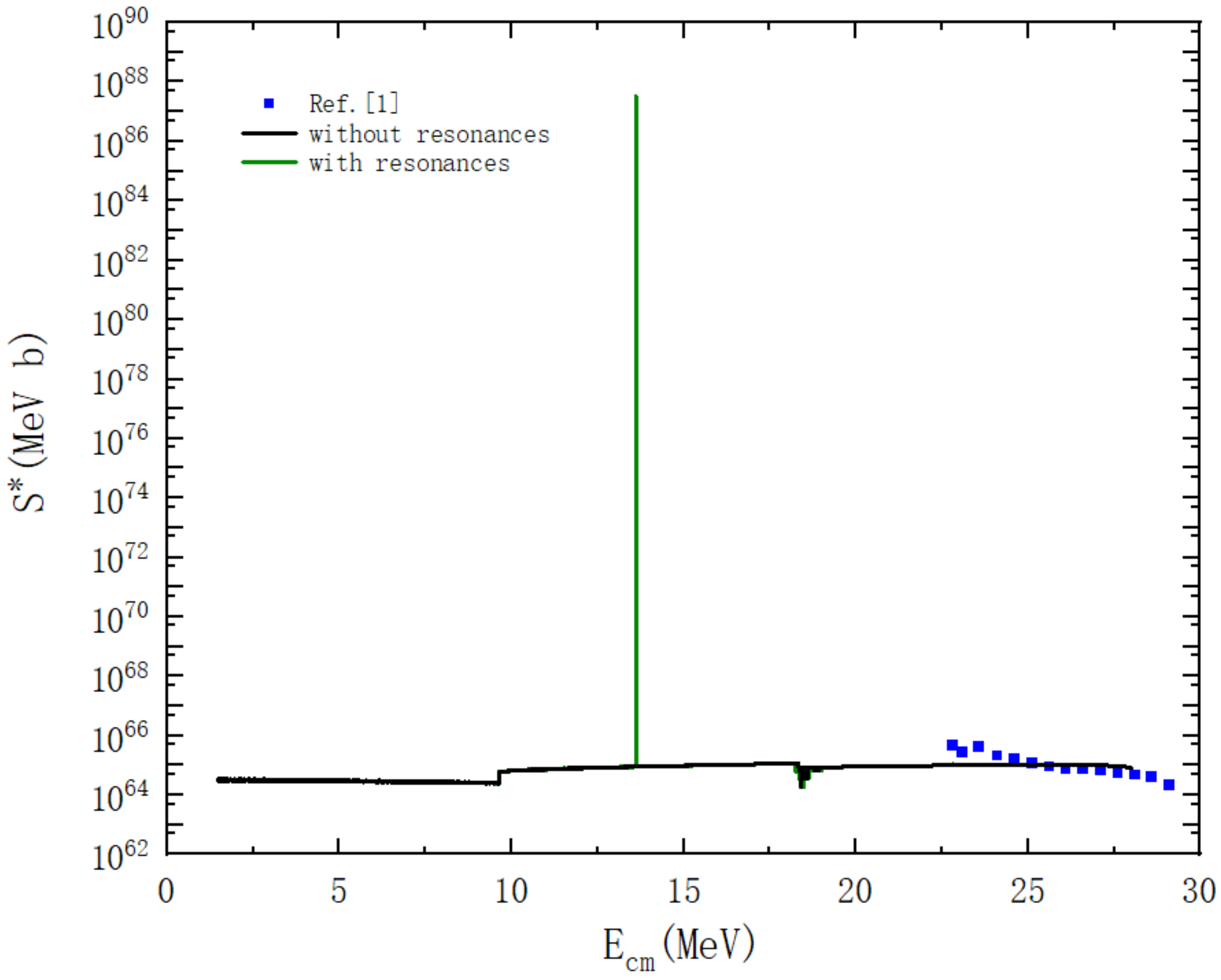
Shuting Sun

Because available resonance parameters are not found in nndc, the calculation of C12+C13 and C13+C13 does not include resonance.



Shuting Sun

Calculation of the Si28+ Si28 sub_barrierfusion cross section



Shuting Sun

Progress in Research, Fall 2022
Dynamical pair production at sub-barrier energies for light nuclei

Thomas Settlemyre, Hua Zheng, and Aldo Bonasera, arXiv:2207.06900 **[nucl-th]**

National Nuclear Security Administration

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1967 - 2017 50 YEARS OF BEAM exploring the nuclear frontier

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Schwinger mechanism

- Dirac particles approximately satisfy the Klein-Gordon Equation

$$\left(p^2+m^2\right)\psi=\left(i\hbar\frac{\partial}{\partial t}-V(x)\right)^2\psi$$

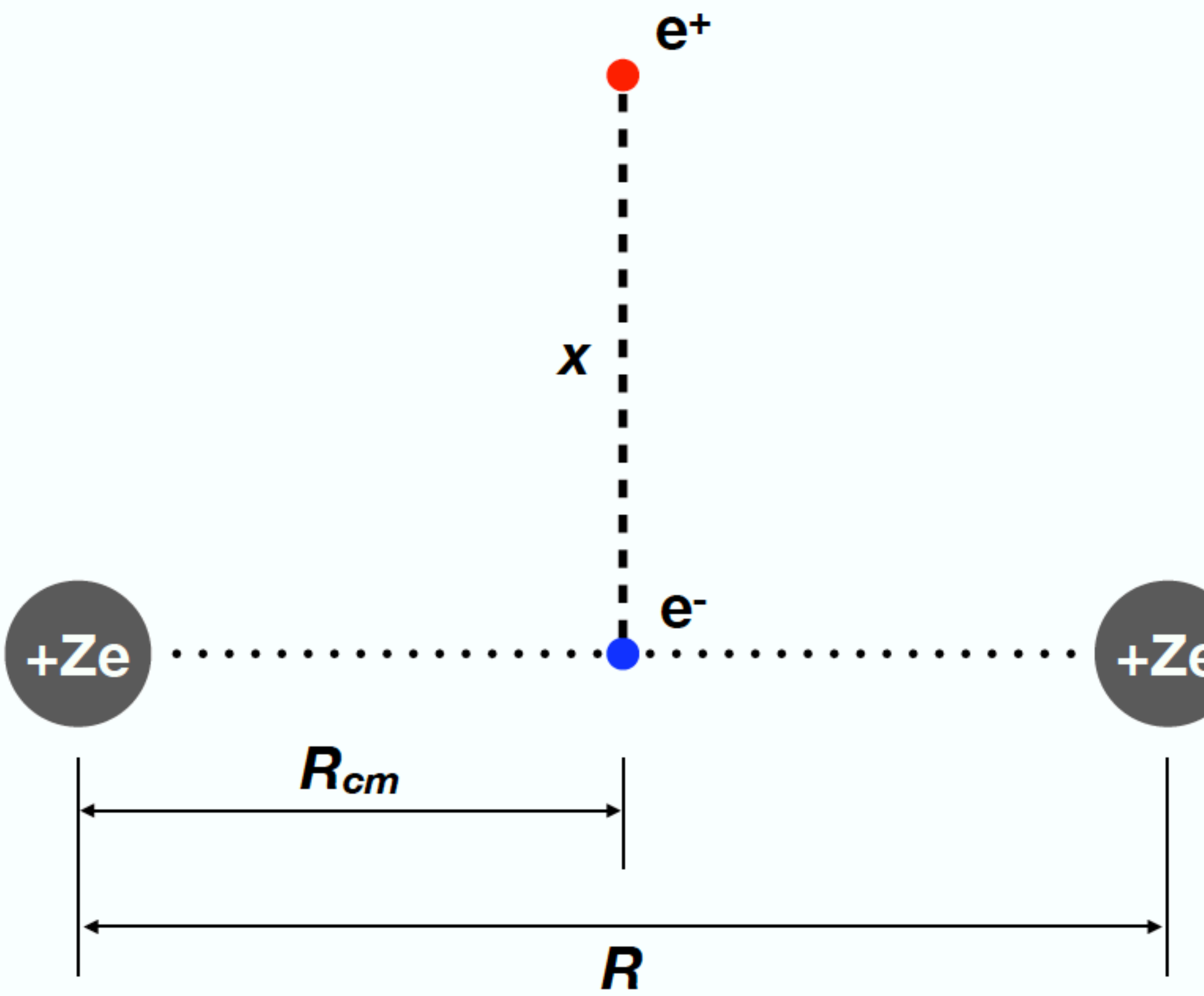
- Manipulate into a Schrödinger equation.

$$\left(\frac{p_x^2}{2m_T}+V_{eff}(x)\right)\psi=0$$

$$V_{eff}(x)=\frac{m_T}{2}-\frac{\left(E-V(x)\right)^2}{2m_T} \qquad m_T=\sqrt{m^2+p_y^2+p_z^2}$$

- Negative energy particles in Dirac sea can tunnel through the effective potential and become real.

- The two nuclei come together with impact parameter zero.
- Suppose the e^- is created at the center of mass of the two nuclei and the e^+ is on an axis perpendicular to the beam axis.
- Symmetric and energetically favorable.
- The ions get accelerated by the e^- in the middle, encouraging fusion.



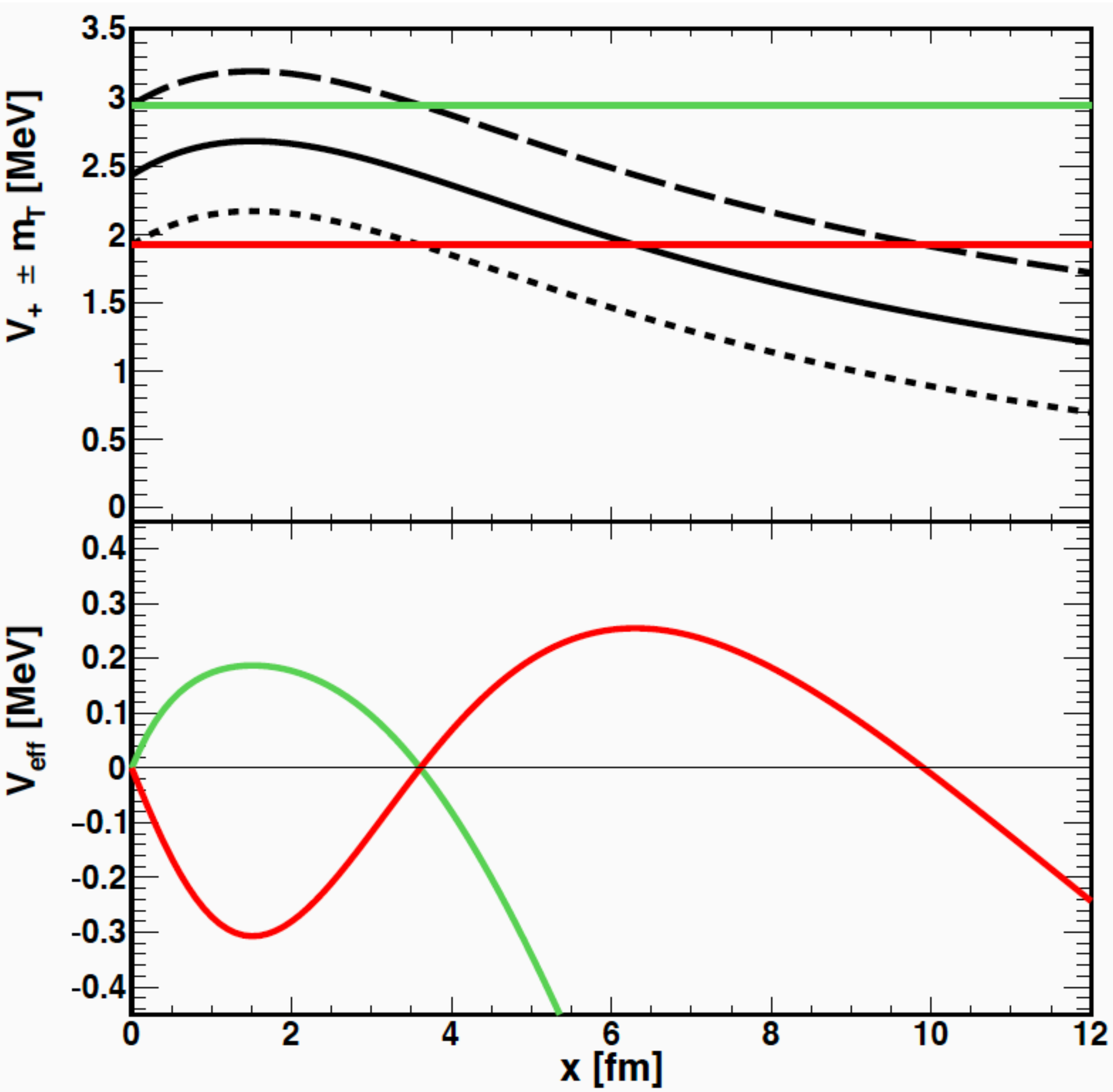
Effective potential

- Potential energy of positron

$$V_+(R,x) = \frac{2Ze^2}{\sqrt{\left(\frac{R}{2}\right)^2 + x^2}} - S(x)\frac{e^2}{x},$$

- Effective potential

$$V_{eff}(x) = \frac{m_T}{2} - \frac{(E_+ - V_+(R,x))^2}{2m_T},$$

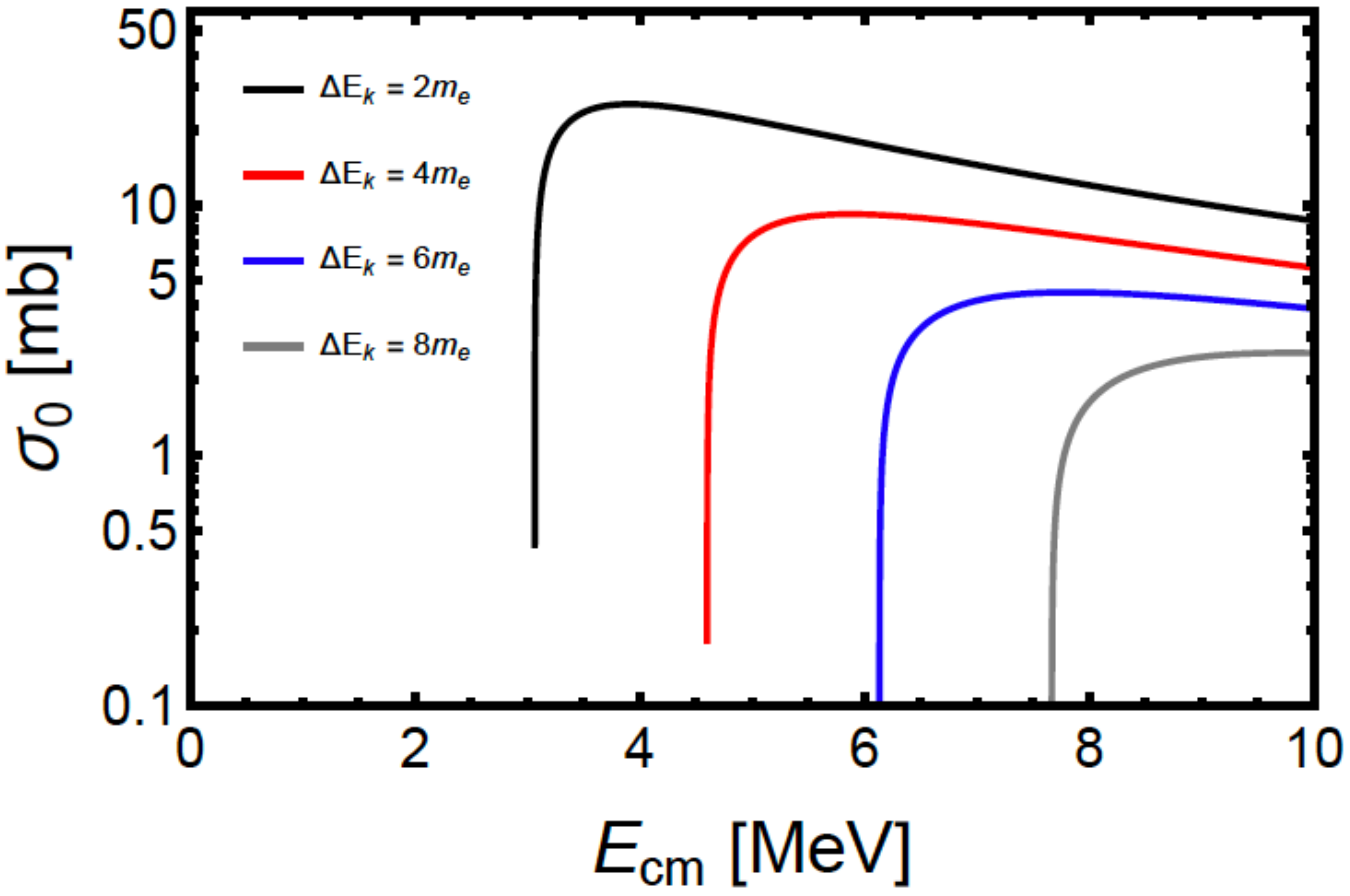


Results

$$\sigma(E_{c.m.}) = \frac{\pi\hbar^2}{2\mu E_{c.m.}} \sum_{l=0}^{n} (2l+1)\Pi_l P_H.$$

$$P_H = \tau/\Delta\tau, \qquad \Delta\tau = \frac{\hbar}{2m_T},$$

- Square root in σ gives minimum value for E_{cm}



$$\begin{aligned}\sigma_0(E_{c.m.}) &= \frac{\pi\hbar^2}{2\mu E_{c.m.}} 0.5 \frac{\tau}{\Delta\tau} \\ &= \frac{1}{N_{\pm}^{max}} \frac{6\pi\hbar Z e^2 m_e}{\sqrt{2\mu} E_{c.m.}^2 (\Delta E_k + 2m_e)} \\ &\times \sqrt{E_{c.m.} - \frac{Z}{4}(\Delta E_k + 2m_e)}.\end{aligned}$$

Pair production as a probe for the dynamics of nuclear fission and α decay

T. Settlemyre,[1] H. Zheng,[2] and A. Bonasera[1,3]

[1] Cyclotron Institute, Texas A&M University, College Station, Texas 77843, USA
[2] School of Physics and Information Technology, Shaanxi Normal University, Xi'an 710119, China
[3] Laboratori Nazionali del Sud, INFN, via Santa Sofia, 62, 95123 Catania, Italy

Electron-positron pairs can be produced via the Schwinger mechanism in the presence of strong electric fields. In particular, the fields involved in α decay and nuclear fission are strong enough to produce them. The energy of the e^+e^- pair is related to the relative distance and velocity of the daughter nuclei. Thus, the energy distribution of the produced pairs can give information about the dynamics of the fission and α decay processes. A neck model of nuclear fission is used to illustrate how the pairs can be used as a probe of the dynamics.

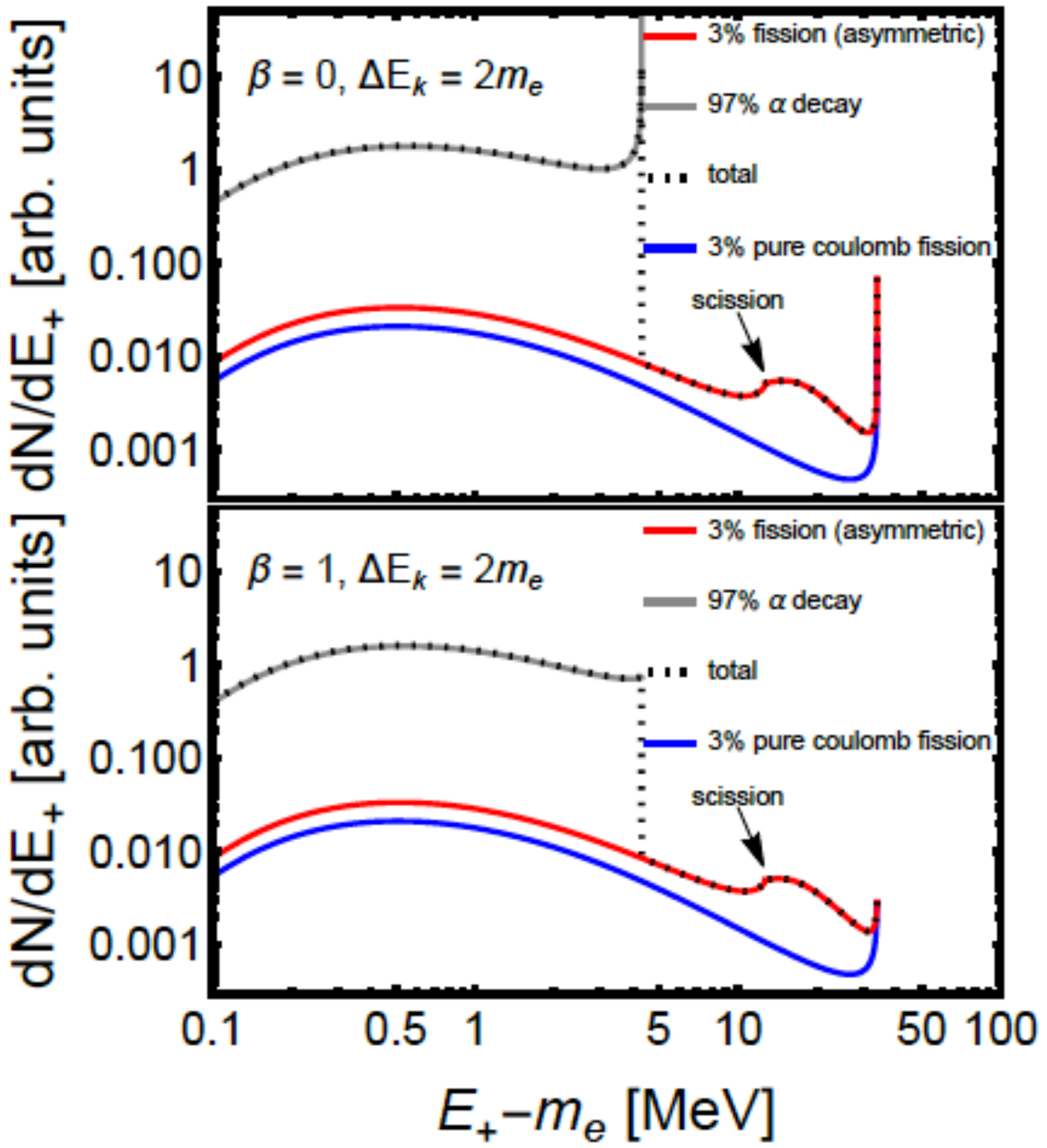


FIG. 3: dN/dE_+ for fission and α decay of ^{252}Cf, weighted by the corresponding branching ratios. Here, $m_T = m_e$ and $\Delta E_k = 2m_e$. In the top panel we assume that the extra acceleration (ΔE_k) is given to the ions after the pair creation resulting in the divergence at time zero, $E_+ \approx 30$ MeV, $\beta = 0$. The Coulomb divergence disappears if $\beta = 1$, i.e. the ions are accelerated at the beginning of the pair production process (bottom panel). Similarly for α decay.

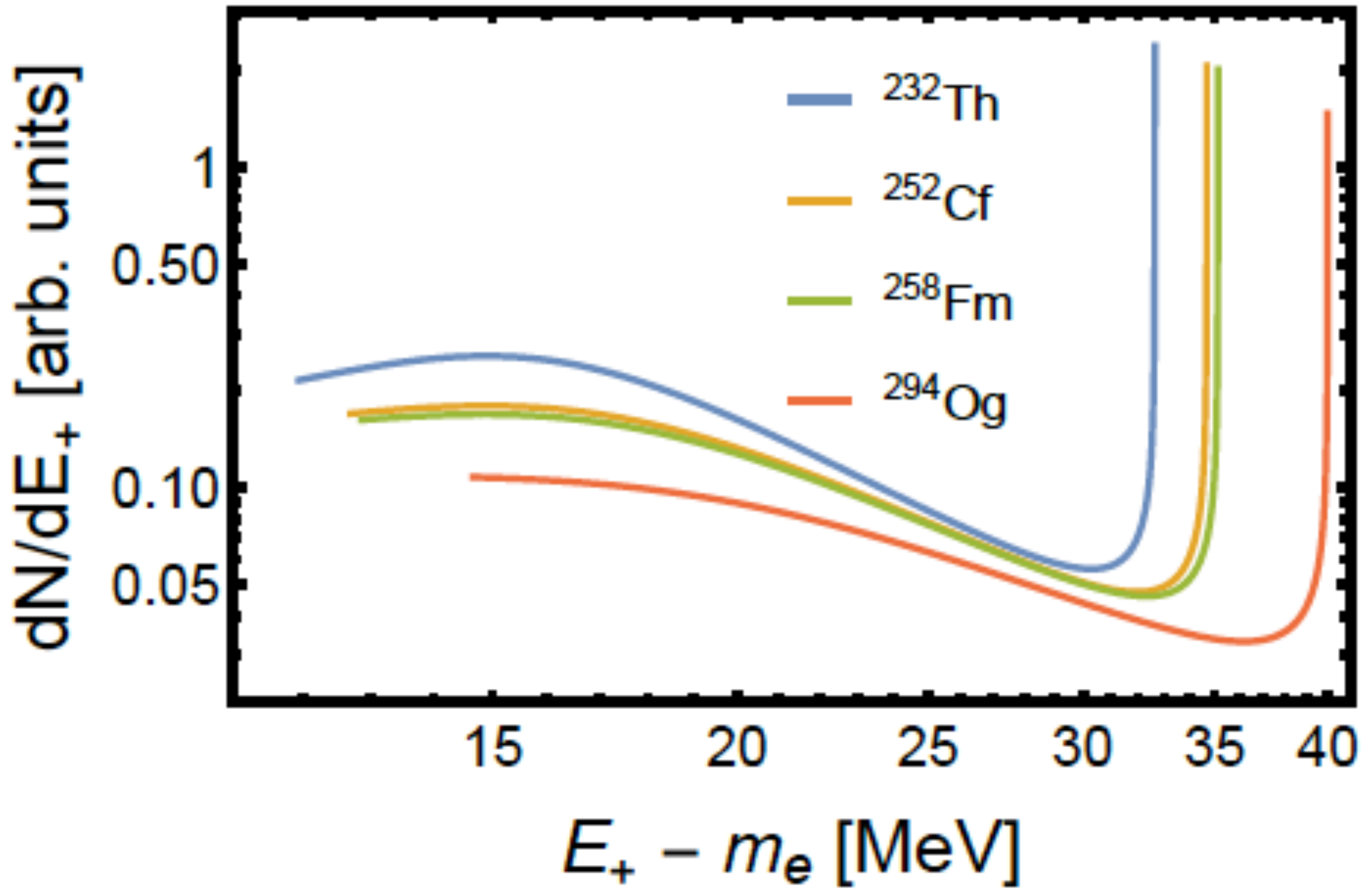


FIG. 4: dN/dE_+ for fission of a selection of nuclei. The fission dynamics are based on the neck model. Here, $m_T = m_e$, $\beta = 0$ and $\Delta E_k = 2m_e$.

Conclusions

12C+12C: The Neck model and the Vlasov approach in imaginary time give S*>e16MeVb for Ecm>0.5 MeV

Adding resonances is in some agreement with the THM

I=0 channel is dominant up to Ecm=3MeV

For heavier systems resonances not so important because of the large Q-value

Microscopic models in progress and some interesting results have been obtained for fusions in C stars.

Pair production in heavy and superheavy nuclei plus other systems

Applications to superheavy formation

THANKS

CENTER FOR EXCELLENCE IN NUCLEAR TRAINING AND UNIVERSITY-BASED RESEARCH

DOE grant: DE-FG03-93ER40773