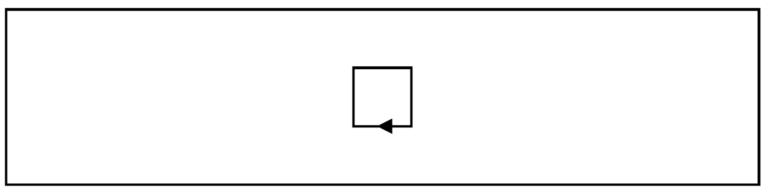
Physics 305 – Final Exam
Tuesday, May 5, 2020

There are four problems on this exam. Each problem is worth 25 points. When you are done, follow the instructions on the exam cover page to submit your papers to eCampus. Be sure to include a signed copy of the cover page (which was previously e-mailed to you, is posted on eCampus and the class web page, and is attached to this exam).

GOOD LUCK, HAVE A GREAT SUMMER, and STAY HEALTHY !!!

(1) (25 pts) A square loop of wire with sides of length a is centered within a rectangular loop of wire with size $L \times 3a$. Assume $L >> 3a$. A current $I(t) = kt$, where k is constant, is flowing around the square in the direction shown in the figure below. The rectangular loop has resistance R. Find the magnitude and direction of the current flowing in the rectangle.



(2) An infinite current sheet is located in the x-y plane. Assume (even though it's not physically possible!) that the surface current density throughout the entire sheet suddenly shifts from $\boldsymbol{K} = 0$ to $\boldsymbol{K} = K_0\,\hat{\boldsymbol{x}}$ at $t = 0$, and then remains constant thereafter. Also assume that this current is established in a manner that preserves the electrical neutrality of the sheet at all times.

a) (10 pts) What is the vector potential at an arbitrary point along the z axis at some later time t?

b) (7 pts) What are the electric and magnetic fields along the z axis at time t produced by the current sheet?

c) (8 pts) Calculate the Poynting vector for an arbitrary point along the z axis at time t. Give a physical explanation for your result in light of the fields you found in part (b).

(3) Consider a particle of mass m and charge q. At $t = 0$, the particle is at rest at the origin. At that moment, a constant electric field $\boldsymbol{E} = E_x\,\hat{\boldsymbol{x}}$ turns on.

a) (17 pts) Find parametric equations that describe the subsequent motion of the particle.

b) (8 pts) Solve your parametric equations to find the position of the particle as a function of time. **Note**: It's possible to do part (b) without doing part (a) first, though I personally think that approach is more difficult. If you find the particle position as a function of time without going the parametric route, you will receive full credit for this problem.

(4) Consider a particle of mass M, initially at rest, which decays into a final state that contains two particles with masses m_a and m_b.

a) (15 pts) Calculate the energy of particle a in the final state.

b) (10 pts) Now assume that M decays into a final state that contains four particles, with masses m_1, m_2, m_3, and m_4. Calculate the *maximum* energy of particle 1 in the final state.
Hint: Don't do a new calculation for part (b). Use your result from part (a).

PHYSICS 305 -- Final Exam

Tuesday, May 5, 2020, 10:30 am – 12:30 pm

General Procedures:

This exam will be given using Zoom and eCampus. A day before the exam, I will e-mail you information to register for the zoom session. You will need to complete the registration to gain access to the zoom meeting link. **No later than 5 minutes before the test begins, you should join the zoom meeting for the exam.** Note that, if I successfully set up the zoom meeting the way I intend, you will need to provide your standard NetID credentials to join. **You will need to have video enabled (even if you don't during normal classes), with the camera located such that you (both face and hands) and your entire workspace are visible.** This is best done by setting up your phone or laptop webcam to one side of your workspace. I will have the zoom session set up for you to be muted on entry. However, I might randomly activate your mic for periods of time during the exam.

Just before 10:30 am, I will e-mail a copy of the exam to all of you who have joined the zoom meeting by that time. If you have a question while taking the test, you will be able to ask me via the zoom chat window. You will have until 12:30 pm to complete the exam. Note that I will record the entire zoom session to my laptop (not the cloud).

After the exam period finishes, I will give you an additional 15 minutes to scan (preferred) or photograph your papers and upload them to the "Final Exam" assignment on eCampus. **The first page of your exam MUST be this cover sheet, including your signed statement below**:

Student Statement:

The Aggie Honor Codes states, *"An Aggie does not lie, cheat, or steal or tolerate those who do."* Consistent with that, I hearby affirm that I have neither given nor received help while taking this exam.

____________________________________ ______________________

Signature Date

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Printed name

Note: You do not need to include the following two pages of vector equations, etc., in your submission to eCampus.

VECTOR DERIVATIVES

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}; \quad d\tau = dx\,dy\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}; \quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient : $$\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Curl : $$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial \phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian : $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial t}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}$$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}; \quad d\tau = s\,ds\,d\phi\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/Nm^2}$

$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/A^2}$

$c = 3.00 \times 10^{8}\ \mathrm{m}/s$

$e = 1.60 \times 10^{-19}\ \mathrm{C}$

$m = 9.11 \times 10^{-31}\ \mathrm{kg}$

VECTOR IDENTITIES

Triple Products

(1) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$

(2) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$

(5) $\nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$

(6) $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$

(7) $\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$

(8) $\nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$

Second Derivatives

(9) $\nabla \cdot (\nabla \times \mathbf{A}) = 0$

(10) $\nabla \times (\nabla f) = 0$

(11) $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem : $\int_{\mathbf{a}}^{\mathbf{b}} (\nabla f) \cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem : $\int (\nabla \cdot \mathbf{A})\, d\tau = \oint \mathbf{A} \cdot d\mathbf{a}$

Curl Theorem : $\int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint \mathbf{A} \cdot d\mathbf{l}$

$$P_l(x) = \frac{1}{2^l l!}\left(\frac{d}{dx}\right)^l (x^2 - 1)^l$$

$$P_l(1) = +1 \qquad \int_{-1}^{+1} P_l(x) P_{l'}(x)\,dx = \frac{2}{2l+1}\delta_{ll'}$$

$$\boxed{\begin{array}{lcl} P_0(x) & = & 1 \\ P_1(x) & = & x \\ P_2(x) & = & (3x^2 - 1)/2 \\ P_3(x) & = & (5x^3 - 3x)/2 \\ P_4(x) & = & (35x^4 - 30x^2 + 3)/8 \\ P_5(x) & = & (63x^5 - 70x^3 + 15x)/8 \end{array}}$$