# Physics 305 – Homework Set 9

**Due submitted to eCampus no later than 5 pm on Wednesday, Apr. 22**

Do the following five problems from Griffiths:

12.6 (pg 510).

12.13 (pg 523).

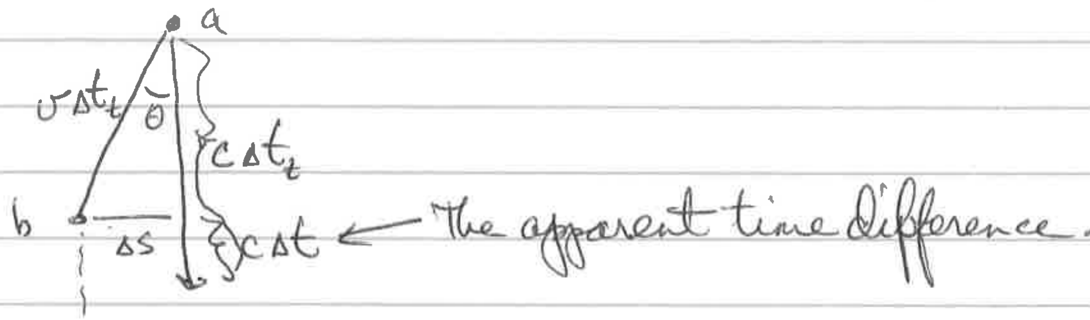
12.14 (pg 523). You may skip the “compare Prob. 12.10” request.

12.16 (pg 524-25).

12.25 (pg 534-35).

Griffiths, Problem 12.6

Assume the true time interval between a and b is $\Delta t_t$. Then we have




$$c\Delta t = c\Delta t_t - v\Delta t_t \cos\theta \Rightarrow \Delta t = \Delta t_t\left(1 - \frac{v}{c}\cos\theta\right)$$

$$\Delta s = v\Delta t_t \sin\theta \Rightarrow$$

The apparent speed is $\frac{\Delta s}{\Delta t} = \frac{v\Delta t_t \sin\theta}{\Delta t_t\left(1-\frac{v}{c}\cos\theta\right)} = \boxed{v\frac{\sin\theta}{1-\frac{v}{c}\cos\theta}}$

$\frac{\Delta s}{\Delta t}$ will be a maximum when $\frac{d}{d\theta}\left(\frac{\Delta s}{\Delta t}\right) = 0 \Rightarrow$

$$0 = \left(1-\frac{v}{c}\cos\theta\right)\cos\theta - \sin\theta\left(\frac{v}{c}\sin\theta\right) = \cos\theta - \frac{v}{c}\left(\cos^2\theta + \sin^2\theta\right) \Rightarrow$$

$\boxed{\cos\theta = \frac{v}{c}}$. Inserting this into $\frac{\Delta s}{\Delta t}$, we find:

$$\frac{\Delta s}{\Delta t}_{MAX} = v\frac{\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v}{c}\left(\frac{v}{c}\right)} = \frac{v}{\sqrt{1-\frac{v^2}{c^2}}}$$

, which can be much larger than c when v is close to c.

Griffiths, Problem 12.13

Assume the brother hits his thumb at $x=0, t=0$ in his frame (S), and also at $x'=0, t'=0$ in the scientist's frame (S').

Note that $\beta = \frac{12}{13} \Rightarrow \gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{144}{169}}} = \frac{1}{\sqrt{\frac{25}{169}}} = \frac{13}{5}$.

Then in the S' frame, Sophie cries out at:

$$ct' = \gamma\left(ct_{sophie} - \beta x_{sophie}\right) = \frac{13}{5}\left(0 - \frac{12}{13}\, 500\text{ km}\right)$$

$$\Rightarrow t' = \frac{-\frac{12}{5}\, 500 \times 10^3 \text{ m}}{3 \times 10^8 \frac{\text{m}}{\text{s}}} = \frac{-4 \times 10^5 \text{ s}}{10^8} \boxed{= -0.004 \text{ s}}$$

In the scientist's frame, Sophie cries out $\boxed{\text{4 ms before}}$ the brother hits his thumb.

12.14.1

Griffiths, Problem 12.14

a) From Ex. 12.6, we still have $d\bar{x} = \gamma(dx - v\,dt)$

$$d\bar{t} = \gamma\left(dt - \frac{v}{c^2}dx\right)$$

But now we also have $d\bar{y} = dy \Rightarrow$

$$\bar{u}_y = \frac{d\bar{y}}{d\bar{t}} = \frac{dy}{\gamma\left(dt - \frac{v}{c^2}dx\right)} = \frac{\frac{dy}{dt}}{\gamma\left(1 - \frac{v}{c^2}\frac{dx}{dt}\right)} = \boxed{\frac{u_y}{\gamma\left(1 - \frac{v u_x}{c^2}\right)}}$$

b) Here we are going from $\bar{S} \to S$, so we interchange bar and unbar and flip the sign of $v$. Thus,

$$u_x = \frac{\bar{u}_x + v}{1 + \frac{\bar{u}_x v}{c^2}} \qquad u_y = \frac{\bar{u}_y}{\gamma\left(1 + \frac{v\bar{u}_x}{c^2}\right)}$$

In $\bar{S}$, the photon velocities are $\bar{u}_x = -c\cos\bar{\theta}$

$$\bar{u}_y = c\sin\bar{\theta}$$

$$\Rightarrow u_x = \frac{-c\cos\bar{\theta} + v}{1 - \frac{v\cancel{c}\cos\bar{\theta}}{c^{\cancel{2}}}} \qquad u_y = \frac{c\sin\bar{\theta}}{\gamma\left(1 - \frac{v\cancel{c}\cos\bar{\theta}}{c^{\cancel{2}}}\right)}$$

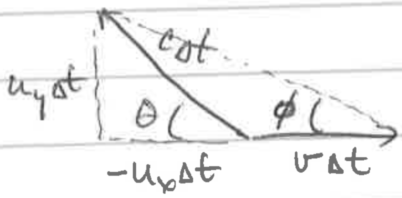
In the S (dock) frame, we have:

$$\tan\theta = \frac{u_y}{-u_x} = \frac{\dfrac{c\sin\bar{\theta}}{\gamma\cancel{\left(1 - \frac{v}{c}\cos\bar{\theta}\right)}}}{\dfrac{c\cos\bar{\theta} - v}{\cancel{\left(1 - \frac{v}{c}\cos\bar{\theta}\right)}}} = \boxed{\frac{\sin\bar{\theta}}{\gamma\left(\cos\bar{\theta} - \frac{v}{c}\right)}}$$

From definition of $\theta$

To figure out the beam direction seen from the dock, let's work entirely in the S frame. In a time $\Delta t$, we have:

12.14.2




$$\tan\phi = \frac{u_y\Delta t}{(-u_x\Delta t)+v\Delta t} = \frac{u_y}{v-u_x}$$

$$= \frac{\dfrac{c\sin\bar\theta}{\gamma\left(1-\frac{v}{c}\cos\bar\theta\right)}}{v+\dfrac{c\cos\bar\theta - v}{\left(1-\frac{v}{c}\cos\bar\theta\right)}}$$

$$= \frac{\sin\bar\theta}{\gamma\left[\frac{v}{c}\left(1-\frac{v}{c}\cos\bar\theta\right)+\cos\bar\theta-\frac{v}{c}\right]} = \frac{\sin\bar\theta}{\gamma\left[\cos\bar\theta-\frac{v^2}{c^2}\cos\bar\theta\right]}$$

$$= \frac{\sin\bar\theta}{\gamma\left(1-\frac{v^2}{c^2}\right)\cos\bar\theta} = \boxed{\gamma\,\frac{\sin\bar\theta}{\cos\bar\theta}} = \tan\phi.$$

12.16.1

Griffiths, Problem 12.16

a) $\gamma = \frac{1}{\sqrt{1-(\frac{4}{5})^2}} = \frac{5}{3}$. The twin brother saw her clock tick more slowly for 18 yrs total. Thus, his faster clock ticked

$\frac{5}{3} * 18\text{yrs} = 30\text{yrs}$. $30+21=51 \Rightarrow$ $\boxed{\text{he is 51 yr old}}$

b) He sees her take 15 yrs out and 15 yrs back. Thus, the star is $(15\text{yrs})\frac{4}{5}c =$ $\boxed{\text{12 light-yr away}}$.

c) $\boxed{(x,t) = (12 \text{ light-yr}, 15\text{yr})}$

d) $\bar{x} = \gamma(x - \beta c t) = \frac{5}{3}\left(12\text{ light-yr} - \frac{4}{5}c * 15\text{yr}\right) = \boxed{0}$

$\bar{t} = \gamma\left(t - \frac{v}{c^2}x\right) = \frac{5}{3}\left(15\text{yr} - \frac{4}{5}\frac{12\text{ light-yr}}{c}\right) = \boxed{9\text{yr}}$

e) $\tilde{x} = \gamma(x + \beta c t) = \frac{5}{3}\left(12\text{ lightyr} + \frac{4}{5}c * 15\text{yr}\right) = \boxed{40\text{ light-yr}}$

$\tilde{t} = \gamma\left(t + \frac{v}{c^2}x\right) = \frac{5}{3}\left(15\text{yr} + \frac{4}{5}\frac{12\text{ light-yr}}{c}\right) = \boxed{41\text{yr}}$

f) She has to add $41\text{yr} - 9\text{yr}$ $\boxed{= 32\text{yr}}$ to synchronize with $\tilde{S}$. It takes another 9yr to get home, so her watch in $\tilde{S}$ will read $41\text{yr} + 9\text{yr}$ $\boxed{= 50\text{yr}}$

g) (i) She sees her brother as moving at $\frac{4}{5}c$ (away from her current location) for 9yr of her time. She sees his clock running slow during this time. Thus she says his age is $21\text{yr} + \frac{3}{5}(9\text{yr})$ $\boxed{= 26.4\text{yr}}$

(ii) Now she sees her brother as having moved <u>toward</u> her current location in $\tilde{S}$ at $\frac{4}{5}c$ for 41 yr of $\tilde{S}$ time. Again, she says his clock ran slow this whole time, so she says his age is $21\,yr + \frac{3}{5}(41\,yr) = 45.6\,yr$

h) It will take 9 yr of her time to return. She will see that taking $\frac{3}{5}(9\,yr) = 5.4\,yr$ of earth time. Thus, when she gets home, she will find her brother to be

$45.6\,yr + 5.4\,yr = 51\,yr$ old, which matches part (a), as it should. ✓

Post-script: A cross check:

From (e) and (f) $(\tilde{x}, \tilde{t}) = (40\,\text{light-yr}, 50\,yr)$ in $\tilde{S}$ when she returns to earth. This implies the earth ~~...~~ frame coordinates of the return are:

$$x = \gamma(\tilde{x} - \beta c \tilde{t}) = \frac{5}{3}\left(40\,\text{light-yr} - \frac{4}{5}c(50\,yr)\right) = 0$$
$$t = \gamma\left(\tilde{t} - \frac{v}{c^2}\tilde{x}\right) = \frac{5}{3}\left(50\,yr - \frac{4}{5}\,\frac{40\,\text{light-yr}}{c}\right) = 30\,yr,$$

Both are consistent with the expectations from part (a). ✓

Griffiths, Problem 12.25

(a) $u_x = u_y = \frac{u}{\sqrt{2}} = \boxed{\sqrt{\frac{2}{5}}c}$

(b) $\gamma_u = \frac{1}{\sqrt{1-\left(\frac{2}{\sqrt{5}}\right)^2}} = \frac{1}{\sqrt{1-\frac{4}{5}}} = \sqrt{5} \Rightarrow \eta_x = \eta_y \boxed{= \sqrt{2}c}$

(c) $\eta^0 = \gamma_u c = \boxed{\sqrt{5}c}$

(d) $v = \sqrt{\frac{2}{5}}c \Rightarrow \gamma = \frac{1}{\sqrt{1-\frac{2}{5}}} = \sqrt{\frac{5}{3}} \Rightarrow$

$u_x' = 0$ by Eq. 12.45 ; $u_y' = \frac{\sqrt{\frac{2}{5}}c}{\sqrt{\frac{5}{3}}\left(1 - \frac{\sqrt{\frac{2}{5}}\cancel{c}\sqrt{\frac{2}{5}}\cancel{c}}{\cancel{c^2}}\right)} = \frac{\sqrt{\frac{2}{5}}c}{\sqrt{\frac{5}{3}}\left(1-\frac{2}{5}\right)} \boxed{= \sqrt{\frac{2}{3}}c}$

(e) $\eta_x' = \gamma(\eta_x - \beta\eta^0) = \sqrt{\frac{5}{3}}\left(\sqrt{2}c - \sqrt{\frac{2}{5}}\sqrt{5}c\right) \boxed{= 0}$

$\eta_y' = \eta_y \boxed{= \sqrt{2}c}$

(f) $\vec{u}' = \sqrt{\frac{2}{3}}c\hat{y}' \Rightarrow \vec{\eta}' = \frac{1}{\sqrt{1-\frac{2}{3}}}\sqrt{\frac{2}{3}}c\hat{y}' = \sqrt{\cancel{3}}\sqrt{\frac{2}{\cancel{3}}}c\hat{y}' \boxed{= \sqrt{2}c\hat{y}'}$. ✓