Transfer Reactions in Nuclear Astrophysics

Phil Adsley
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What is a transfer reaction?

Intro to transfer reactions

Reactions in which something is transferred

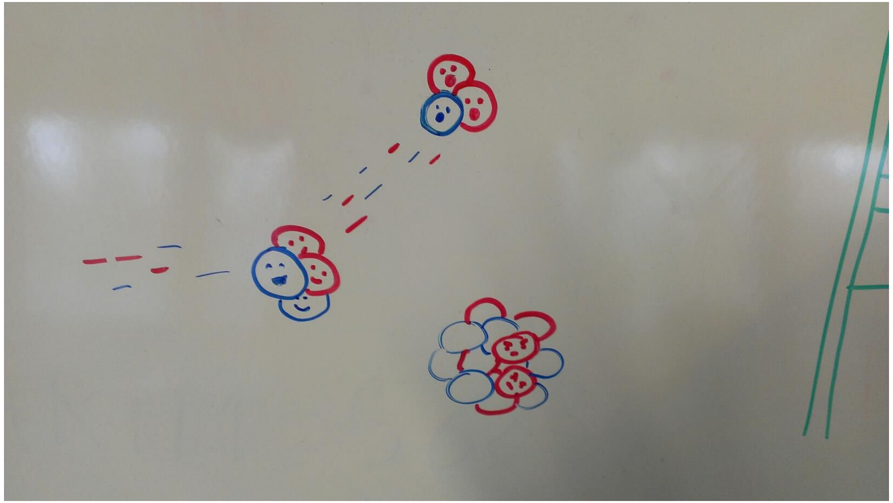
Can be single nucleons (protons, neutrons) e.g. (d,p)

Clusters (neutron pairs, α particles, ^{3}He) e.g. (^{7}Li,t)

Charge-exchange (swapping a proton for a neutron or a neutron for a proton) e.g. (^{3}He,t)

The reaction is "direct" - it happens quickly (~10^{-22} s)

Dominates at small angles - peripheral reactions



Richard Longland

Measuring transfer reactions - good kinematics

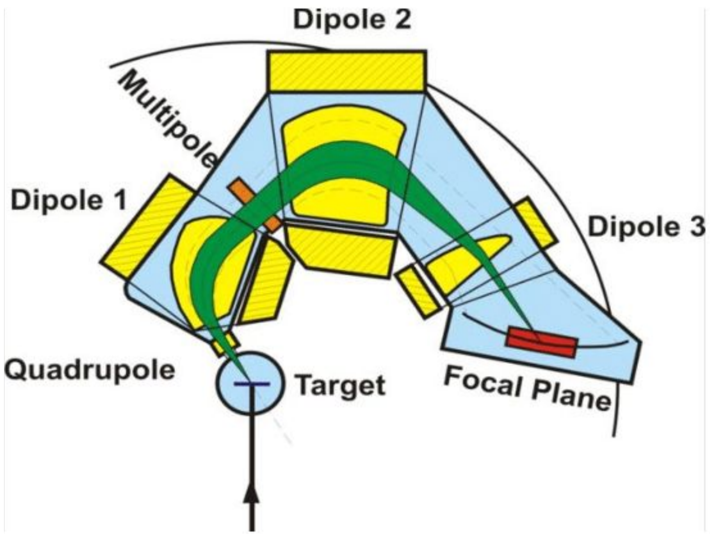
Transfer reactions in normal/forward kinematics (light beam on heavy target) is using a magnetic spectrometer

Momentum-analyse the reaction products

Measure the position at which the particles hit the focal plane -> tells us how easily bent in the magnetic field gives the rigidity (momentum/charge)

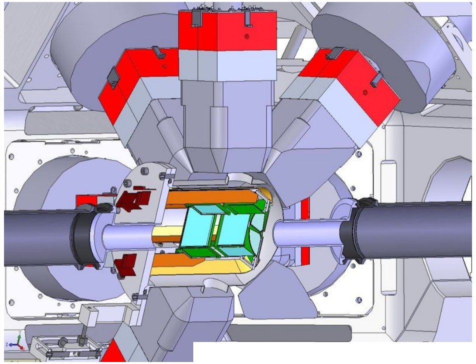
Reconstruct excitation energy with two-body kinematics

Can also do this with solid-state detectors e.g. silicon arrays



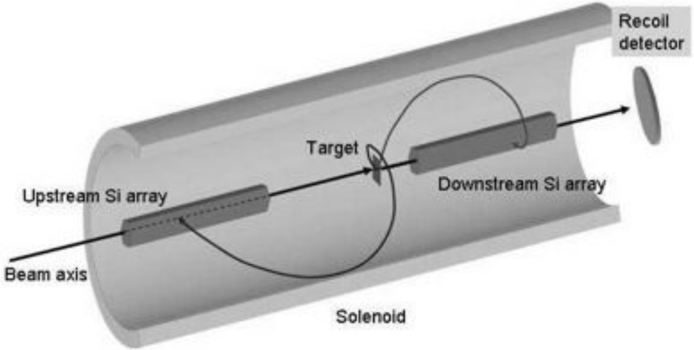
Q3D spectrometer (formerly) at Munich
The edges of the pole pieces and the multipole correct the kinematic aberrations

Measuring transfer reactions - bad kinematics



Silicon+HPGe
SHARC/TIGRESS(+EMMA
)
T-REX/Miniball
GODDESS (Argonne/FRIB)

HELIOS
ISS



See Matt Williams' talk on Friday

Forward kinematics requires a stable (or long-lived) target isotope

Also problems with target contamination, noble gases, weird compounds and chemistry

Alternative is to go to inverse kinematics - heavy beam on light target

Can access new nuclei but generally at the cost of energy resolution due to angular and target effects

Detect charged particles with silicon detectors (usually)

Improve energy resolution by detecting the γ rays from reactions at the same

Why bother?

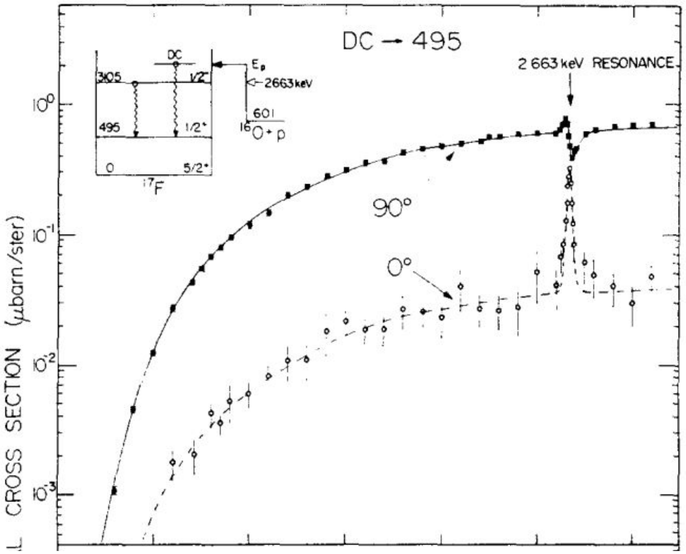
What do we need to calculate the reaction rate?

Fold probability of interaction (cross section) with the probability of having two particles with that certain energy (Maxwell-Boltzmann distribution) and sum up all of those probabilities:

$$\langle \sigma v \rangle = \sqrt{\frac{8}{\pi\mu}} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E) E \exp\left(\frac{-E}{kT}\right) dE$$

Split the cross section into two main parts
(1) Direct capture
(2) Resonance capture

Cross section is small - direct measurements are hard so calculate cross section based on indirect measurements



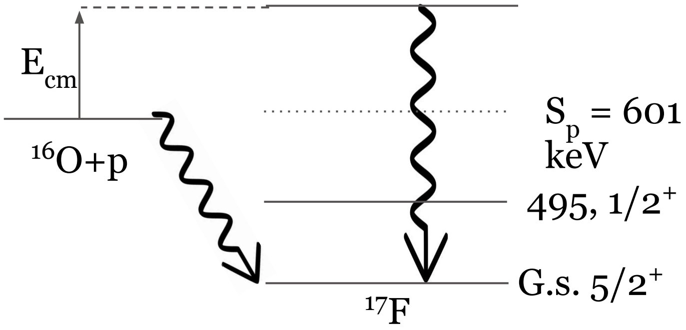
Rolfs Nuclear Physics A **217** 29-70 (1973)

Direct capture

What is direct capture?

Transition from projectile in initial continuum state to a final bound state within a nucleus through an electromagnetic interaction

Or my personal way of thinking about it:



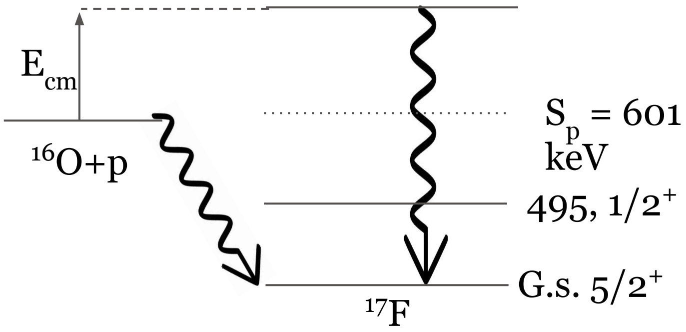
How does the particle decide what levels it likes to capture into?

The levels need to look like target + particle and we quantify how much the levels look like that by the spectroscopic factor C^2S

$$\sigma_{\mathrm{exp}} = \sum_{\ell_f} C^2 S(\ell_f)\sigma_{\mathrm{theo}}(\ell_f)$$

Resonances

$$\sigma_r(E) \propto \frac{2J+1}{(2j_1+1)(2j_2+1)} \frac{\Gamma_{\mathrm{in}}\Gamma_{\mathrm{out}}}{(E-E_r)^2+\Gamma^2/4}$$



Cross section depends on certain nuclear properties

We can work out these properties with transfer reactions (I'll explain how in a moment)

Note that the J - the spin of the state - has two impacts, both in terms of the statistical factor and also for the barrier penetrability

Orbital angular momentum L to go from an initial state of j_1 to final state J but higher L means a larger barrier and less chance of interaction

Theory of transfer reactions

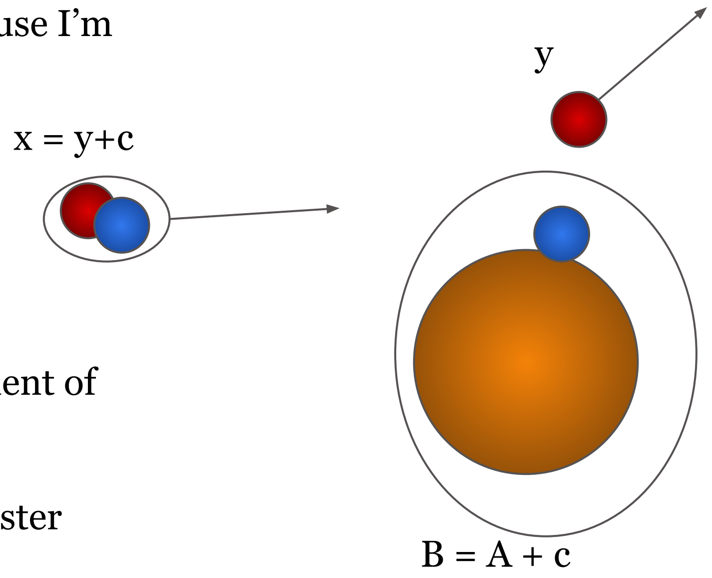
I'm really only going to talk about one theory (DWBA) because I'm an dumb experimentalist

Assume the following:

Entrance and exit channels dominated by elastic scattering
Transfer is weak - treat as first-order perturbation
Transfer proceeds directly between two channels
Direct transfer into the final state with no other rearrangement of the core

Consider a reaction like A(x,y)B where x = y+c and c is a cluster which gets transferred

Using potentials to describe interactions e.g. x = y+v is held together with a potential, A and x have some interaction between them



Theory of transfer reactions

Some necessary ingredients

Optical-model potential for incoming distorted waves (from elastics)

Overlap function - usually single-particle states in a Woods-Saxon potential with adjusted depth

Optical-model potential for outgoing distorted waves (from elastics)

Transfer operator - what potential describes the actual transferring part

Initial and final states (energies, spins, parities), masses, beam energy etc

Spectroscopic Factors

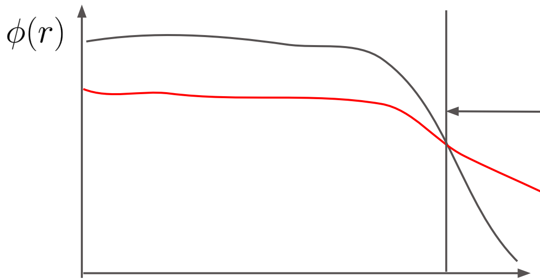
$$\left[\frac{d\sigma}{d\Omega}\right]_{\text{exp}} = C^2 S \left[\frac{d\sigma}{d\Omega}\right]_{\text{DWBA}}$$

Normalisation of the single-particle computed transfer cross section to the experimental data

Depends on the potential used in the calculation

My handwavy explanation - we only get the magnitude of the wavefunction * C^2S at the nuclear surface

The size of the tail is the ANC - less/not sensitive to the potential



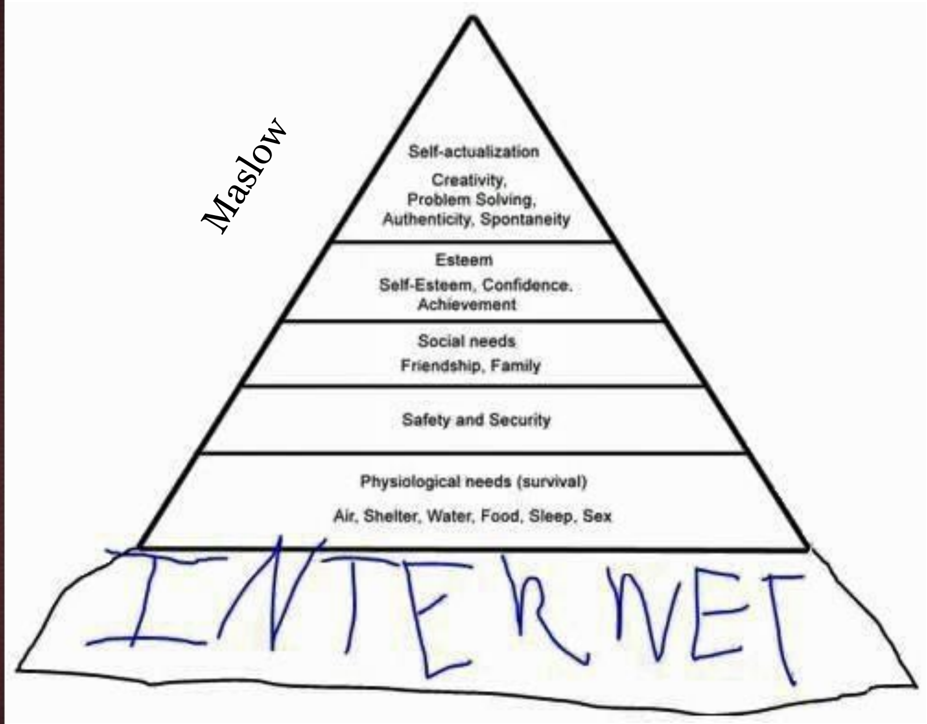
We only measure this radius - which function is right? Black of red?

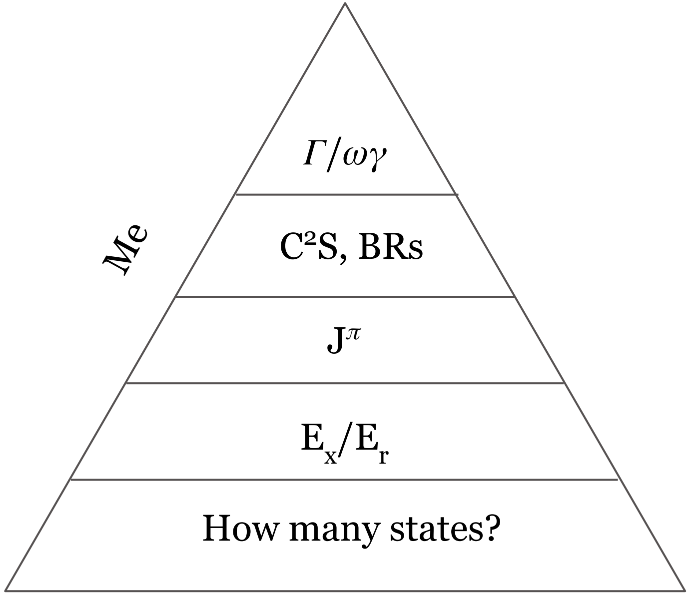
ANCs: See GF D'Agata talk on Saturday

r

Using transfer reactions to get useful information

Maslow and me





Transfer reactions for E_r

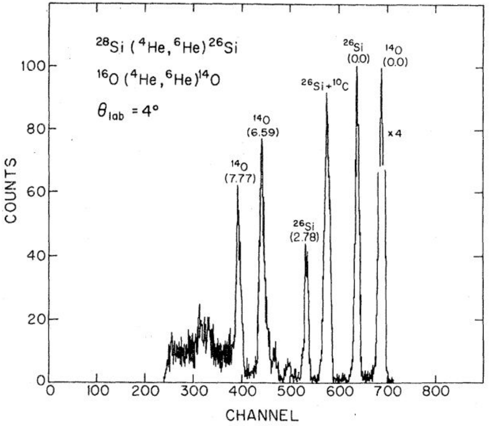


FIG. 1. Typical spectrum from the (^{4}He, ^{6}He) reaction on a SiO target showing the population of the low-lying states in ^{26}Si and ^{14}O. The dominant peaks have been identified as belonging to either ^{14}O of ^{26}Si.

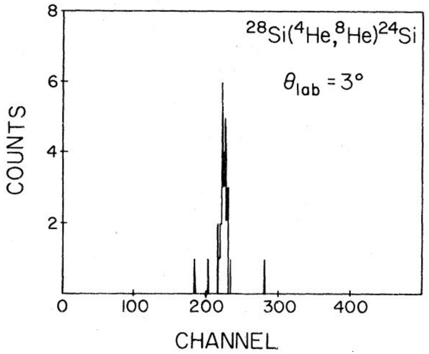


FIG. 2. Spectrum of ^{8}He events. The peak represents the yield to the ground state of ^{24}Si.

PHYSICAL REVIEW C VOLUME 22, NUMBER 1 JULY 1980

Mass of ^{24}Si

R. E. Tribble, D. M. Tanner, and A. F. Zeller*
Cyclotron Institute and Physics Department, Texas A&M University, College Station, Texas 77843
(Received 11 January 1980)

The first and most basic task is to work out if there are states in a nucleus!

Generally we can do this at the same time as working out some other things like spins and parities but not always

Back before RIB+Penning traps were a thing - mass measurements using transfer reactions

Can find excited states by the same process (which traps generally cannot)

Transfer reactions for J^{π}

The shape of the angular distribution is characteristic of the orbital angular momentum transferred

The momentum transferred (q) is the difference between momentum in (p_1) and out (p_2), and the orbital AM is L=rq

Can do a quick classical calculation (I think the full derivation is in Krane):

$$\ell = \frac{c}{197\ \mathrm{MeVfm}} 1.25\ \mathrm{fm} A^{1/3} \times 2\sqrt{p_1 p_2} \sin\left(\frac{\theta}{2}\right)$$

Then choose the L-value of interest and work out the angle at which the classical condition is satisfied… which works surprisingly well

Interference: gap between maxima:

$$\Delta\theta = \frac{\pi}{qR}$$

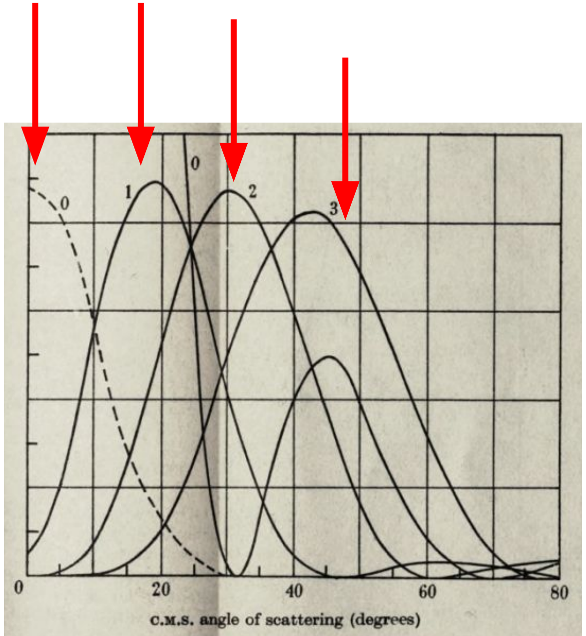


FIGURE 10. Incident 8·8 MeV, outgoing 19·4 MeV.

BUTLER, S. T., Proc. R. Soc. (London) A208, 559 (1951)

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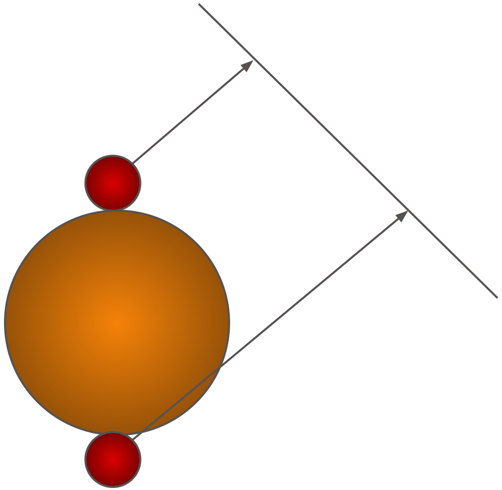


Path difference for constructive interference = 2λ

$2R\theta = n\pi$

Then choose the L-value of interest and work out the angle at which the classical condition is satisfied… which works surprisingly well

Interference: gap between maxima:

$$\Delta\theta = \frac{\pi}{qR}$$

J^π and L

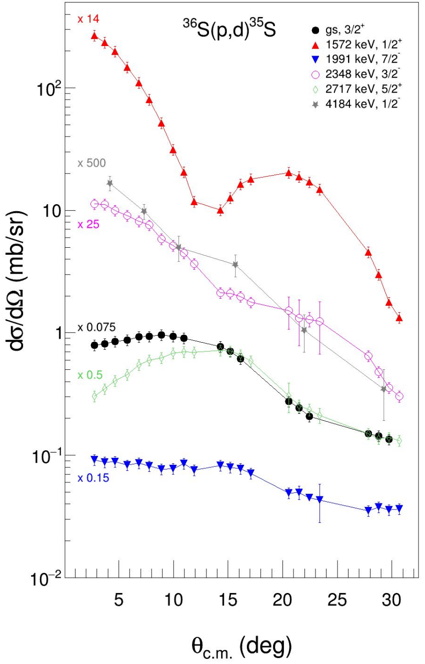


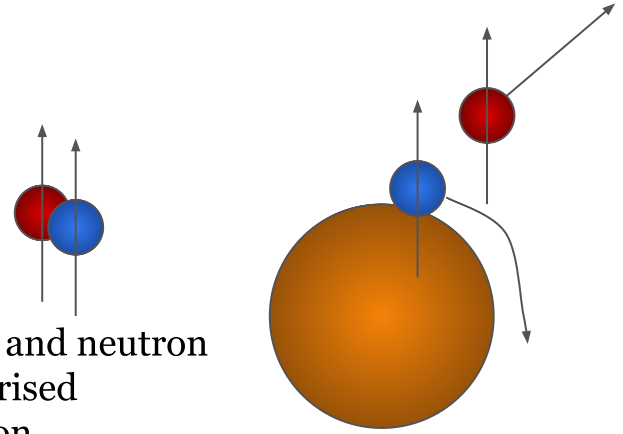
Fig courtesy Sandile Jongile+Retief Neveling

Telling different spins apart from single-nucleon transfer is difficult

L transfer much more clear but then J=L±½

Can get around this by using polarised beams!

Some differences due to spin-orbit effects but don't rely on it

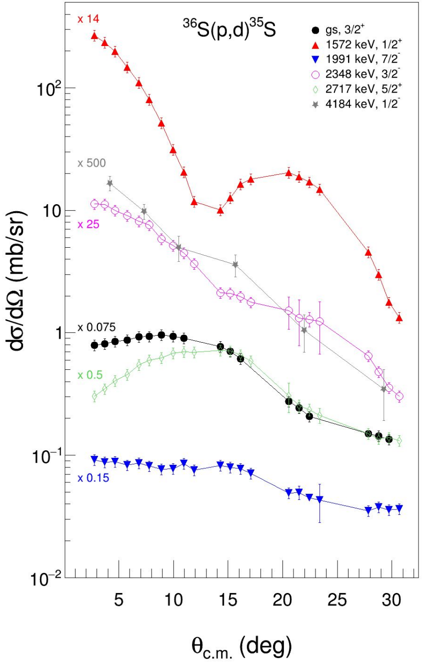


Proton and neutron in polarised deuteron

Target populating J = L-½ state

Cross section for a spin-up deuteron is higher than for the spin-down deuteron for an L-½ state

J^{π} and L



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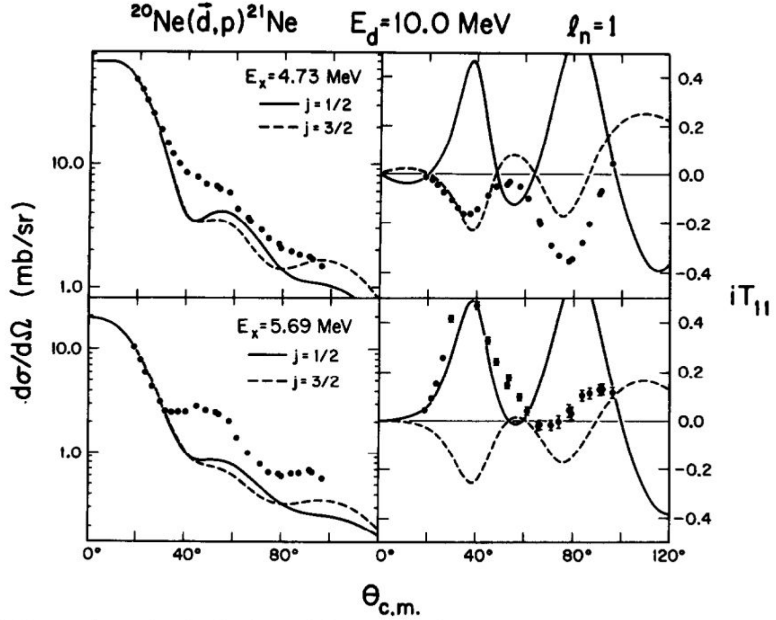
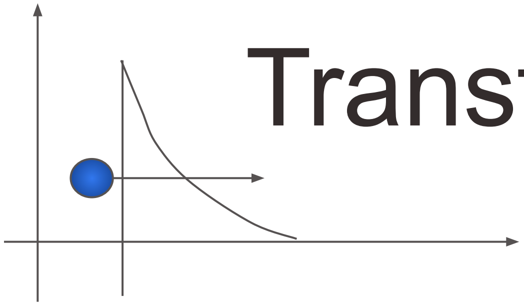


$$iT_{11} = \frac{1}{2it_{11}}\frac{r-1}{r+1} \quad r = \sqrt{\frac{C_{L,up}C_{R,down}}{C_{L,down}C_{R,up}}}$$

Transfer reactions for widths



$$\left[\frac{d\sigma}{d\Omega}\right]_{\mathrm{exp}} = C^2S \left[\frac{d\sigma}{d\Omega}\right]_{\mathrm{DWBA}}$$

How likely we are to get through the barrier

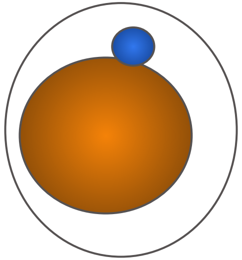
$$\Gamma = 2P_\ell(E,R)\frac{\hbar^2 R}{2\mu}C^2S|\phi(R)|^2$$

Some boring constants

The scaling factor for the wavefunction

The size of the single-particle wavefunction at the nuclear surface

Can determine the partial width for a particle to decay from a nucleus



Hand-wavy explanation: how likely the particle is to appear at the surface of the nucleus x how likely the particle is to make it out through the barrier

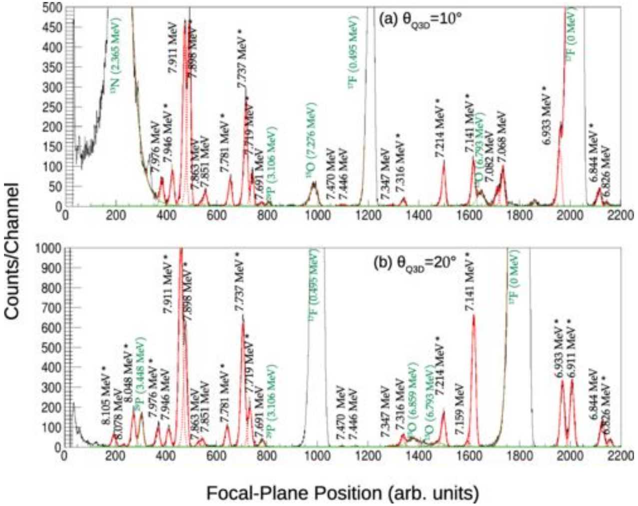
Examples of transfer reactions

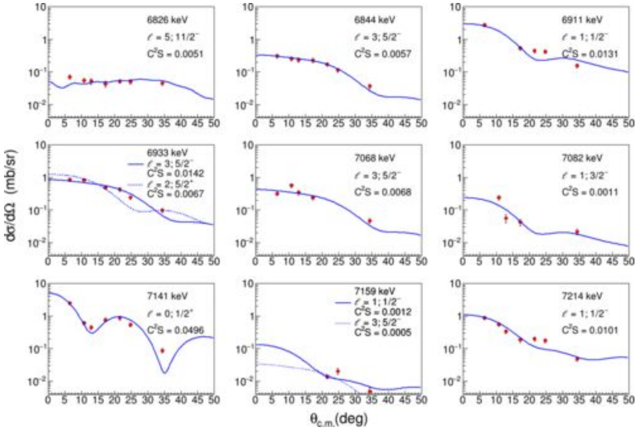
Proton transfer

Traditionally the tool for e.g. (p,γ) or (p,α) reactions has been proton transfer

In normal kinematics: (^{3}He,d) with magnetic spectrometers

D. S. Harrouz++
Phys. Rev. C 105, 015805

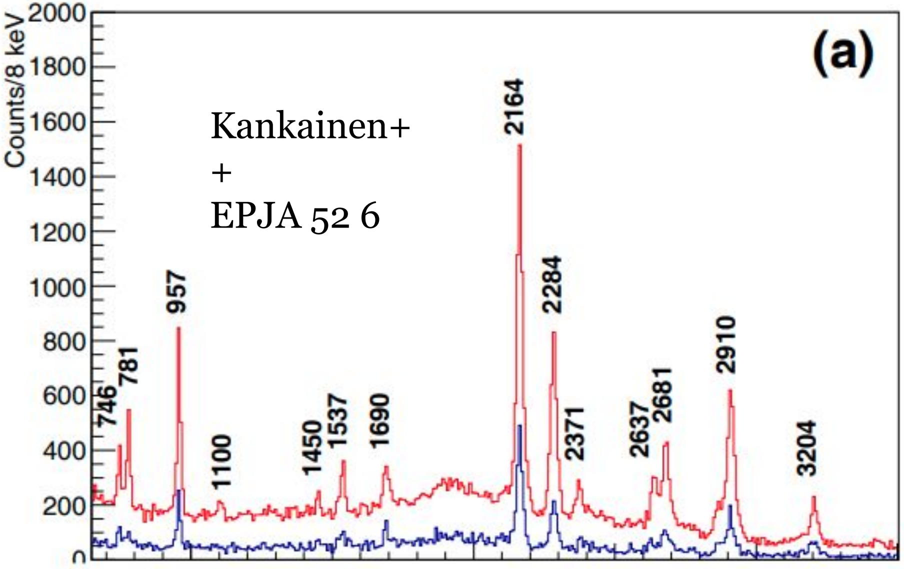




Proton Transfer

Inverse kinematics is harder:
^{3}He targets are difficult
(d,n) is an alternative but then either need to detect the neutron (boooo) or do an angle-integrated measurement in which case there's potentially a significant systematic uncertainty in the reaction mechanism

This experiment here was a ^{26}Al beam on a CD_2 target - γ rays detected in GRETINA with ^{27}Si recoils detected in the S800



Kankainen+
+
EPJA 52 6

Neutron transfer

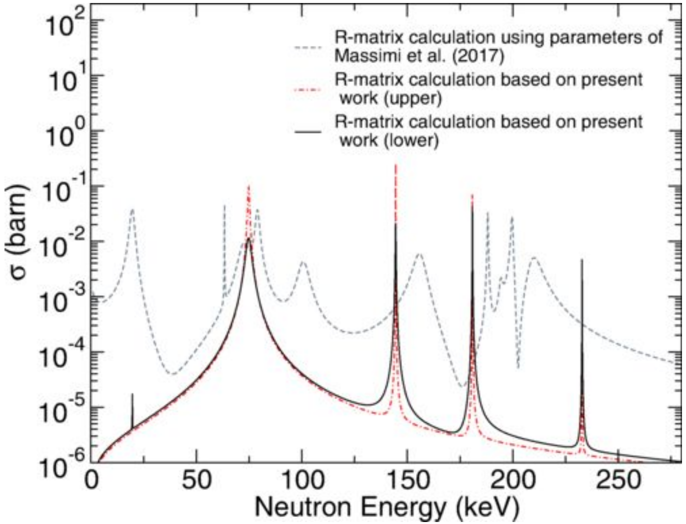
Neutron transfer is used to compute (n,γ) cross sections

E.g. ^{60}Fe(d,p)^{61}Fe for ^{60}Fe(n,γ)^{61}Fe direct capture S. Giron et al. Phys. Rev. C 95, 035806

Some caveats - it's important to remember that the rate of a reaction is determined by the slowest step

For (p,γ) it's usually the proton width since the Coulomb barrier makes capture hard

For (n,γ) it's usually the γ width since the lack of a Coulomb barrier means the neutron penetrability is higher



Y. Chen++
Phys. Rev. C 103, 035809

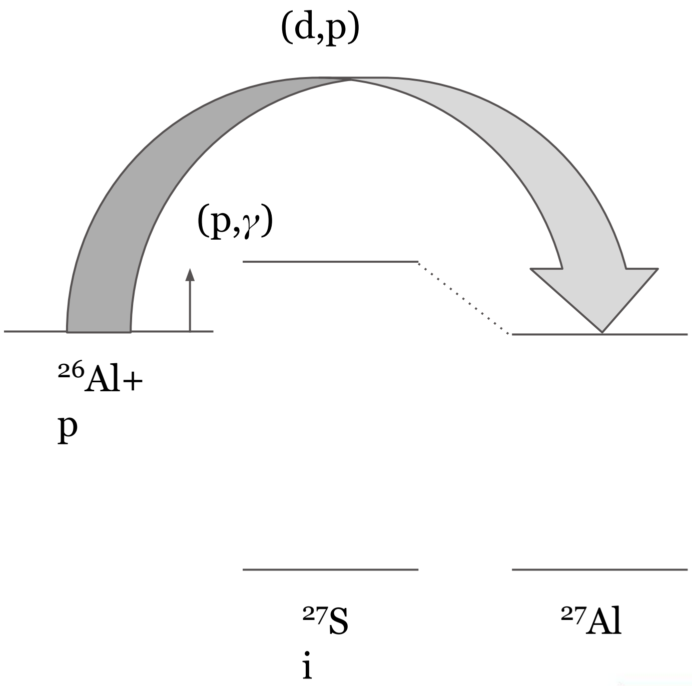
Comparison of ^{25}Mg+n and ^{25}Mg(d,p) reactions

Mirror transfer

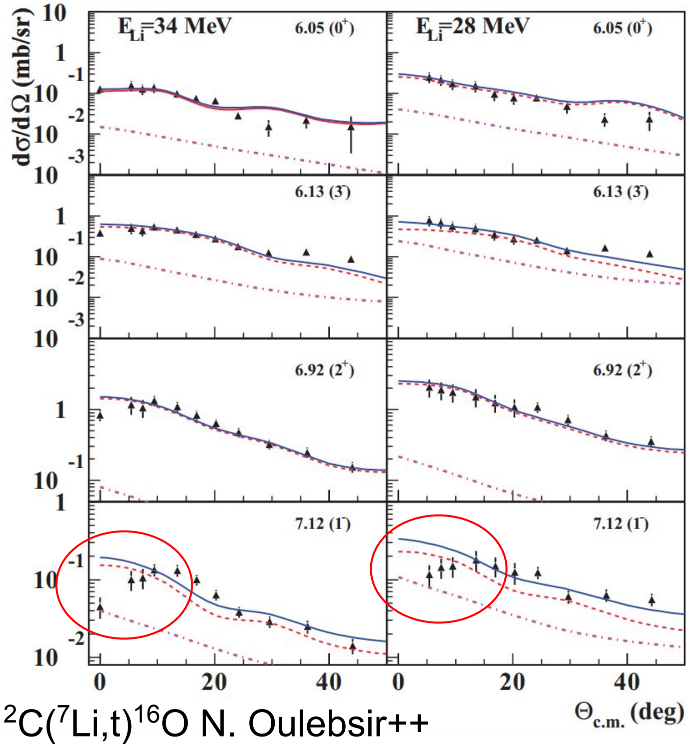
Proton transfer in inverse kinematics is hard - one alternative is to exploit isospin symmetry

(d,p) is easier to measure and, if isospin symmetry approximately holds, then we can use (d,p) to get the spectroscopic factor for a state, assign it to the mirror and use that the calculate the proton width

Caveats - what are the systematic uncertainties? Does this symmetry still hold for pairs of states when one is bound and the other is not? Or one of them is weakly bound? Coulomb effects?



α-particle transfer



$^{12}C(^7Li,t)^{16}O$ N. Oulebsir++ Phys. Rev. C 85, 035804

Many important α particle-induced reactions

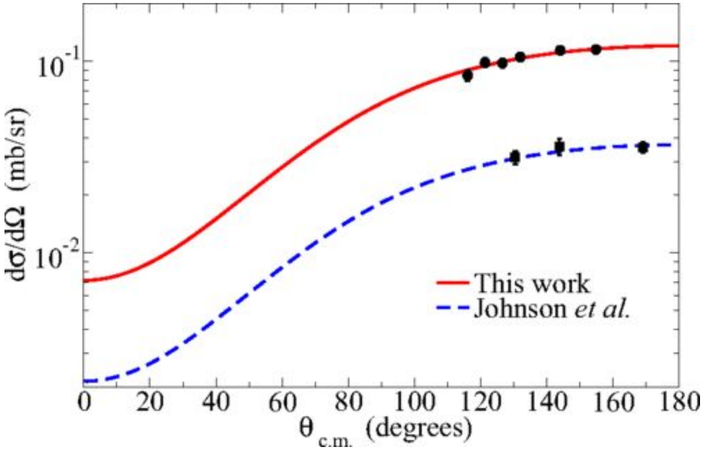
E.g. $^{13}C(\alpha,n)$ and $^{22}Ne(\alpha,n)$ are the neutron sources for the s-process

Usually need to know the α-particle width to compute these

(^{6}Li,d) and (^{7}Li,t) are common tools

A couple of caveats:
The differential cross sections don't usually show the nice shape differences required for spin/parity assignments
Some odd behaviour in some of the differential cross sections for (^{7}Li,t) may indicate incomplete understanding of the reaction - structure also definitely has an impact

Sub-Coulomb transfer



M. L. Avila, G. V. Rogachev++
Phys. Rev. C 91, 048801

Problems due to the beam energy change from 7.81->7.72 MeV due to target build-up gives factor 3(!) difference

One big problem with transfer is the systematic uncertainty in the results due to the optical model potentials chosen for the reaction (though mitigate with consistent treatment)

Sub-Coulomb transfer is less sensitive to the OMPs since it really only measures the tail outside the nucleus (the Asymptotic Normalisation Coefficient - ANC)

Advantages: less model-dependence in the results

Disadvantages: maybe more experiment-dependence in the results... the cross sections are very small (higher systematic uncertainty) and e.g. there is a very strong energy dependence which has been a problem in the past - this is a fixable problem but you have to be very very careful

Charge-exchange reactions

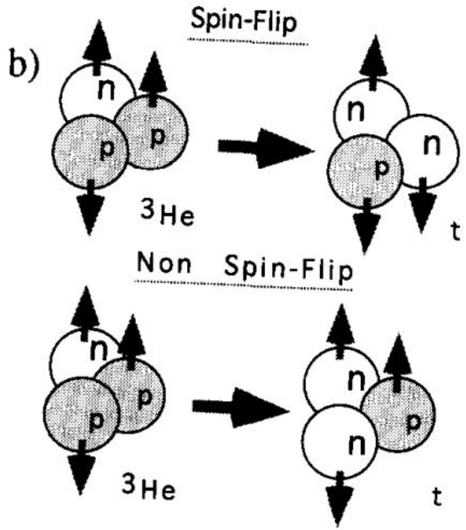
A special kind of transfer which swaps a proton and a neutron to probe weak interaction rates (β decay, electron capture etc)

(p,n) historically used but resolution is a problem

(^{3}He,t), (p,p') and (t,^{3}He) are used as a triplet set of reactions to probe Gamow-Teller transitions in nuclei

Choice of bombarding energy is important - at high energy one proton in ^{3}He is a spectator since other reactions proceed to unbound final states

At high energy (^{3}He,t) looks like a 1-step process where a single proton is interacting



M. Fujiwara et al. / Nuclear Physics A599 (1996) 223c–244c

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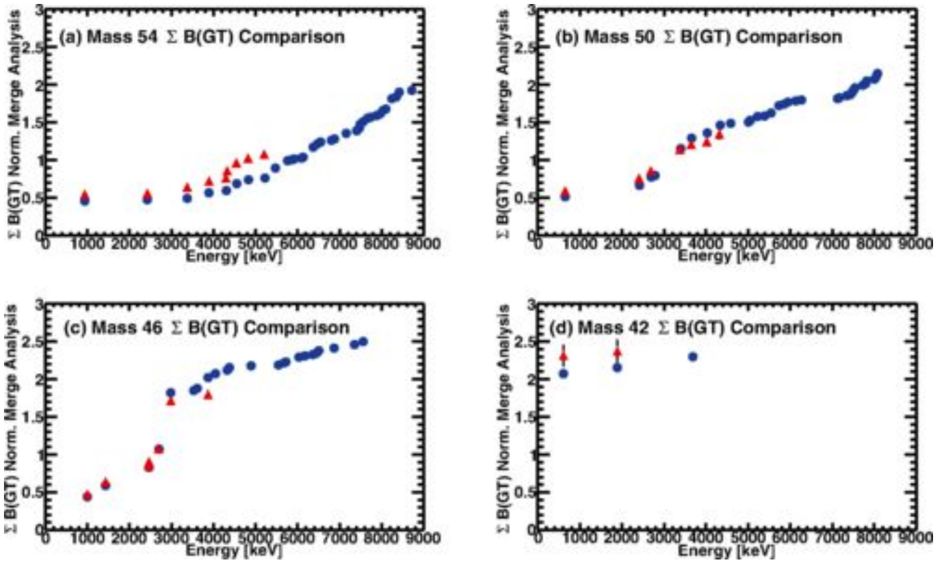


F. Molina et al. Phys. Rev. C 91, 014301
Sum of GT strength for (^{3}He,t) and beta decay in three different T_z = -1 nuclei

Transfer and decay

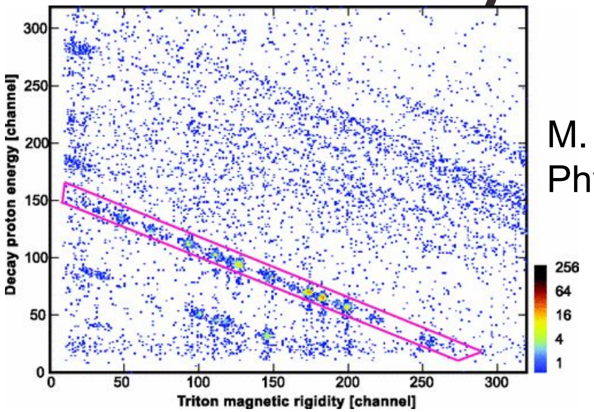
Populate states with a transfer reaction and look at the decay

Branching ratios - useful when competing final channels e.g. (n,p) vs (n,α)

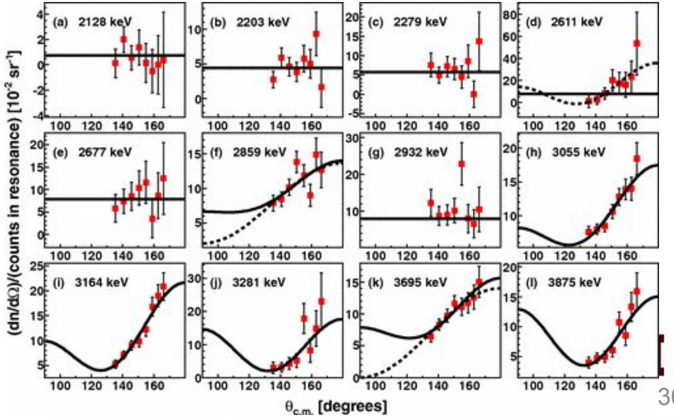
Angular correlations - the act of transferring the particle polarises the nucleus and this polarisation manifests as preferential decay directions - can use this to work out useful nuclear information

Transfer information should be consistent with R-matrix methods for e.g. the reduced widths

Can do a proxy reaction e.g. (d,p+α) as a way of indirectly determining (n,α)

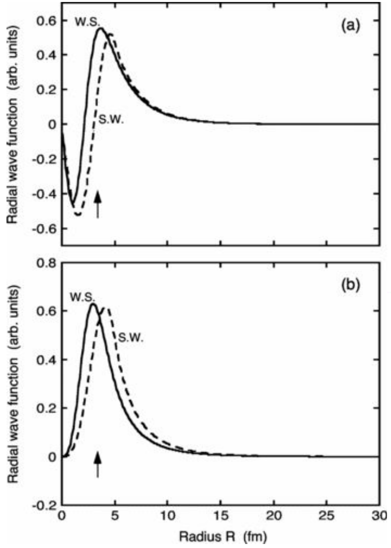


M. Matoš++
Phys. Rev. C 84, 055806



Computin stuff

Computing direct capture



$$\sigma_{\rm DC,calc}(E1) = 0.0716\mu^{\frac{3}{2}}\left(\frac{Z_p}{A_p} - \frac{Z_t}{A_t}\right)^2 \times \frac{E_\gamma^3}{E^{\frac{3}{2}}} \frac{(2J_f+1)(2\ell_i+1)}{(2j_p+1)(2j_t+1)(2\ell_f+1)} \times (\ell_i 010|\ell_f 0)^2 R^2_{n\ell_i 1\ell_f},$$

$$R_{n\ell_i 1\ell_f} = \int_0^\infty u_c(r) O_{E1}(r) u_b(r) r^2 dr,$$

C. Iliadis and M. Wiescher
Phys. Rev. C 69, 064305

There's a recipe for this given in various papers but generally it's angular momentum coupling and a radial overlap integral that needs to be calculated

There are significant uncertainties in the computation of the DC cross section due e.g. to the choice of the potentials

I think as a general rule of thumb: try to use the same optical-model potential to do the DWBA calculation for the transfer and the direct capture reaction

Dircap, Radcap and TEDCA are available codes

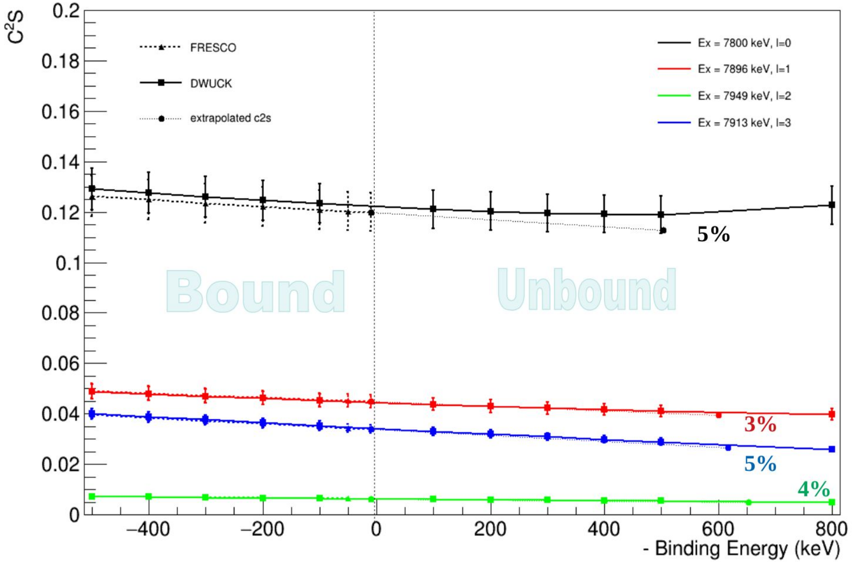
Resonance widths from transfer data

Can calculate the spectroscopic factor at some slightly bound energy and then use it with the penetrability to calculate the partial width

There's another version of this which includes computing the spectroscopic factor at a number of different positive binding energies and extrapolating to the unbound state of interest

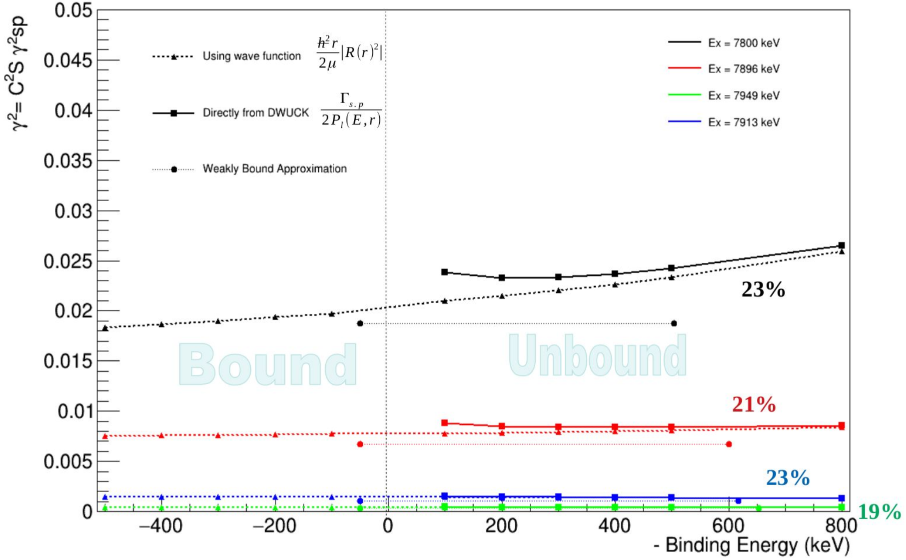
This seems fairly consistent but there's a ~20% uncertainty from the choice of OMP

Better to calculate the width directly from the wavefunction and spectroscopic factor from the DWBA calculation



Sarah Harrouz but just for one OMP

Resonance widths from transfer data



Sarah Harrouz

As a general rule of thumb:

Use the same OMP for the binding potential for the resonance width calculation and the DWBA calculation

The resulting uncertainty is a bit lower

In FRESCO we used the weak-binding extrapolation method

DWUCK4 can calculate the width of an unbound resonance (as can FRESCO)

For FRESCO - can also use the slope of the phase shift to get the single-particle width

Tools for transfer reactions

TWOFNR - https://nucleartheory.eps.surrey.ac.uk/NPG/code.htm
-There's a "front" code which helps you to prepare inputs for the transfer code

DWUCK4 - https://github.com/padsley/DWUCK4
-Does unbound states fairly easily
-The gentleman who wrote it may no longer be active (which is why it's on my Github)

FRESCO - this is (probably) the most powerful reaction code on the market http://www.fresco.org.uk/
-You can do loads of stuff (which can be very confusing)
-Well documented and Ian is very friendly (the textbook he and Filomena Nunes wrote is excellent and includes documented FRESCO inputs)
-Can do unbound states but it doesn't seem to adjust the well depth automatically to get the right resonance energy

Other Resources

Transfer Reactions As a Tool in Nuclear Astrophysics
Faïrouz Hammache and Nicolas de Séréville
Front. Phys., 30 March 2021 https://doi.org/10.3389/fphy.2020.602920

I have to recommend this since they're my old bosses :)

Proton single-particle reduced widths for unbound states
Christian Iliadis Nuclear Physics A 618 1-2 166-175 (1997)
Plus "Nuclear Physics of Stars"

Nuclear Reactions for Astrophysics
Thompson and Nunes