

---

---

# Physics 606 — Spring 2017

## Homework 1

Instructor: Rainer J. Fries

Turn in your work by January 26

---

---

[1] **Bohr-Sommerfeld Quantization** (30 points)

Using the quantization criterion given by Bohr and Sommerfeld calculate the possible energies allowed for a particle of mass  $m$

- (a) in an harmonic oscillator potential  $U(x) = \frac{1}{2}kx^2$ .
- (b) in an infinitely deep 1-D potential well of width  $L$ , i.e.  $U(x) = 0$  for  $-L/2 < x < L/2$  and  $U(x) \rightarrow \infty$  everywhere else.

[2] **Gaussian Wave Packets – Part I** (40 points)

Consider the real-valued 1-D Gauss function

$$f(x) = Ce^{-\frac{(x-x_0)^2}{4\sigma^2}}. \quad (1)$$

with real parameters  $x_0$  and  $\sigma$ .

*Note: All integrals in this problem can be done with basic real and complex calculus. Attempt to solve them yourself with basic techniques instead of just giving the solution to receive full credit.*

- (a) How does one have to choose the normalization factor  $C$  such that the Gaussian has  $L^2$ -norm 1, i.e.

$$\int_{\mathbb{R}} |f(x)|^2 dx = 1? \quad (2)$$

- (b) Calculate the expectation value of position and the variance around it, i.e. evaluate

$$\langle x \rangle = \int_{\mathbb{R}} x |f(x)|^2 dx, \quad (3)$$

$$(\Delta x)^2 = \langle (x - x_0)^2 \rangle = \int_{\mathbb{R}} (x - x_0)^2 |f(x)|^2 dx. \quad (4)$$

What are thus the interpretations of  $x_0$  and  $\sigma$  in  $f$ ?

- (c) Calculate the Fourier transform  $\hat{f}(k)$  of the Gauss function. What are the average value and variance?
- (d) What do you get for the Fourier transform and its average and variance if the Gaussian wave packet is modified to contain a phase factor with wave number  $k_0$

$$f(x) = Ce^{-\frac{(x-x_0)^2}{4\sigma^2}} e^{ik_0x} ? \quad (5)$$

- (e) Show that Gaussian wave packets exhaust the inequality of the uncertainty relation, i.e. they have a minimal uncertainty

$$\Delta x \Delta k = \frac{1}{2} \quad (6)$$

[3] **Refresher: A Simple Hamilton-Jacobi Problem** (30 points)

Consider a particle of mass  $m$  moving in one dimension with potential energy  $U(x) = -bx$ . Write down the Hamilton-Jacobi equation, solve for the action  $S$  and derive the motion  $x(t)$  of the system with initial conditions  $x(0) = x_0, \dot{x}(0) = v_0$ .

*Note: You can consult your favourite Mechanics book or my Mechanics manuscript on the website.*