Calculation of light nuclei sub-barrier fusion cross section in an imaginary time-dependent mean field theory

Aldo Bonasera
Cyclotron Institute Texas A&M University, College Station, TX-USA
LNS-INFN, Catania-Italy

Introduction

Microscopic calculations: Vlasov dynamics in imaginary times.
Future directions-Constrained Molecular Dynamics (CoMD)

Macroscopic calculations: Neck model in imaginary times.

Experimental challenges: pair production in sub barrier fusion reactions

Conclusions

Calculation of the $^{12}C + ^{12}C$ sub-barrier fusion cross section in an imaginary-time-dependent mean field theory

PHYSICAL REVIEW C **102**, 061602(R) (2020)

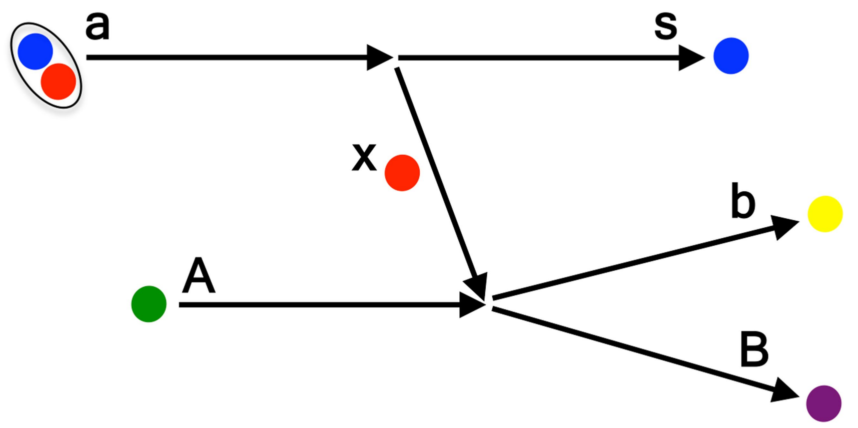
A. Bonasera [1,2] and J. B. Natowitz[1]
[1]*Cyclotron Institute, Texas A&M University, College Station, Texas 77843, USA*
[2]*Laboratori Nazionali del Sud-INFN, v. Santa Sofia 64, 95123 Catania, Italy*

INTRODUCTION&MOTIVATION

Letter 688| Nature | VOL557 | 31MAY2018

An increase in the $^{12}C + ^{12}C$ fusion rate from resonances at astrophysical energies

A. Tumino[1,2*], C. Spitaleri[2,3], M. La Cognata[2], S. Cherubini[2,3], G. L. Guardo[2,4], M. Gulino[1,2], S. Hayakawa[2,5], I. Indelicato[2], L. Lamia[2,3], H. Petrascu[4], r. G. Pizzone[2], S. M. r. Puglia[2], G. G. rapisarda[2], S. romano[2,3], M. L. Sergi[2], r. Spartá[2] & L. Trache[4]



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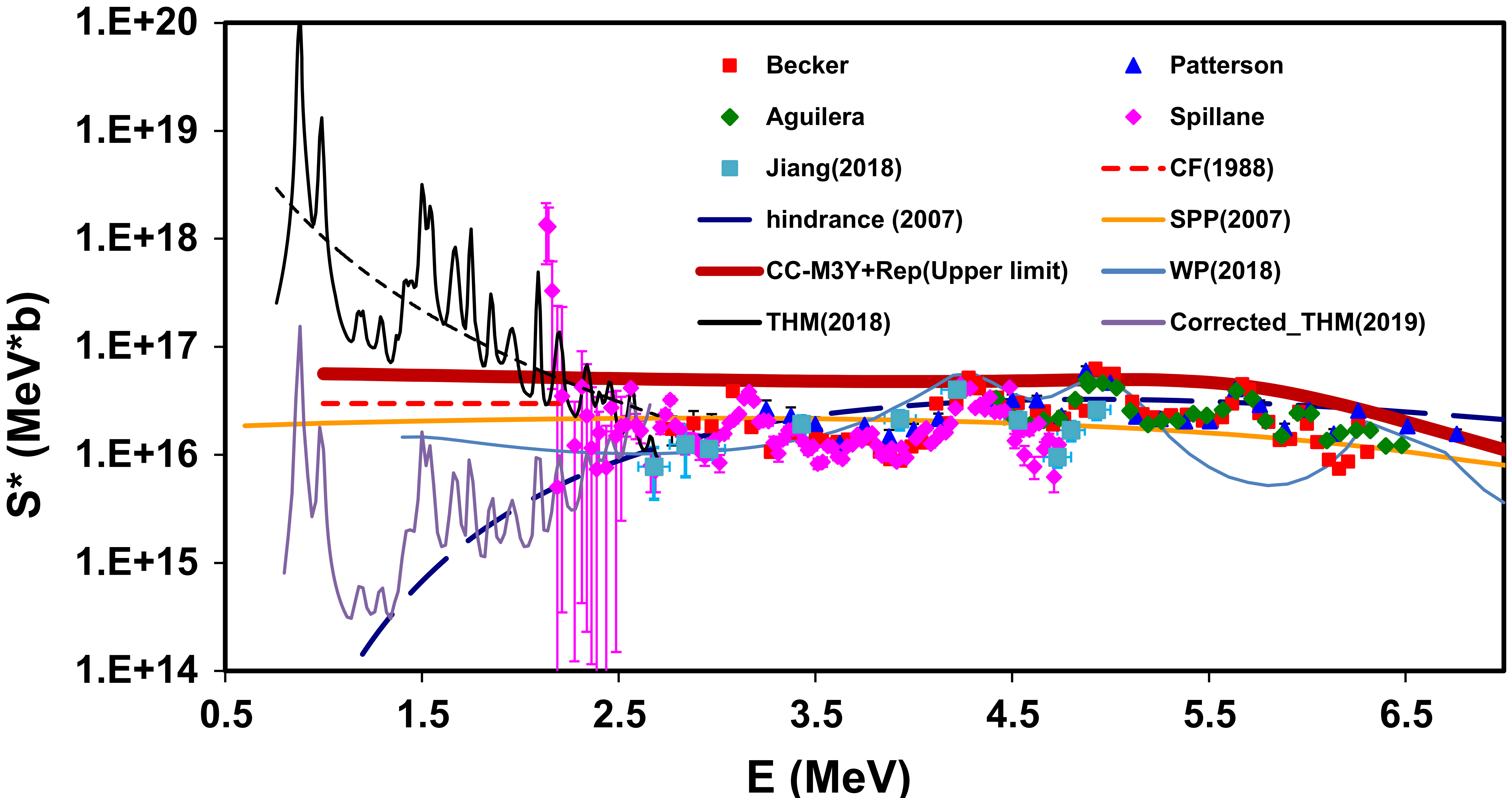


Status on $^{12}C + ^{12}C$ fusion at deep subbarrier energies: impact of resonances on astrophysical S^* factors

C. Beck[1,a], A. M. Mukhamedzhanov[2,b], X. Tang[3,4,c]

Eur. Phys. J. A (2020) 56:87

$$S^*(E_{c.m.}) = E_{c.m.}\sigma(E_{c.m.})\exp\left(87.12E_{c.m.}^{-1/2} + 0.46E_{c.m.}\right)$$
$$= S(E_{c.m.})\exp(0.46E_{c.m.}) \qquad (1)$$

Feynman path integration in phase space

Physics Letters B 339 (1994) 207–210

Aldo Bonasera, Vladimir N. Kondratyev [1]

Laboratorio Nazionale del Sud, INFN, v. S. Sofia 44, 95125 Catania, Italy

Phys.Rev.Lett.78(1997)187

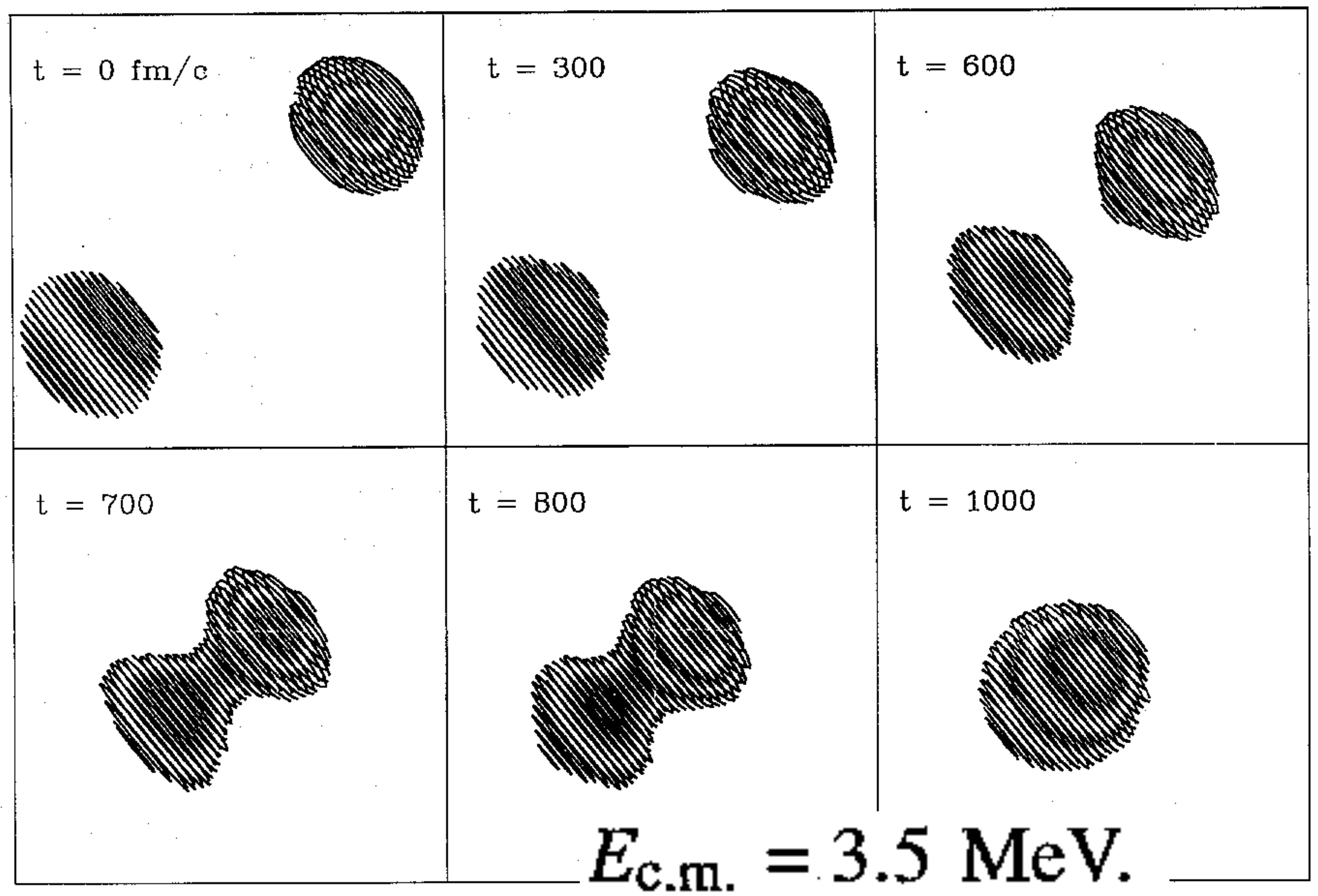
Solve the Vlasov equation in imaginary time. Define collective variables R&P

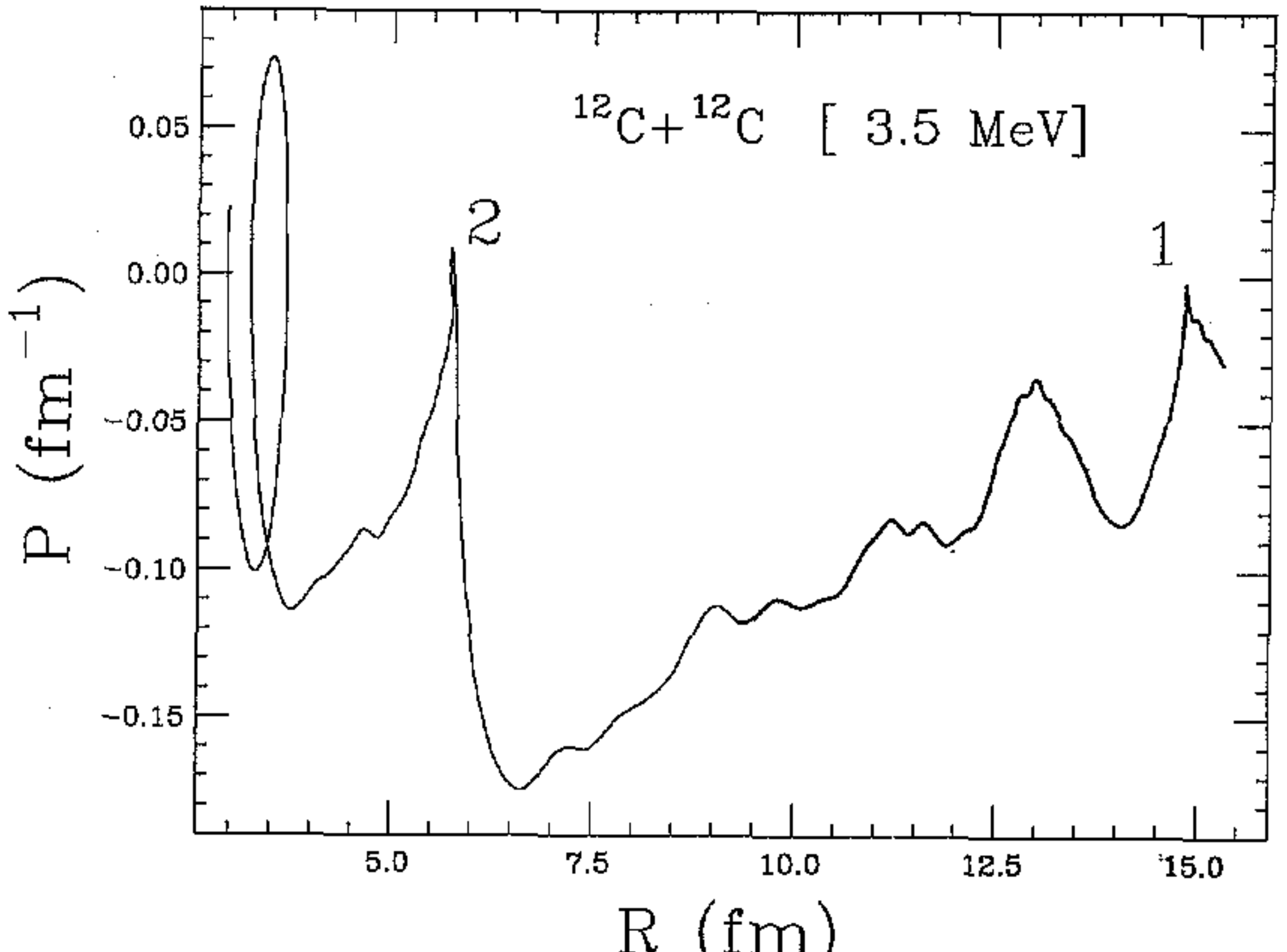
$$\begin{Bmatrix} \boldsymbol{R} \\ \boldsymbol{P} \end{Bmatrix} = \int_A d\boldsymbol{r}d\boldsymbol{p} \begin{Bmatrix} \boldsymbol{r} \\ \boldsymbol{p} \end{Bmatrix} f(\boldsymbol{r},\boldsymbol{p};\,t) - \int_B d\boldsymbol{r}d\boldsymbol{p} \begin{Bmatrix} \boldsymbol{r} \\ \boldsymbol{p} \end{Bmatrix} f(\boldsymbol{r},\boldsymbol{p};\,t)$$

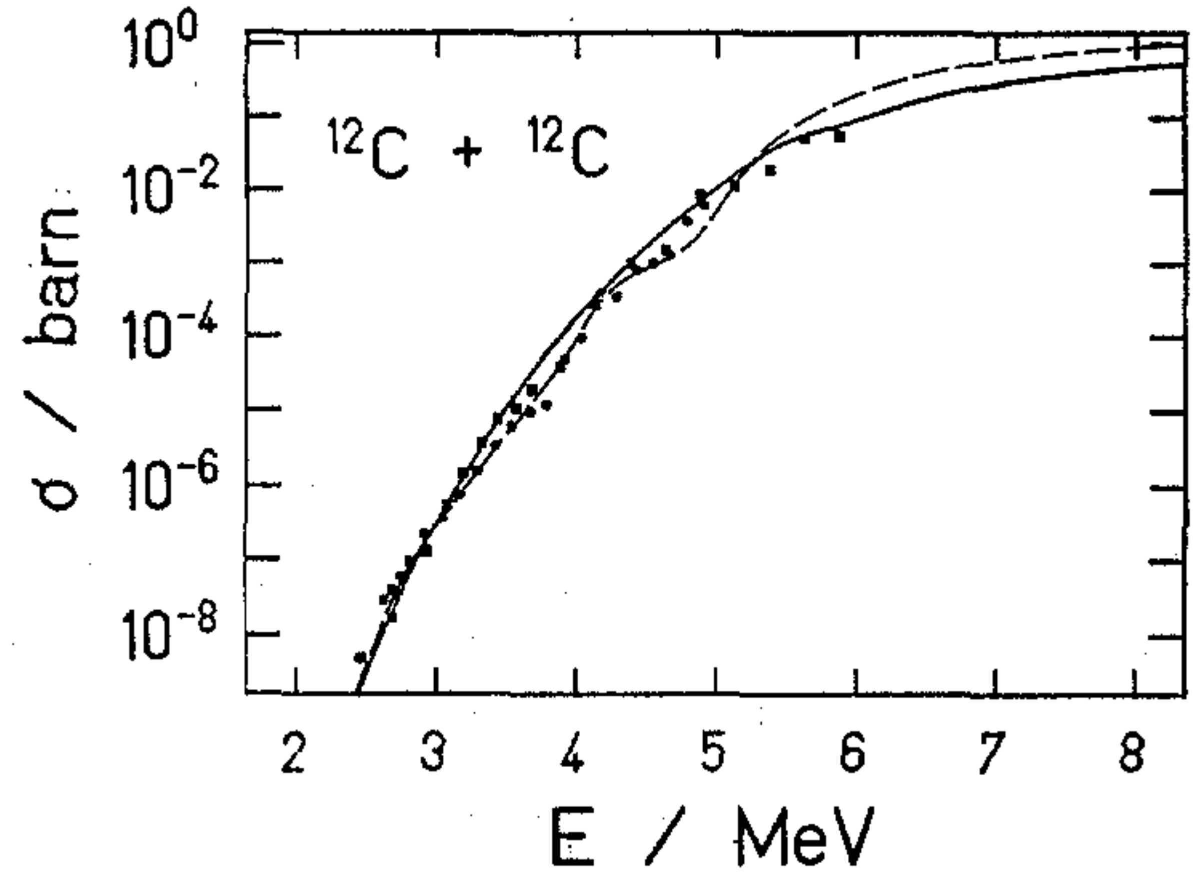
$$\frac{d\boldsymbol{R}_{A(B)}}{dt} = \frac{\boldsymbol{P}_{A(B)}}{m}; \quad \frac{d\boldsymbol{P}_{A(B)}}{dt} = \boldsymbol{F}_{A(B)}$$

in imaginary time t—>it

$$\frac{d\boldsymbol{R}^i_{A(B)}}{dt} = \frac{\boldsymbol{P}^i_{A(B)}}{m}; \quad \frac{d\boldsymbol{P}^i_{A(B)}}{dt} = -\boldsymbol{F}_{A(B)}$$







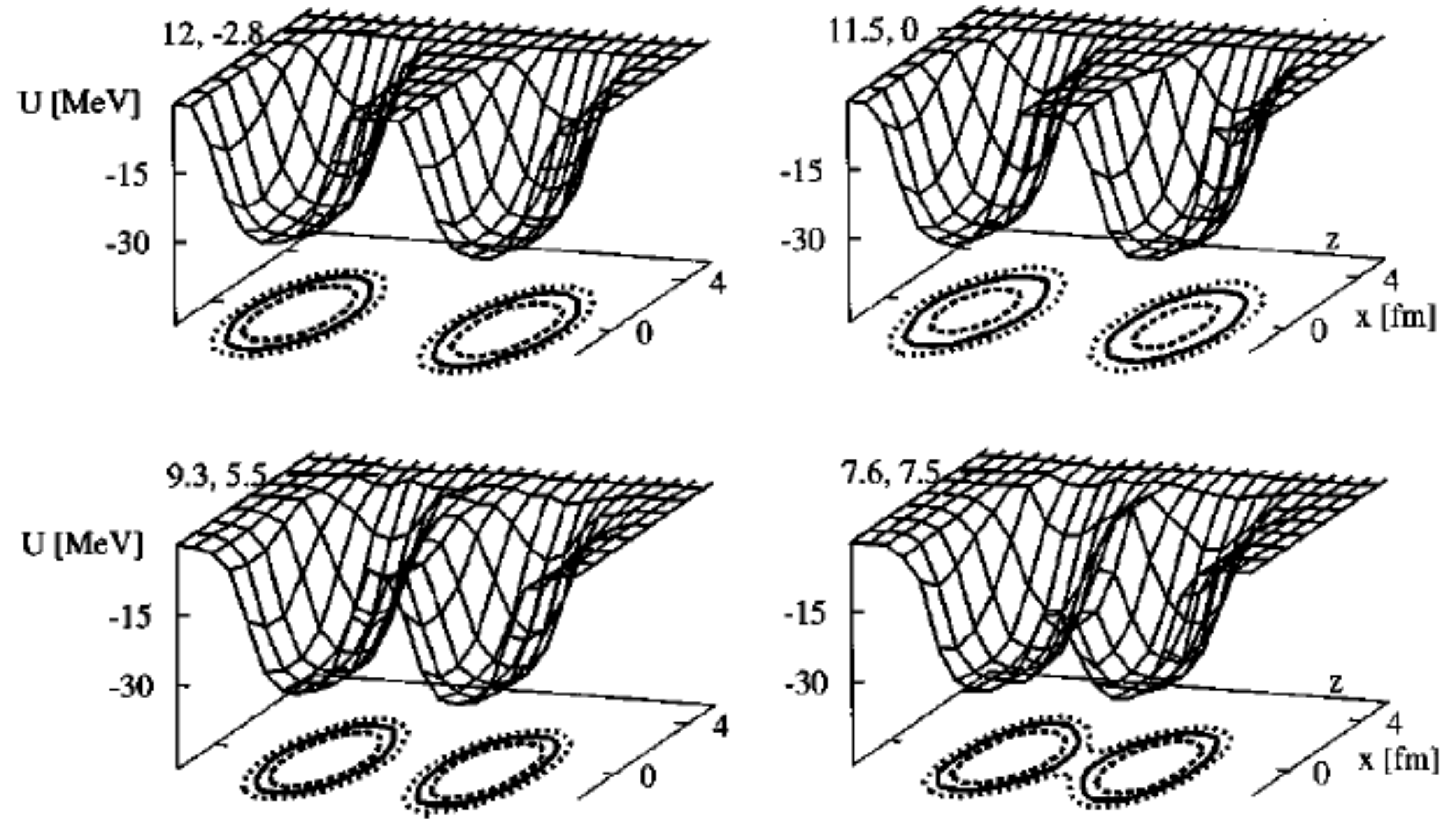


FIG. 1. Snapshots from the mean-field simulation of the fusion reaction for a head-on $^{16}O+^{16}O$ collision at the energy 8 MeV. The

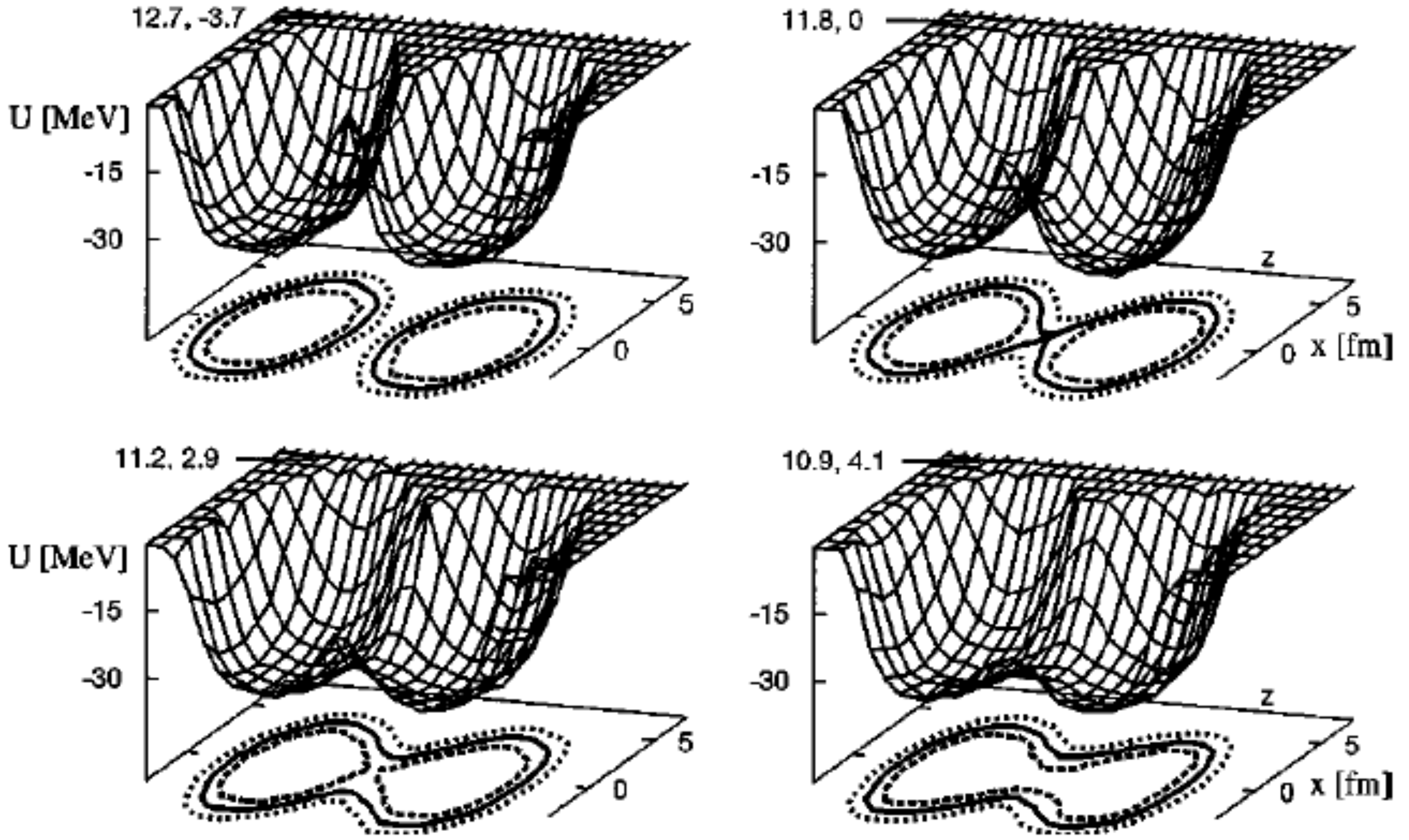


FIG. 2. The same as Fig. 1 for a $^{58}Ni+^{58}Ni$ collision at the energy 93 MeV.

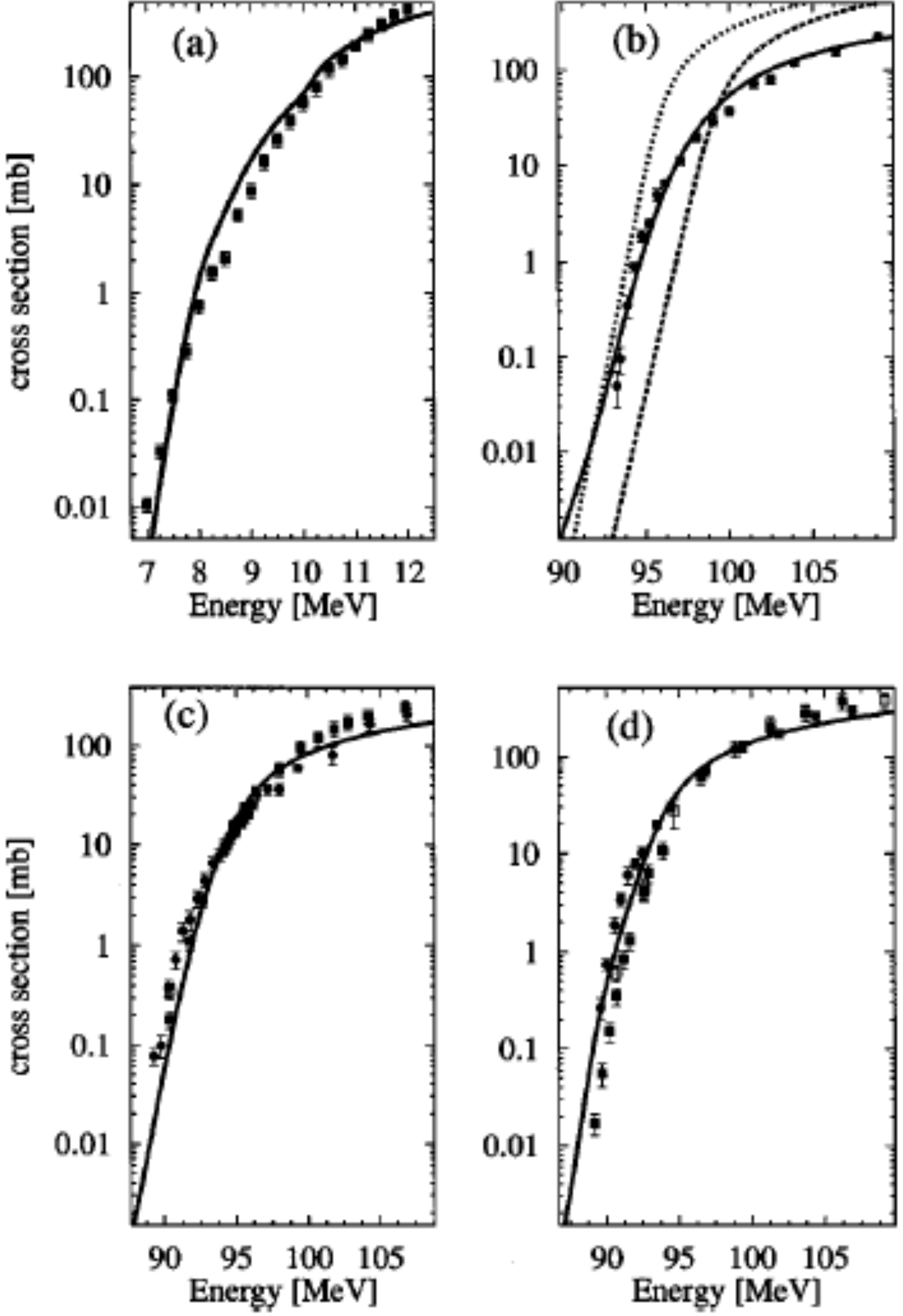
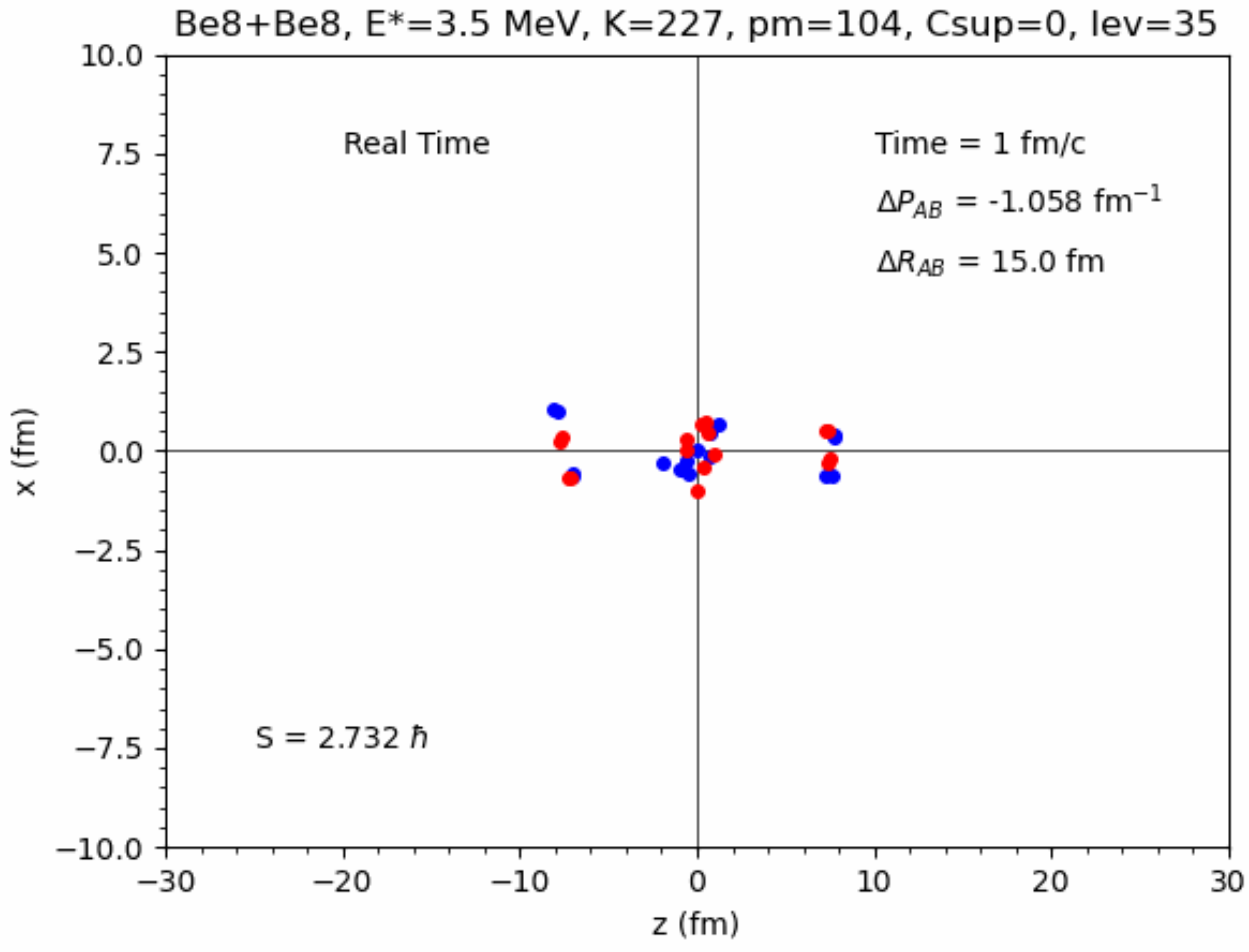


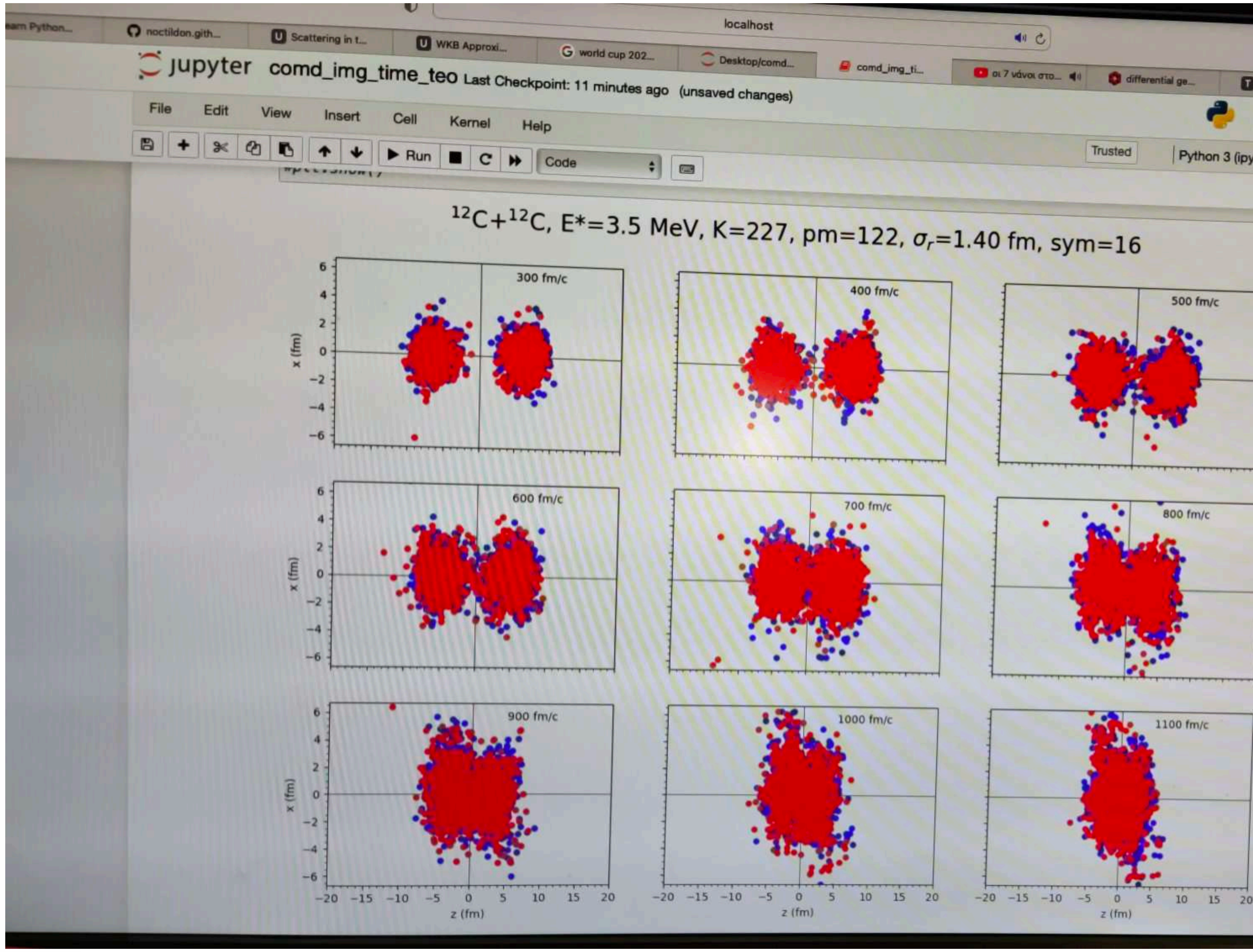
FIG. 5. The fusion excitation functions. Part (a) represents the $^{16}O+^{16}O$ system with the experimental data from Ref. [47]. Parts (b)–(d) display the results for the isotope pairs: (b) $^{58}Ni+^{58}Ni$; (c) $^{58}Ni+^{64}Ni$; and (d) $^{64}Ni+^{64}Ni$. The circles show the experimental

Microscopic calculations in progress- T. Depastas

Preliminary attempts using CoMD

possibly wrong!





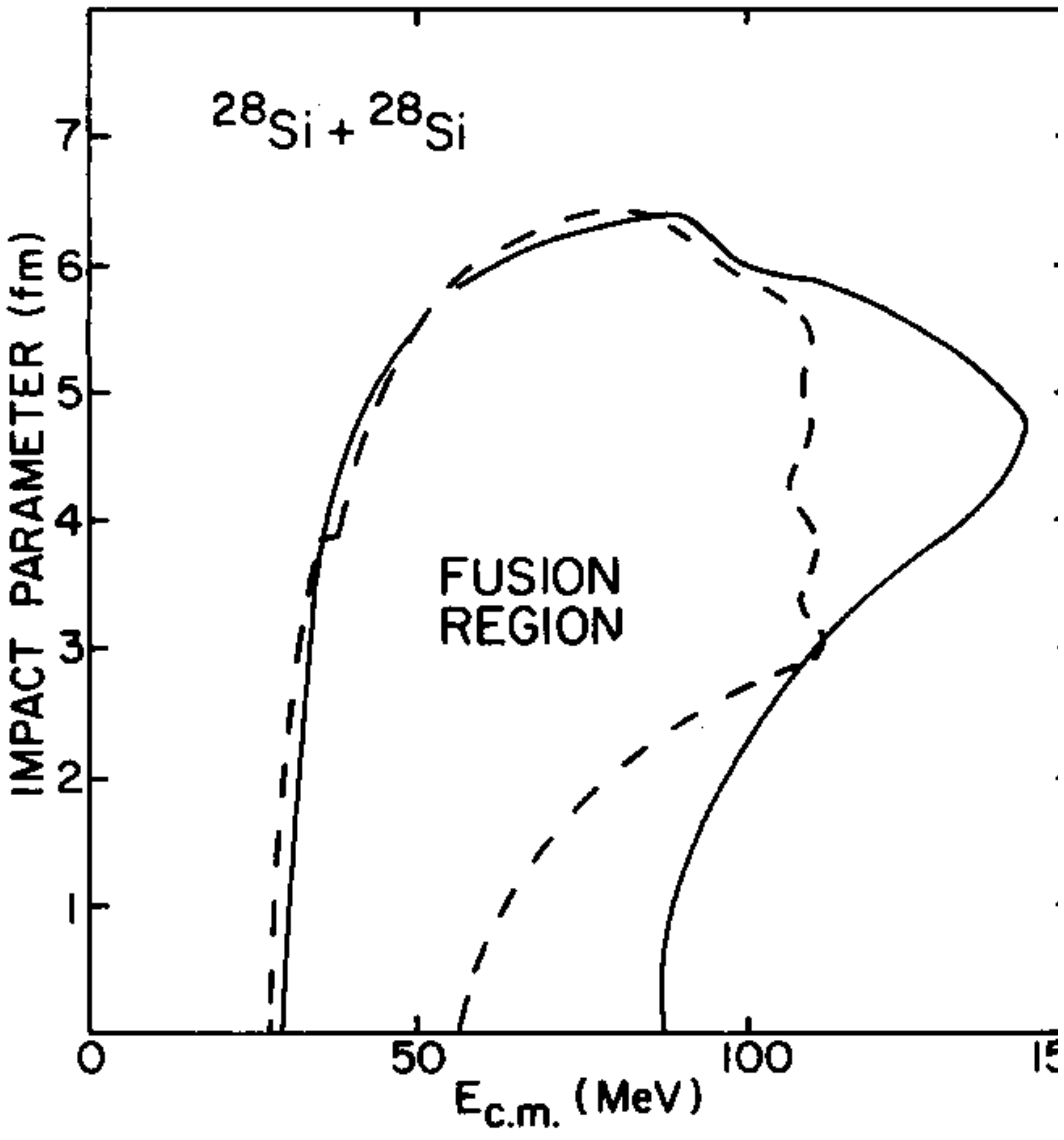
NEWTONIAN DYNAMICS OF TIME-DEPENDENT MEAN FIELD THEORY

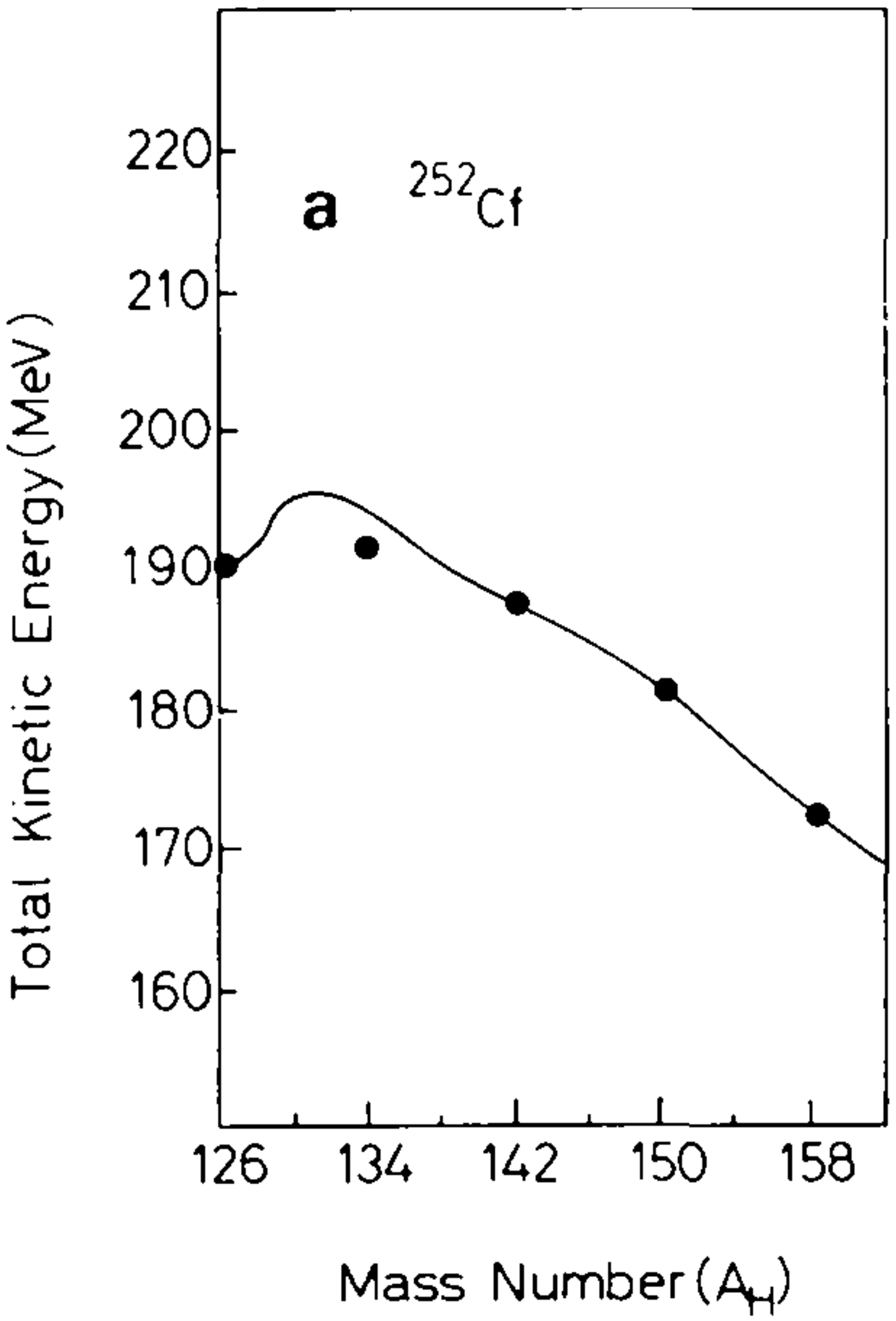
Phys.Lett.B141(1984)9; 168B(1986)35.

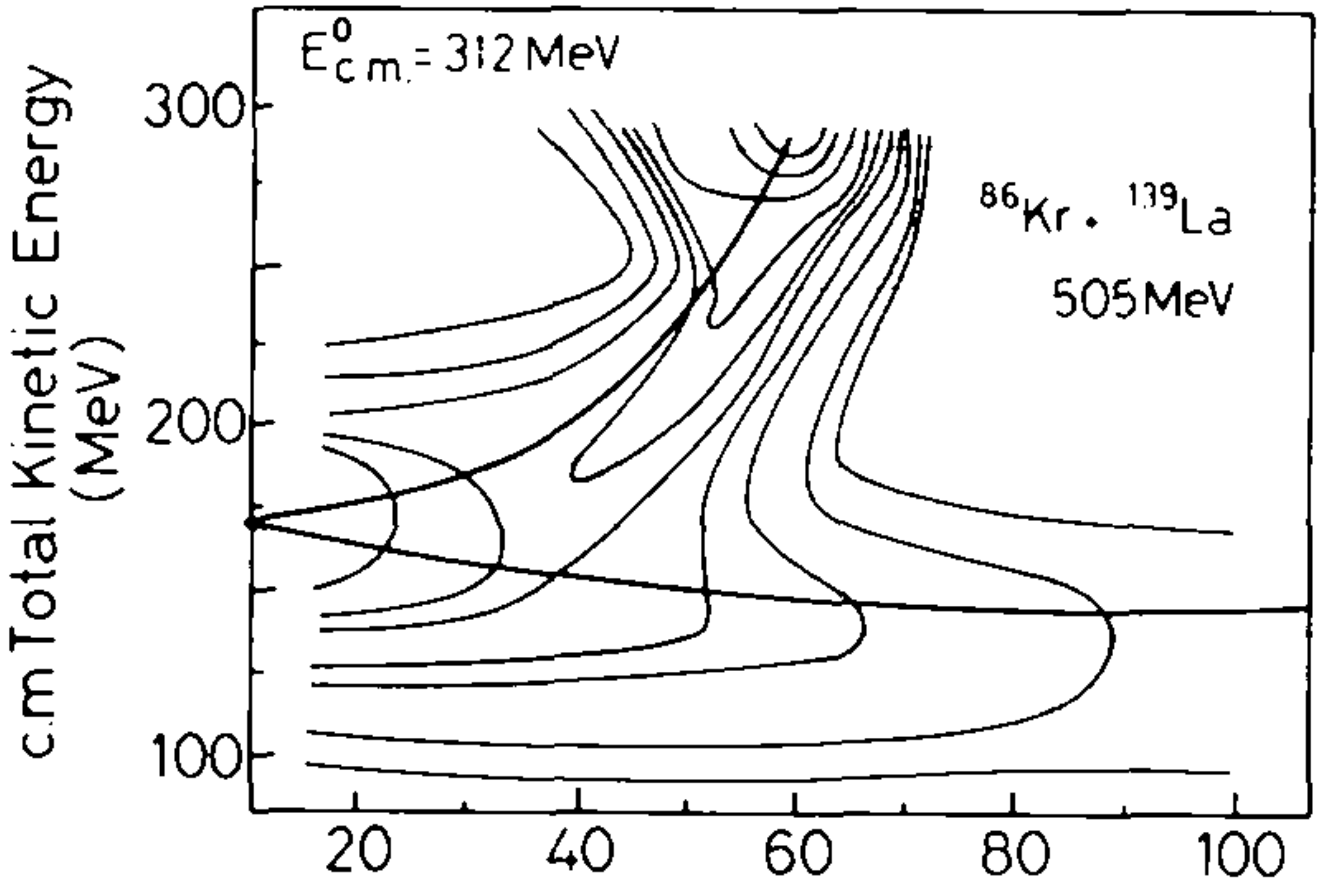
Nuclear Physics **A439** (1985) 353–370

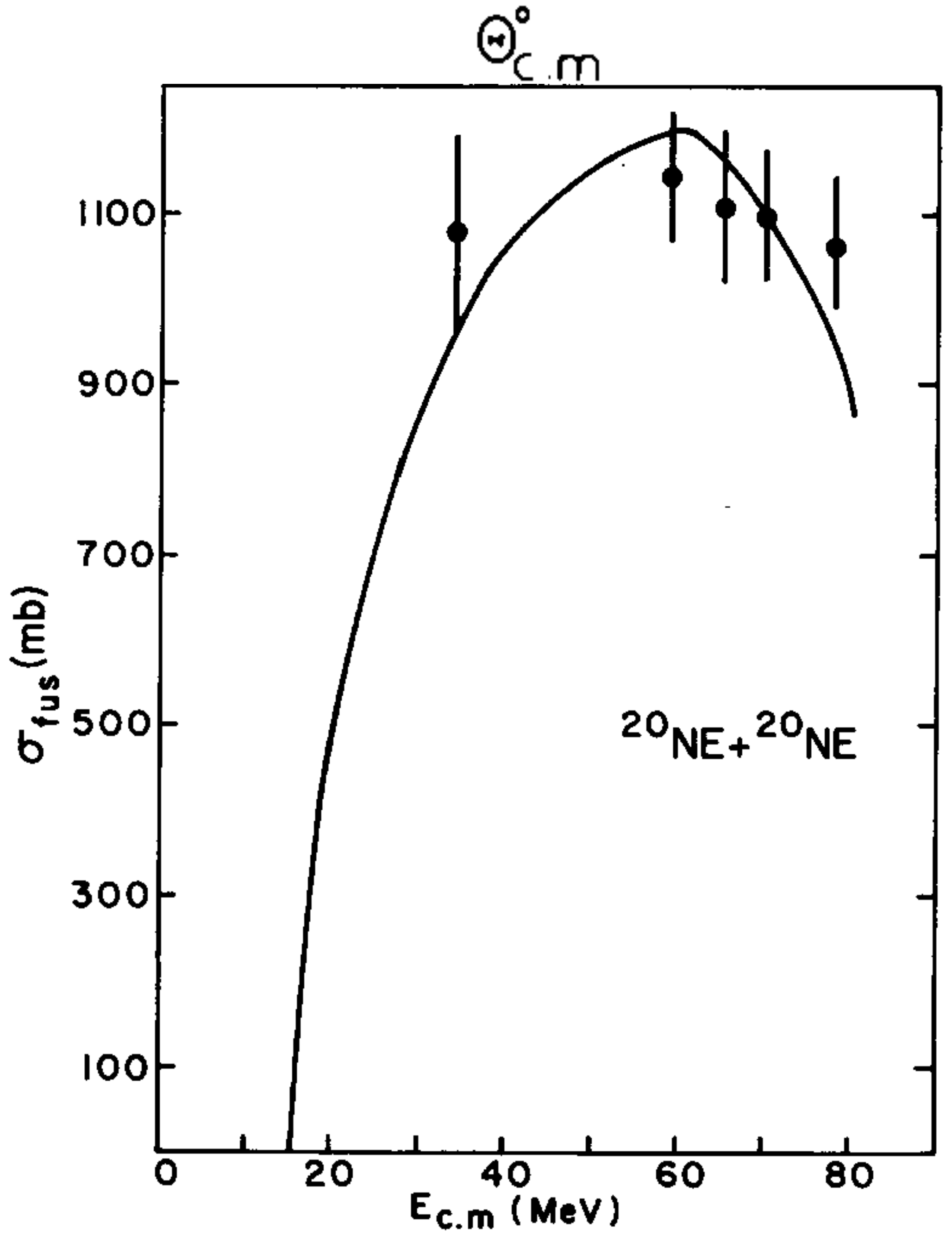
A. BONASERA, G.F. BERTSCH and E.N. EL-SAYED

Cyclotron Laboratory, Michigan State University, East Lansing, MI 48824, USA





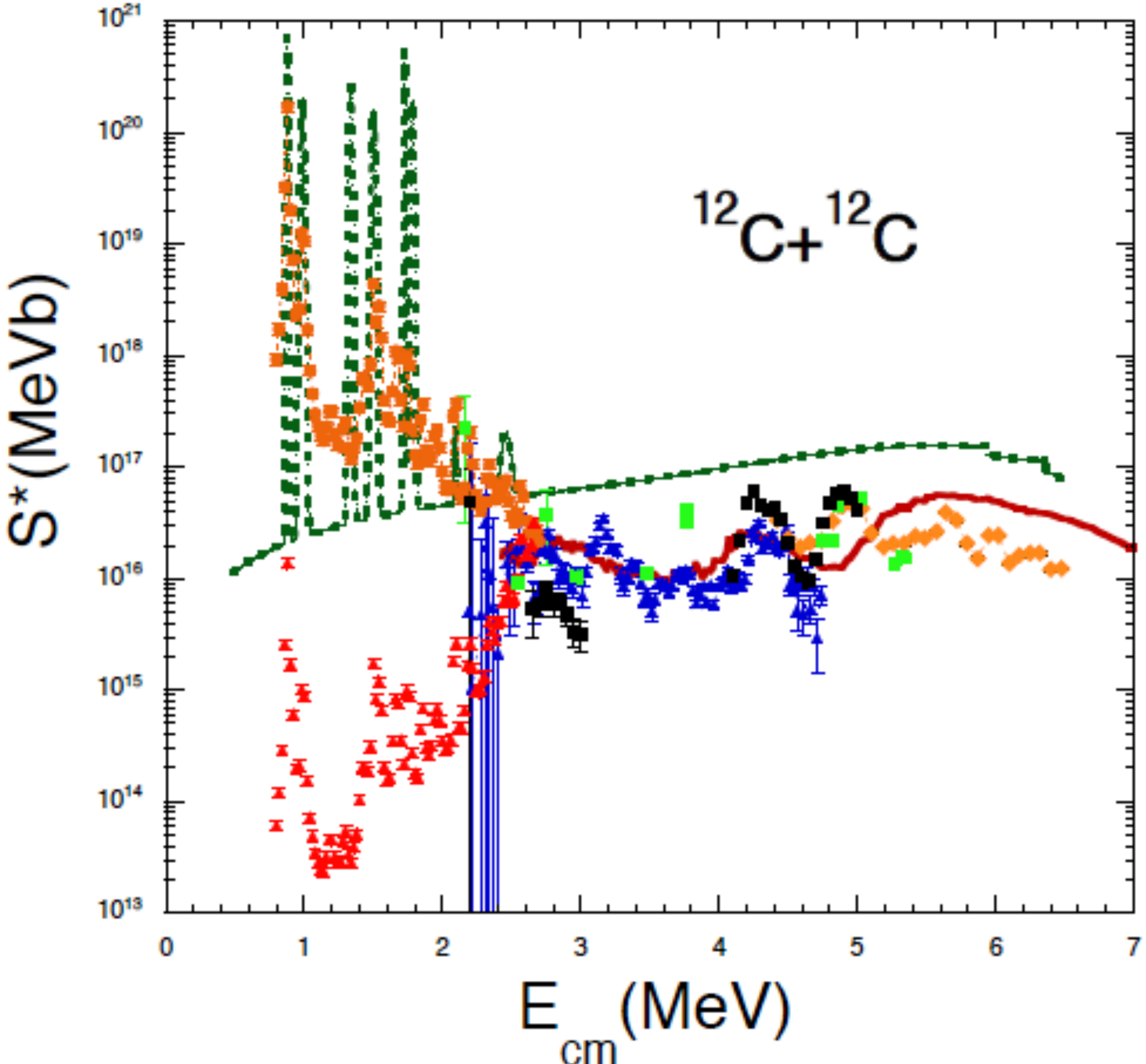




Neck model in imaginary times

PHYSICAL REVIEW C **102**, 061602(R) (2020)

The probability of fusion for the *l* th-partial wave is given by $Tl = 1/(1 + \exp\{2A\})$, $A = \int_1^2 P\, dR$.



To take into account resonances modify the Bass potential as:

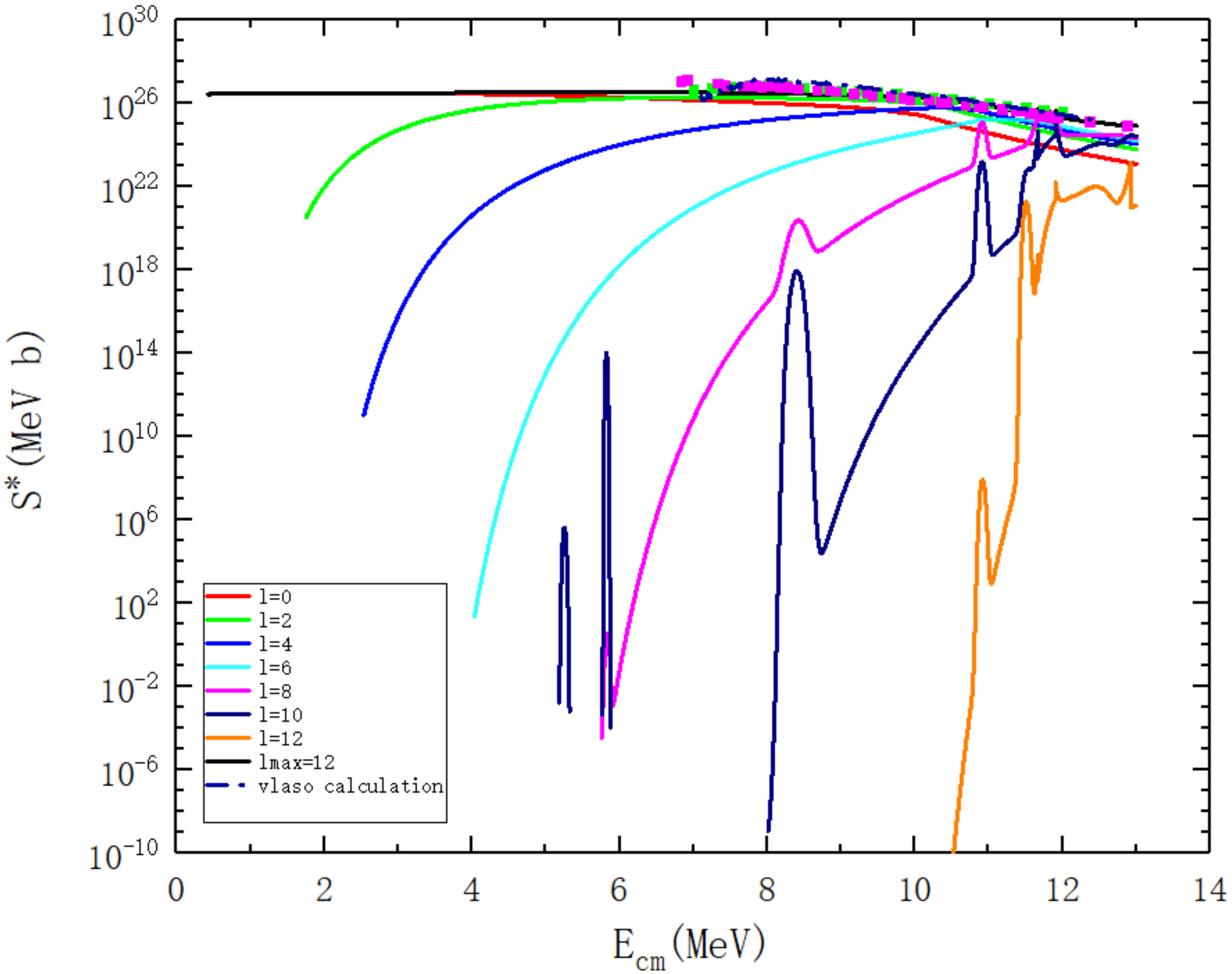
$$V_B \to V_B[1 + g(x, \gamma, \sigma)],$$

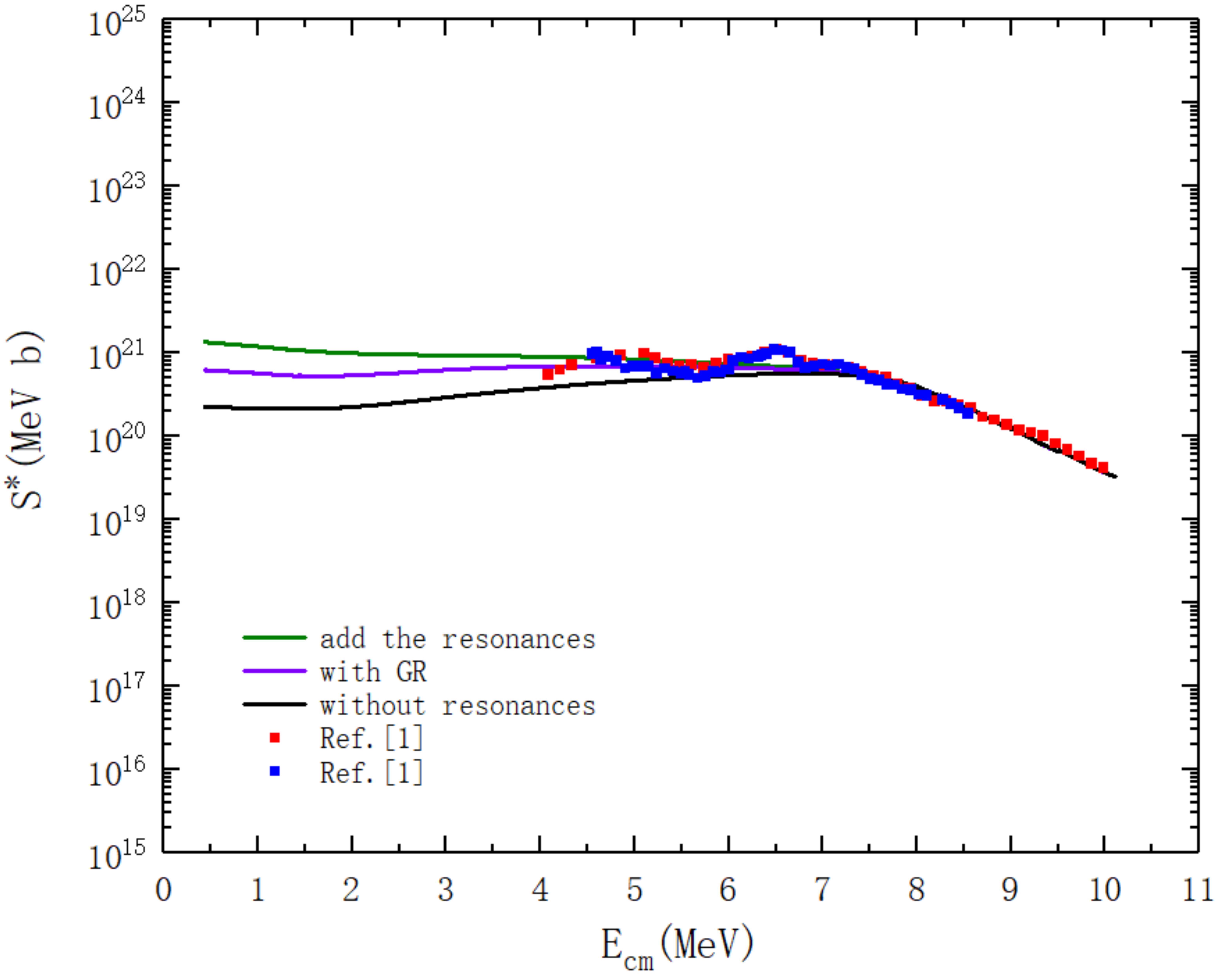
Calculation of the O16+O16 sub_barrier fusion cross section

Shuting Sun

supervisors: H. Zheng and A. Bonasera

School of Physics & Information Technology , Shaanxi Normal University





add the resonances
with GR
without resonances
Ref. [1]
Ref. [1]
S*(MeV b)
E_{cm}(MeV)

12C+16O

Progress in Research, Fall 2022
Dynamical pair production at sub-barrier energies for light nuclei

Thomas Settlemyre, Hua Zheng, and Aldo Bonasera, arXiv:2207.06900 **[nucl-th]**

Schwinger mechanism

- Dirac particles approximately satisfy the Klein-Gordon Equation

$$\left(p^2 + m^2\right)\psi = \left(i\hbar\frac{\partial}{\partial t} - V(x)\right)^2 \psi$$

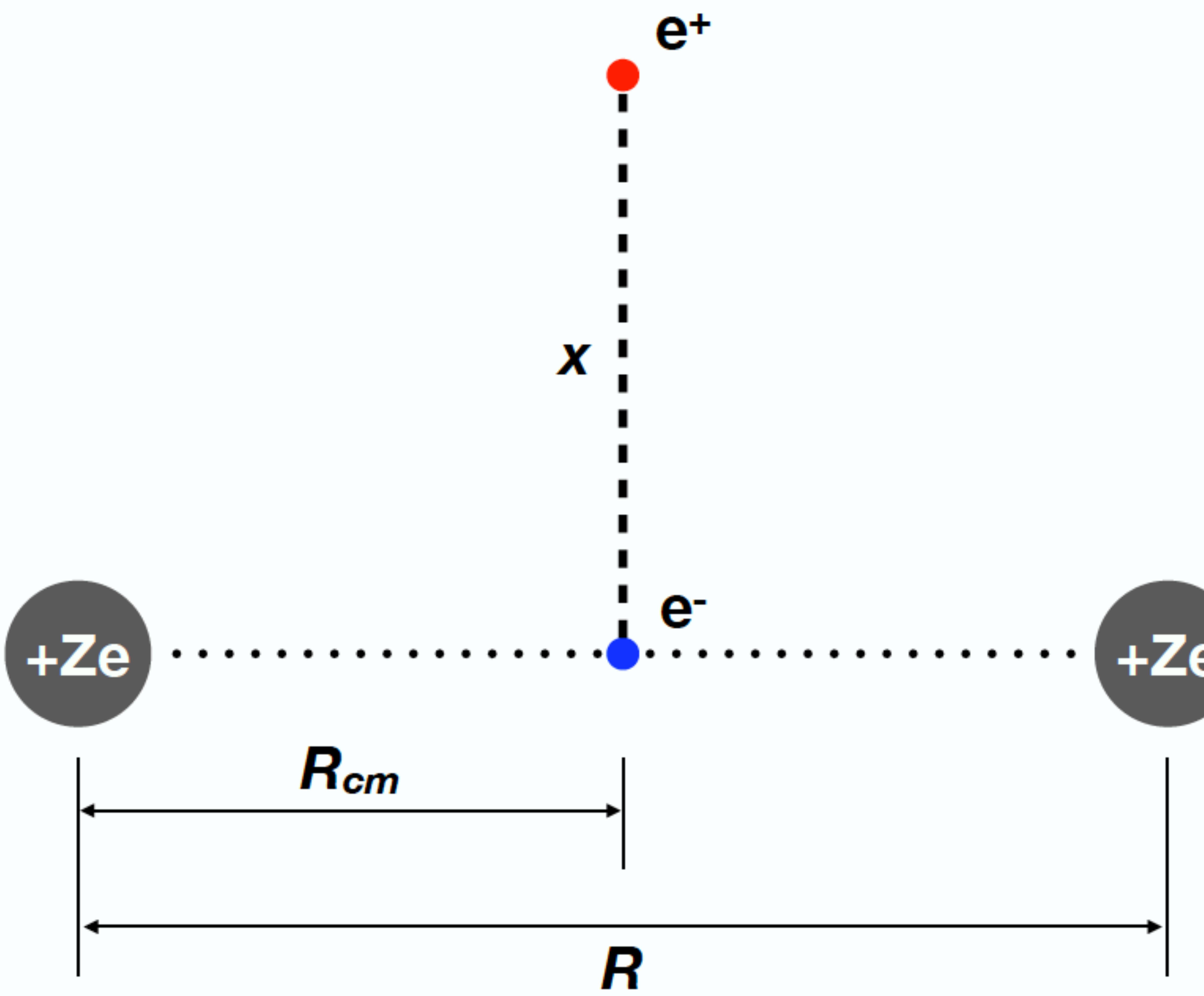
- Manipulate into a Schrödinger equation.

$$\left(\frac{p_x^2}{2m_T} + V_{eff}(x)\right)\psi = 0$$

$$V_{eff}(x) = \frac{m_T}{2} - \frac{\left(E - V(x)\right)^2}{2m_T} \qquad m_T = \sqrt{m^2 + p_y^2 + p_z^2}$$

- Negative energy particles in Dirac sea can tunnel through the effective potential and become real.

- The two nuclei come together with impact parameter zero.
- Suppose the e^- is created at the center of mass of the two nuclei and the e^+ is on an axis perpendicular to the beam axis.
- Symmetric and energetically favorable.
- The ions get accelerated by the e^- in the middle, encouraging fusion.



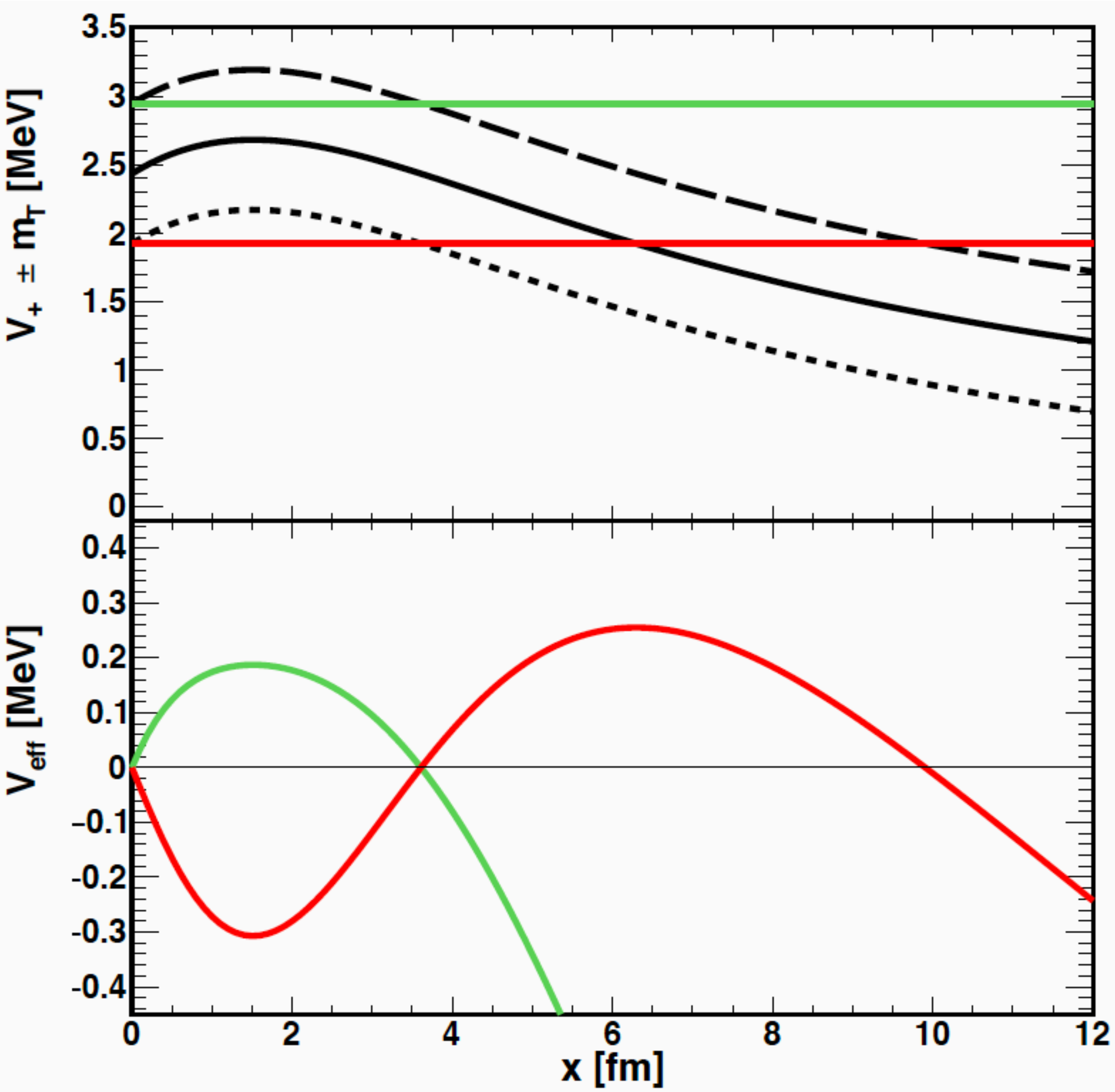
Effective potential

- Potential energy of positron

$$V_+(R,x) = \frac{2Ze^2}{\sqrt{\left(\frac{R}{2}\right)^2 + x^2}} - S(x)\frac{e^2}{x},$$

- Effective potential

$$V_{eff}(x) = \frac{m_T}{2} - \frac{(E_+ - V_+(R,x))^2}{2m_T},$$

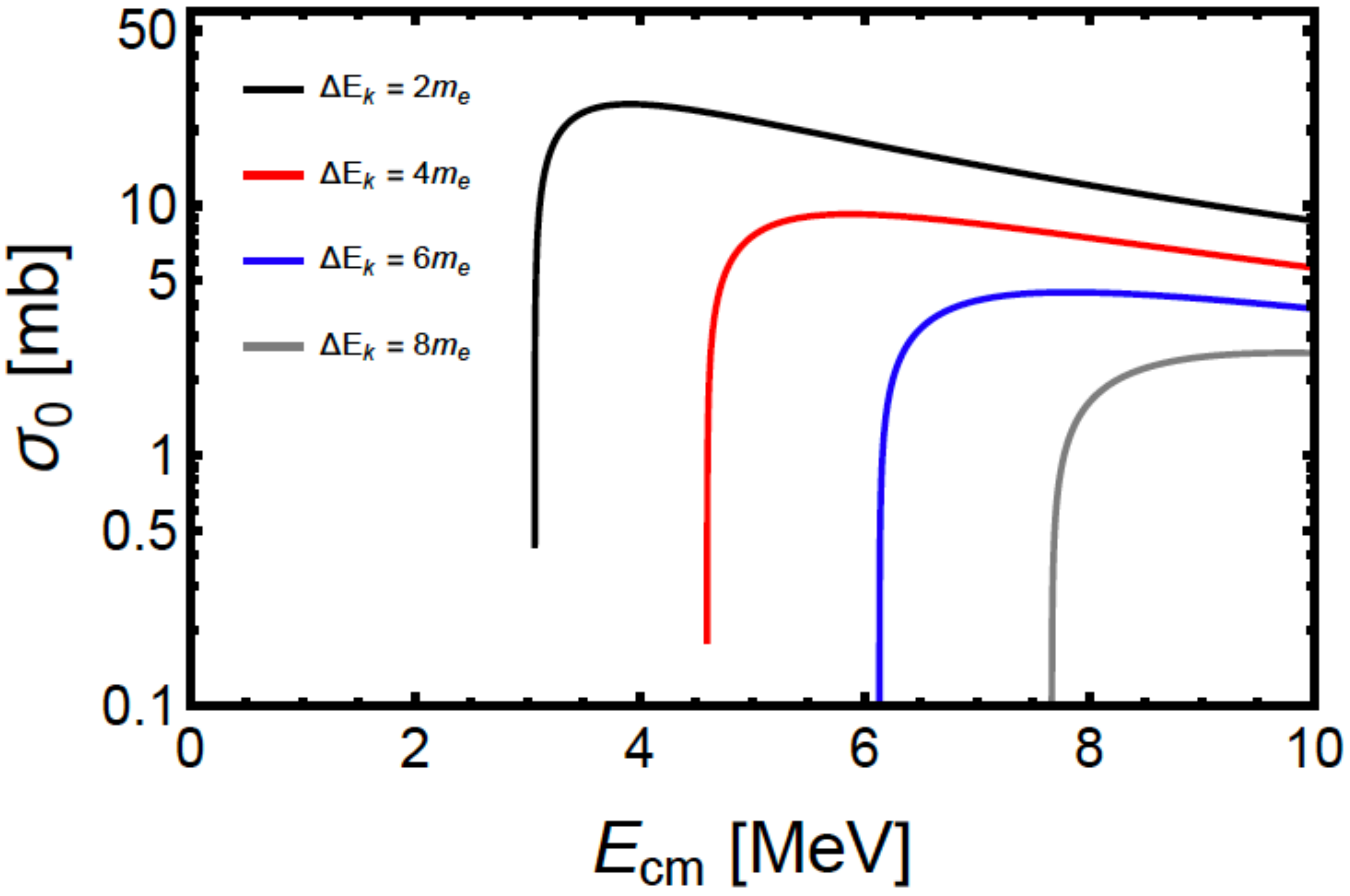


Results

$$\sigma(E_{c.m.}) = \frac{\pi\hbar^2}{2\mu E_{c.m.}} \sum_{l=0}^{n} (2l+1)\Pi_l P_H .$$

$$P_H = \tau/\Delta\tau, \qquad \Delta\tau = \frac{\hbar}{2m_T},$$

- Square root in σ gives minimum value for E_{cm}



$$\begin{aligned}\sigma_0(E_{c.m.}) &= \frac{\pi\hbar^2}{2\mu E_{c.m.}} 0.5 \frac{\tau}{\Delta\tau} \\ &= \frac{1}{N_\pm^{max}} \frac{6\pi\hbar Z e^2 m_e}{\sqrt{2\mu} E_{c.m.}^2 (\Delta E_k + 2m_e)} \\ &\times \sqrt{E_{c.m.} - \frac{Z}{4}(\Delta E_k + 2m_e)}.\end{aligned}$$

Conclusions

12C+12C: The Neck model and the Vlasov approach in imaginary time give S*>e16MeVb for Ecm>0.5 MeV

Adding resonances is in some agreement with the THM

l=0 channel is dominant up to Ecm=3MeV

For heavier systems resonances not so important because of the large Q-value

THANKS

CENTER FOR EXCELLENCE
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IN NUCLEAR TRAINING AND UNIVERSITY-BASED RESEARCH

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