

## Physics 305 – Exam 2

Thursday, April 16, 2020

There are four problems on this exam. Each problem is worth 25 points. Start each problem on a new sheet of paper, and use only one side of each sheet. When you are done, follow the instructions on the exam cover page to submit your papers to eCampus. Be sure to include a signed copy of the cover page (which was previously e-mailed to, is posted on eCampus and the class web page, and is attached to this exam).

### GOOD LUCK !!!

- (1) (25 pts) A thin slab of material has parallel surfaces. Light incident from air strikes the slab at Brewster's angle. Show that the refracted wave also strikes the second interface (*i.e.*, from inside the slab back to air) at Brewster's angle. Note: You may assume that  $\mu_1 = \mu_2 = \mu_0$ .
  
- (2) A plane electromagnetic wave with  $\vec{E} = E_0 \hat{x} \cos(kz - \omega t)$  interacts with an electron that is bound to the origin of the coordinate system by a harmonic restoring force  $\vec{F} = -m\omega_0^2 \vec{r}$ . You may ignore any damping force felt by the electron. Also, assume that  $E_0$  is "not too large". Calculate the time-averaged angular distribution of the scattered radiation and the total radiated power for two limiting cases (17 pts for the first case, plus 3 for the second):
  - a)  $\omega_0 \ll \omega$ . (This limit would be appropriate for a nearly free electron in a plasma.)
  - b)  $\omega_0 \gg \omega$ . (This limit would be appropriate for a tightly bound atomic electron.)
  - c) (5 pts) Explain what would be necessary for  $E_0$  to be "too large" for your answers in parts (a) and (b) to be valid?
  
- (3) A time-dependent point charge  $q(t)$  at the origin,  $\rho(\vec{r}, t) = q(t)\delta^{(3)}(\vec{r})$ , is fed by a current
$$\vec{J}(\vec{r}, t) = \frac{-1}{4\pi r^2} \frac{dq}{dt} \hat{r}.$$
  - a) (10 pts) Find the scalar and vector potentials in the Coulomb gauge.
  - b) (5 pts) Find the electric and magnetic fields.
  - c) (10 pts) Show that your answers in part (b) satisfy Maxwell's equations.
  
- (4) (25 pts) A hollow rectangular wave guide, which propagates waves in the  $+z$  direction, has perfect conducting walls at  $x = 0$ ,  $x = a$ ,  $y = 0$ , and  $y = b$ , where  $a > b$ . A  $TM_{11}$  wave, with angular frequency  $\omega$ , is propagating down the wave guide. The maximum value of the  $z$  component of the electric field within the wave is  $E_0$  (which you can assume to be real valued). Calculate the surface current flowing along the wall at  $y = 0$  as a function of position and time.