



New Evaluation of the γW -Box Correction to Neutron and Nuclear β -Decay

Misha Gorchtein

Johannes Gutenberg-Universität Mainz

Collaborators:

Chien-Yeah Seng,
Hiren Patel,
Michael Ramsey-Musolf

Based on 3 papers:

arXiv: 1807.10197

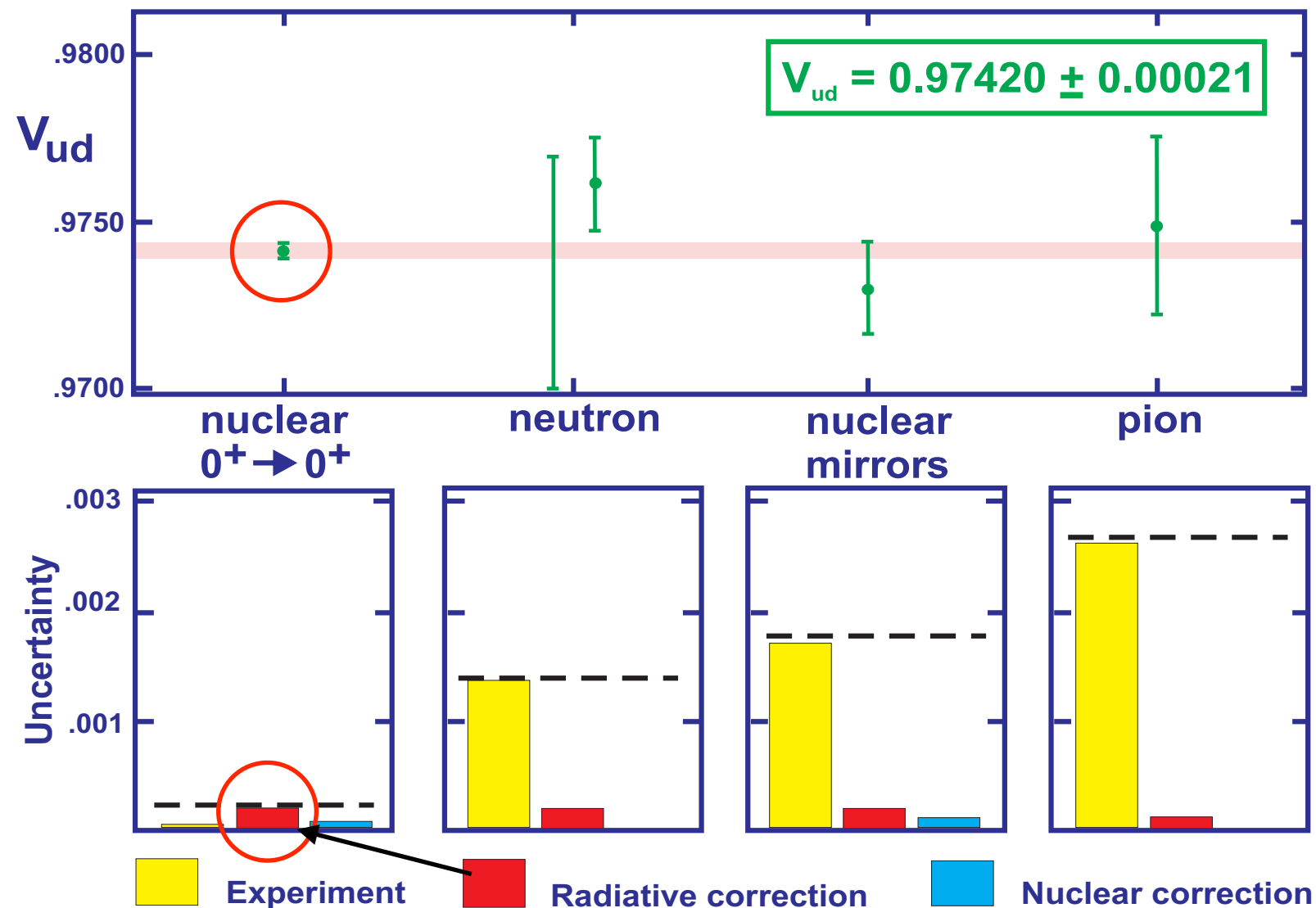
arXiv: 1812.03352

arXiv: 1812.04229



Top Row CKM Unitarity Workshop - John Hardy's Career Celebration
January 7-8, 2019 - The Mitchell Institute, Texas A&M U., College Station, TX USA

Current status of V_{ud} and top-row CKM unitarity



* Slide stolen from one of John's talks

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994 \pm 0.0005$$

$$0^+-0^+ \text{ nuclear decays } |V_{ud}|^2 = 0.94906 \pm 0.00041$$

CKM unitarity: V_{ud} the main contributor to the sum and to the uncertainty

$$\text{K decays } |V_{us}|^2 = 0.05031 \pm 0.00022$$

$$\text{B decays } |V_{ub}|^2 = 0.00002$$

Why are superallowed decays special?

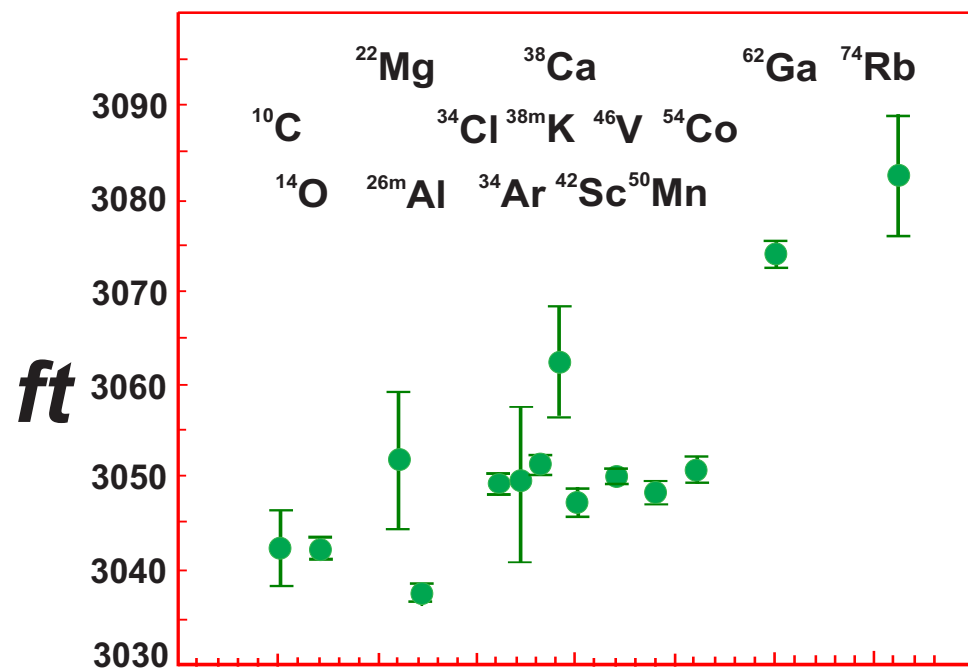
Superallowed 0^+-0^+ nuclear decays:

- only conserved vector current (unlike the neutron decay and other mirror decays)
- many decays (unlike pion decay)
- all decay rates should be the same modulo phase space

Experiment:

f - phase space (Q value + spectrum profile with Coulomb distortion) and

t - partial half-life ($t_{1/2}$, branching ratio)



ft values: same within $\sim 2\%$ but not exactly!

Reason: SU(2) slightly broken

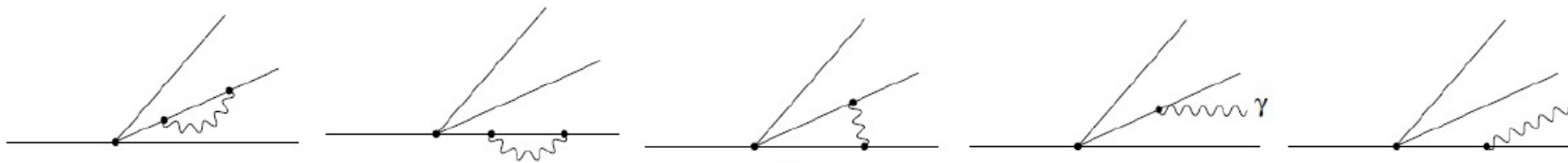
- RC (e.m. interaction does not conserve isospin)
- Nuclear WF are not SU(2) symmetric (proton and neutron distribution not the same)

Why are superallowed decays special?

Modified ft-values to include these effects

$$\mathcal{F}t = ft(1 + \delta'_R)[1 - (\delta_C - \delta_{NS})]$$

δ'_R - “outer” correction (depends on e-energy) - QED



δ_C - SU(2) breaking in the nuclear matrix elements

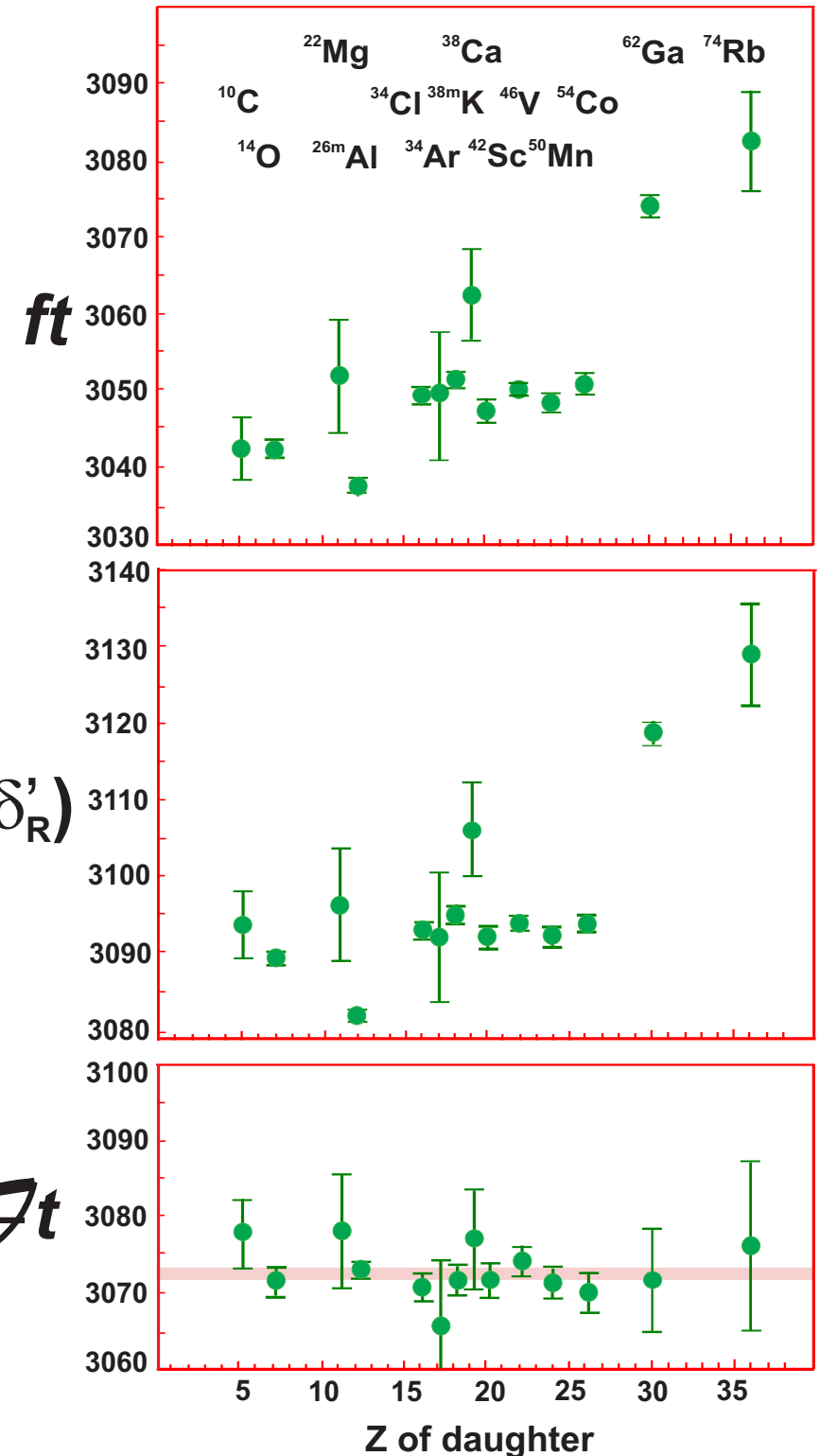
- mismatch of radial WF in parent-daughter
- mixing of different isospin states

δ_{NS} - RC depending on the nuclear structure

δ_C, δ_{NS} - energy independent

Average

$$\overline{\mathcal{F}t} = 3072.1 \pm 0.7$$



Hardy, Towner 1973 - 2018

V_{ud} from free neutron decay

Free neutron decay: axial coupling
- requires additional measurements

$$|V_{ud}|^2 = \frac{5024.49(30) \text{ s}}{\tau_n (1 + 3\lambda^2)(1 + \Delta_R^V)}$$

$$\lambda = g_A/g_V$$

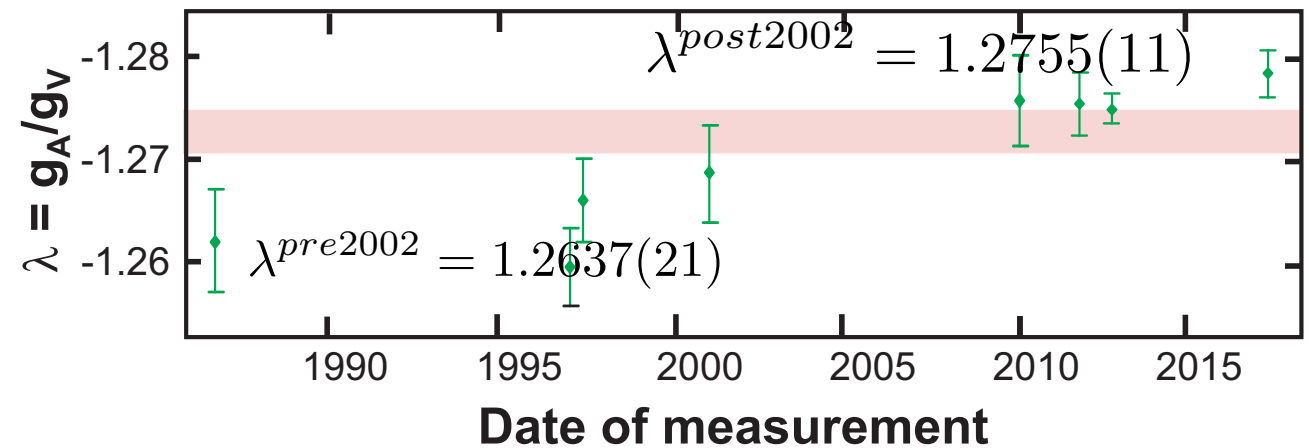
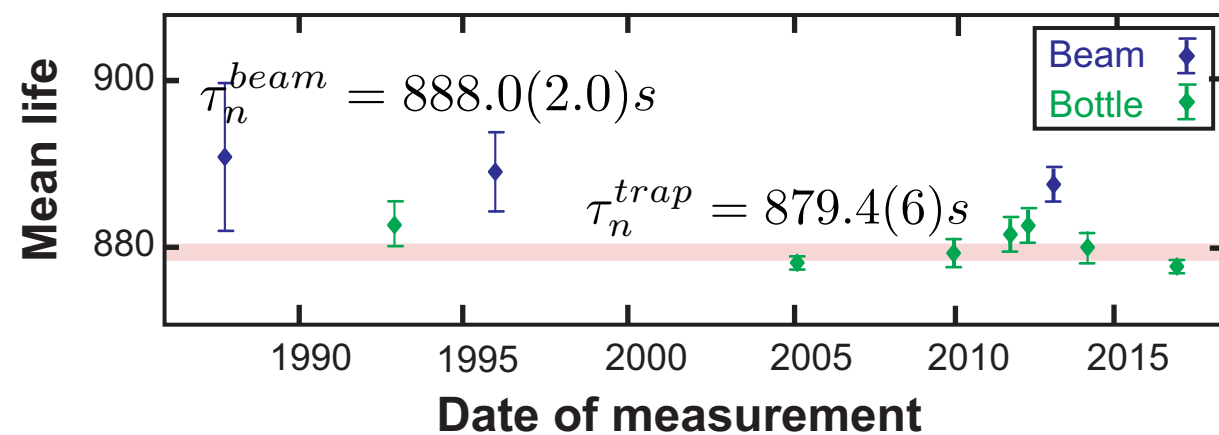
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Unfortunate discrepancy between decay in flight vs. trapped UCN



If using bottle τ_n + post-2002 λ : consistent but 7 times less precise $|V_{ud}|^n = 0.9743(15)$

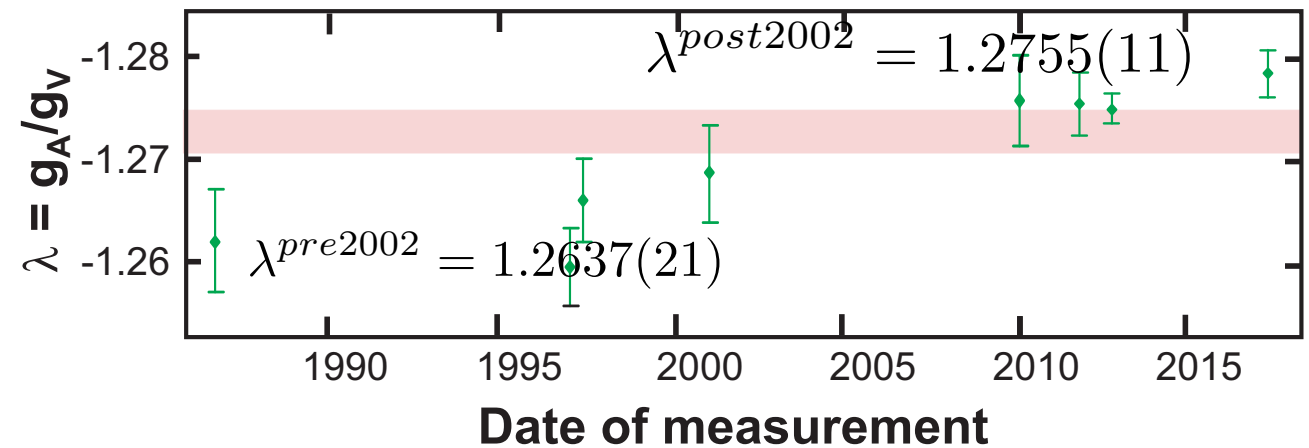
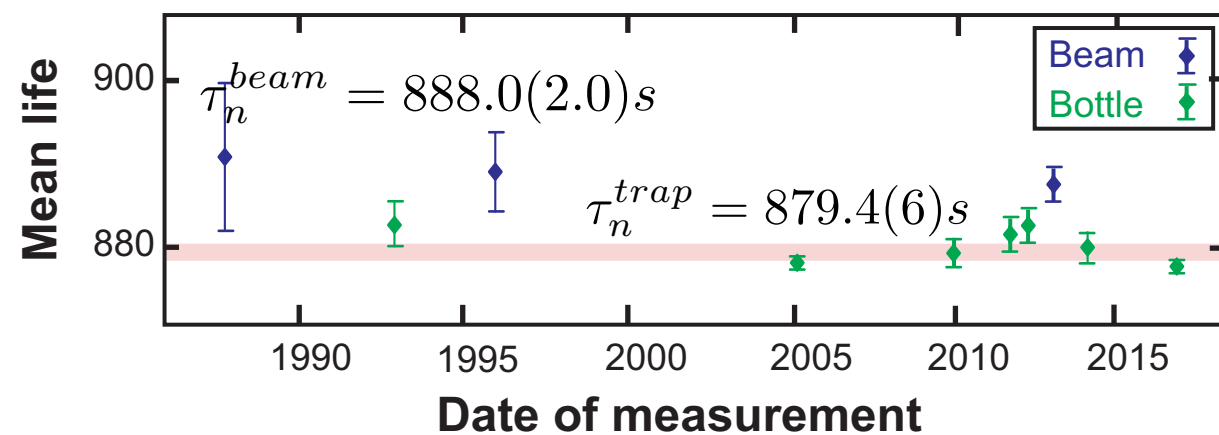
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May be combined with V_{ud} from 0^+-0^+ decays
eliminate RC (the same for n and nuclei)
 λ and lifetime are correlated

$$\tau_n(1 + 3\lambda^2) = 5172.0(1.1) \text{ s}$$

Czarnecki, Marciano, Sirlin, PRL '18

See Bill's talk

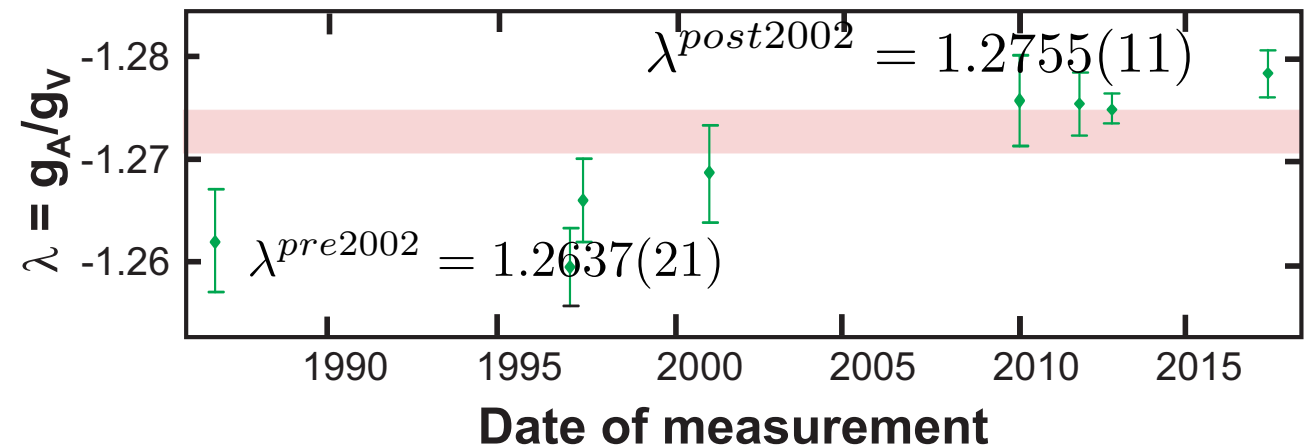
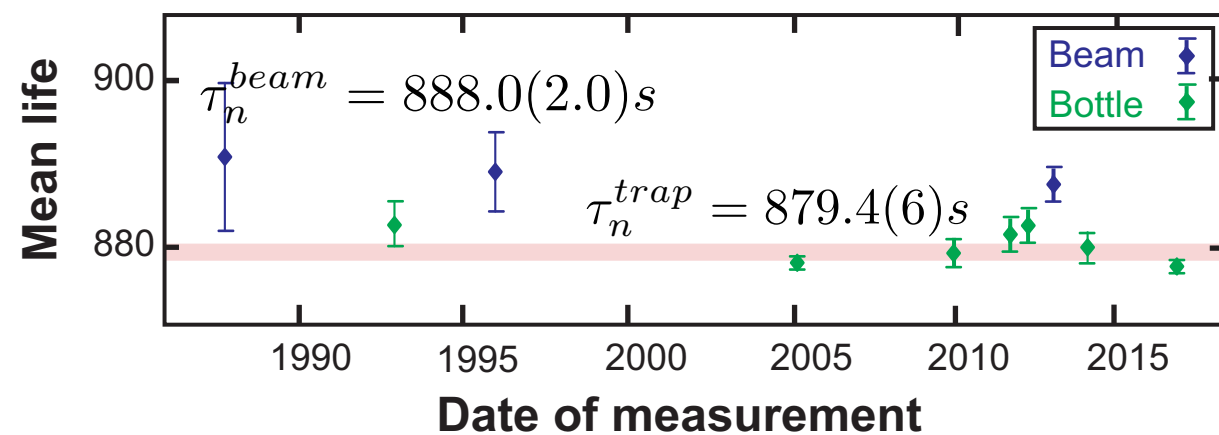
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Recently: PERKEO-III halved the uncertainty of λ $\lambda = -1.27641(45)_{\text{stat}}(33)_{\text{sys}}$

Märkisch et al, arXiv:1812.04666

Ongoing effort worldwide on improving τ_n to 0.2-0.3s

See Albert's talk

Outline: RC to Beta Decay

$$|V_{ud}| = 0.97420(10)_{Ft}(18)_{\Delta_R^V}$$

$$|V_{ud}|^2 = \frac{2984.432(3)}{\mathcal{F}t(1 + \Delta_R^V)}$$

$$\mathcal{F}t = ft(1 + \delta'_R)[1 - (\delta_C - \delta_{NS})]$$

Three caveats:

1. Calculation of the universal free-neutron RC Δ_R^V
2. Splitting the full nuclear RC into free-neutron Δ_R^V and nuclear modification δ_{NS}
3. Splitting the full RC into “outer” (energy-dependent but pure QED: no hadron structure) and “inner” (hadron&nuclear structure-dependent but energy-independent)
 - nucleon and nuclear case

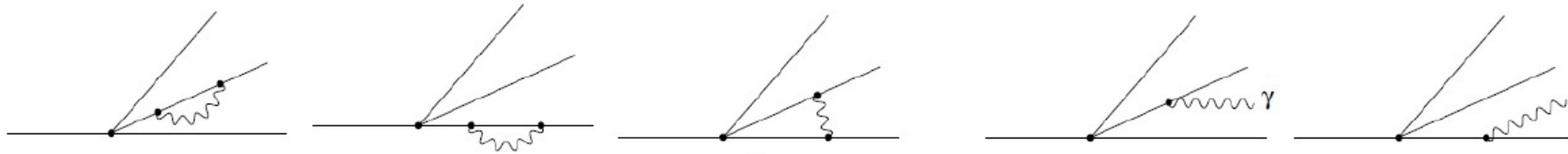
Will address each point

1. Check radiative corrections to the free neutron decay

C-Y Seng, MG, H Patel, M J Ramsey-Musolf, arXiv: 1807.10197

RC on the free neutron

Outer (depend on e-energy): retain only IR divergent pieces



Separation due to scale hierarchy: $m_e = 0.511$ MeV, $Q = M_n - M_p = 1.3$ MeV;
 Q/m_e not small, need to account for exactly.

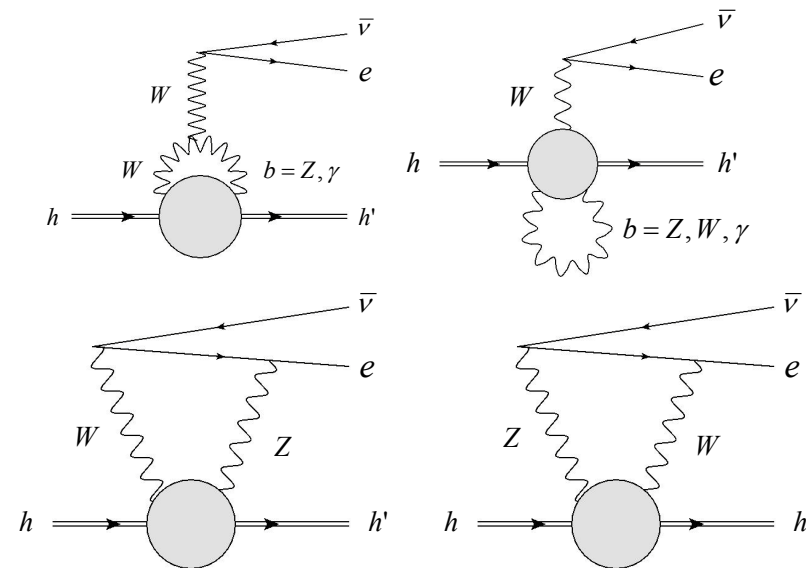
Coulomb distortion: resummation of $(Z\alpha)^n \rightarrow$ Dirac equation in the Coulomb field

IR finite piece: can set $m_e=0 \rightarrow$ if energy-dependent $\sim (\alpha/2\pi) \times (E/\Lambda_{\text{had}})$

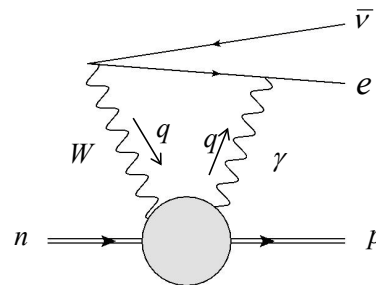
Hadronic structure: relevant scale $\sim m_\pi = 140$ MeV - on top of $\alpha/2\pi \sim 10^{-3} \rightarrow 10^{-5}$ effect \ll

Inner (energy-independent - take $E=0$)

W,Z-exchange:
 UV-sensitive, pQCD;
 model-independent



When γ involved:
 sensitive to long range physics;
 model-dependent!

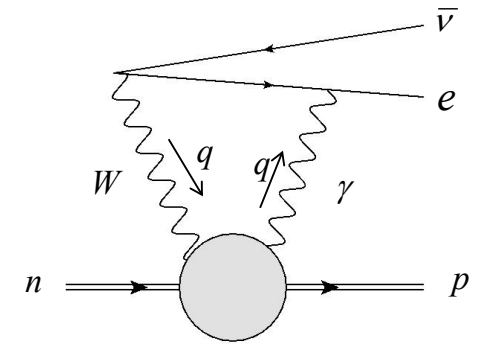


Until recently: best determination Marciano & Sirlin 2006

$$\Delta_R^V = 0.02361(38)$$

γW -box

Box at zero energy and momentum transfer



$$T_{\gamma W} = \sqrt{2}e^2 G_F V_{ud} \int \frac{d^4 q}{(2\pi)^4} \frac{\bar{u}_e \gamma^\mu (\not{k} - \not{q} + m_e) \gamma^\nu (1 - \gamma_5) v_\nu}{q^2 [(k - q)^2 - m_e^2]} \frac{M_W^2}{q^2 - M_W^2} T_{\mu\nu}^{\gamma W}$$

Hadronic tensor: two-current correlator

$$T_{\gamma W}^{\mu\nu} = \int dx e^{iqx} \langle p | T[J_{em}^\mu(x) J_W^\nu(0)] | n \rangle$$

General gauge-invariant decomposition (spin-independent)

$$T_{\gamma W}^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) T_1 + \frac{1}{(p \cdot q)} \left(p - \frac{(p \cdot q)}{q^2} q \right)^\mu \left(p - \frac{(p \cdot q)}{q^2} q \right)^\nu T_2 + \frac{i\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2(p \cdot q)} T_3$$

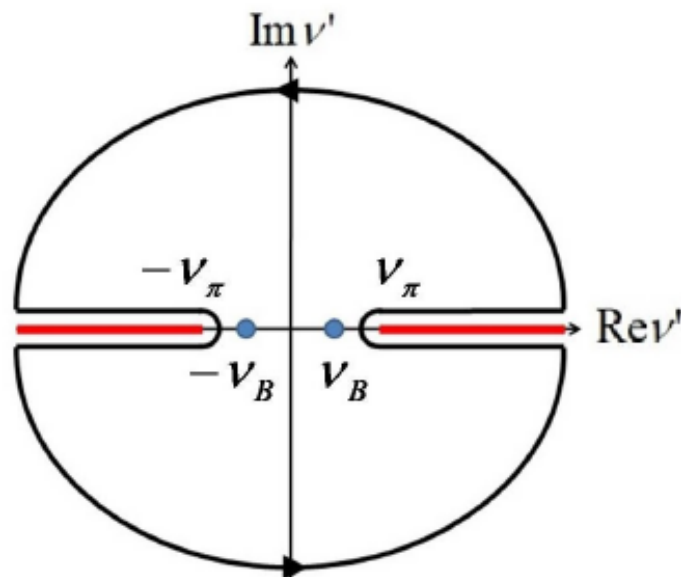
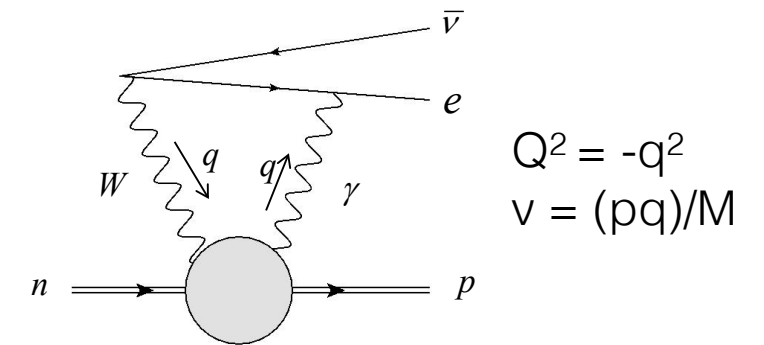
V-V correlator $T_{1,2}$: conserved vector-isovector current - model-independent
Sirlin 1967 - current algebra

Axial current not conserved \rightarrow A-V correlator T_3 - model-dependent

γW -box from Dispersion Relations

Check MS result + uncertainty independently

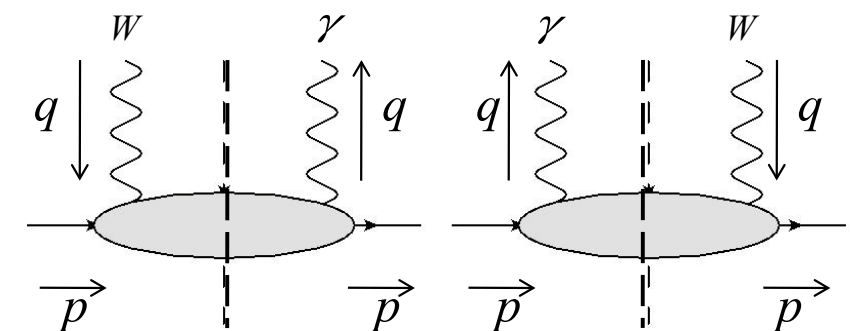
$$\Box_{\gamma W}^{VA} = 4\pi\alpha \text{Re} \int \frac{d^4 q}{(2\pi)^4} \frac{M_W^2}{M_W^2 + Q^2} \frac{Q^2 + \nu^2}{Q^4} \frac{T_3(\nu, Q^2)}{M\nu}$$



T_3 - analytic function inside the contour C in the complex ν -plane determined by its singularities on the real axis - poles + cuts

$$T_3(\nu, Q^2) = \frac{1}{2\pi i} \oint_C \frac{T_3(z, Q^2) dz}{z - \nu} \quad \nu \in C$$

Forward amplitude T_3 - unknown;
Its absorptive part can be related to
production of on-shell intermediate states
a γW -analog of the SF F_3



$$\text{Im } T_3^{\gamma W}(\nu, Q^2) = 2\pi F_3^{\gamma W}(\nu, Q^2)$$

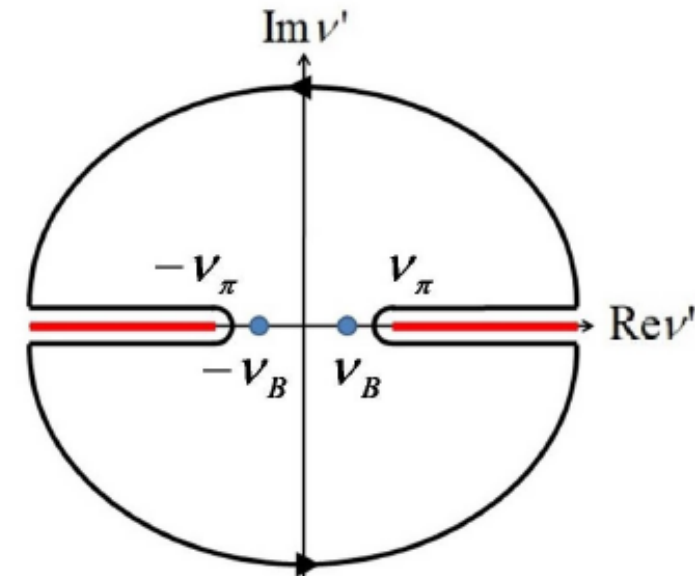
γW -box from Dispersion Relations

Crossing behavior: photon is isoscalar or isovector

Different isospin channels behave differently

$$T_3^{\gamma W} = T_3^{(0)} + T_3^{(3)}$$

$$T_3^{(0)}(-\nu, Q^2) = -T_3^{(0)}(\nu, Q^2), \quad T_3^{(3)}(-\nu, Q^2) = +T_3^{(3)}(\nu, Q^2)$$



Dispersion representation of the γW -box correction at zero energy

$$\square_{\gamma W}^{VA(0)} = \frac{\alpha}{\pi M} \int_0^\infty \frac{dQ^2 M_W^2}{M_W^2 + Q^2} \int_0^\infty \frac{d\nu(\nu + 2q)}{\nu(\nu + q)^2} F_3^{(0)}(\nu, Q^2)$$

$$\square_{\gamma W}^{VA(3)} = 0 \quad q = \sqrt{\nu^2 + Q^2}$$

First Nachtmann moment of F_3

$$M_3^{(0)}(1, Q^2) = \frac{4}{3} \int_0^1 dx \frac{1 + 2\sqrt{1 + 4M^2 x^2 / Q^2}}{(1 + \sqrt{1 + 4M^2 x^2 / Q^2})^2} F_3^{(0)}(x, Q^2)$$

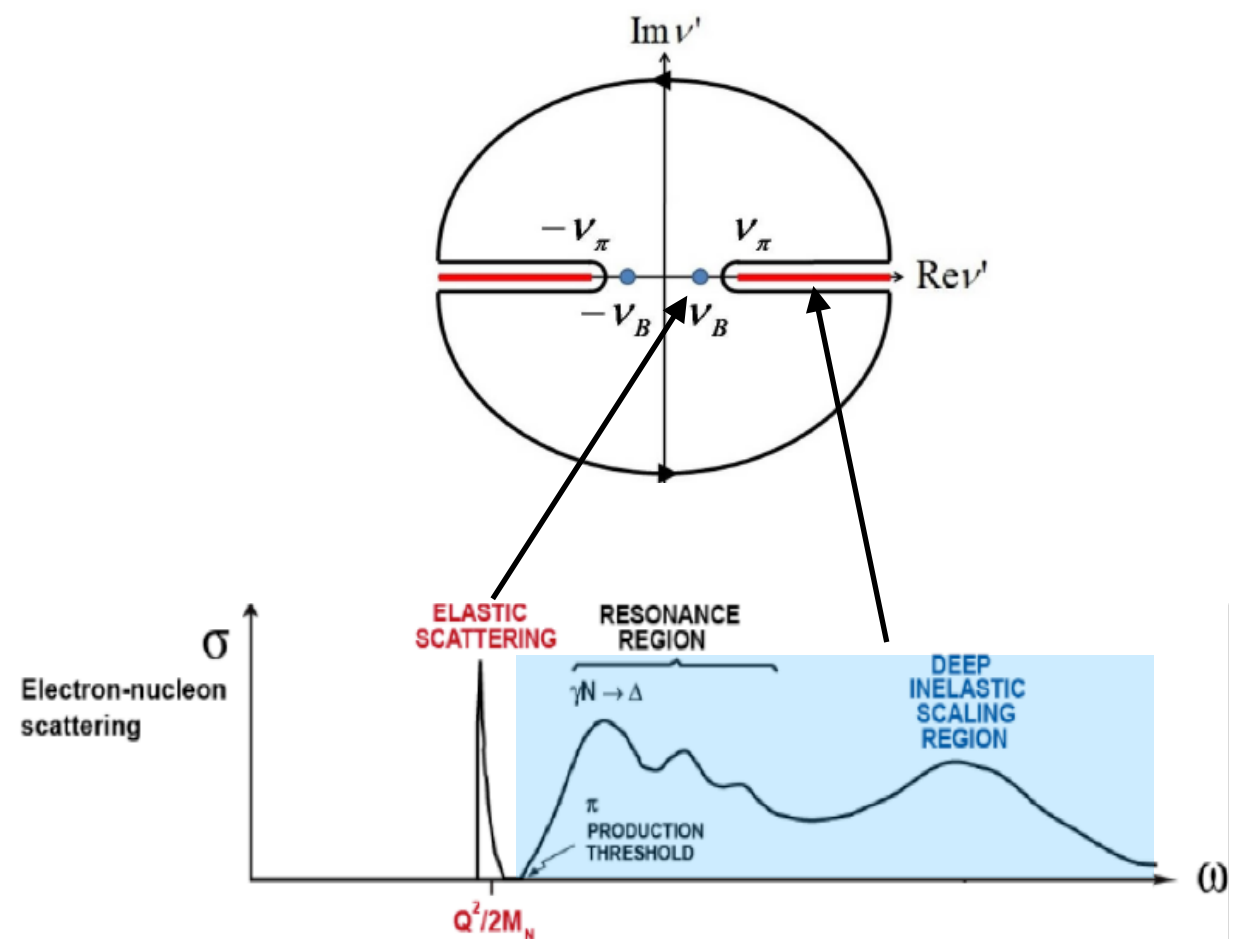
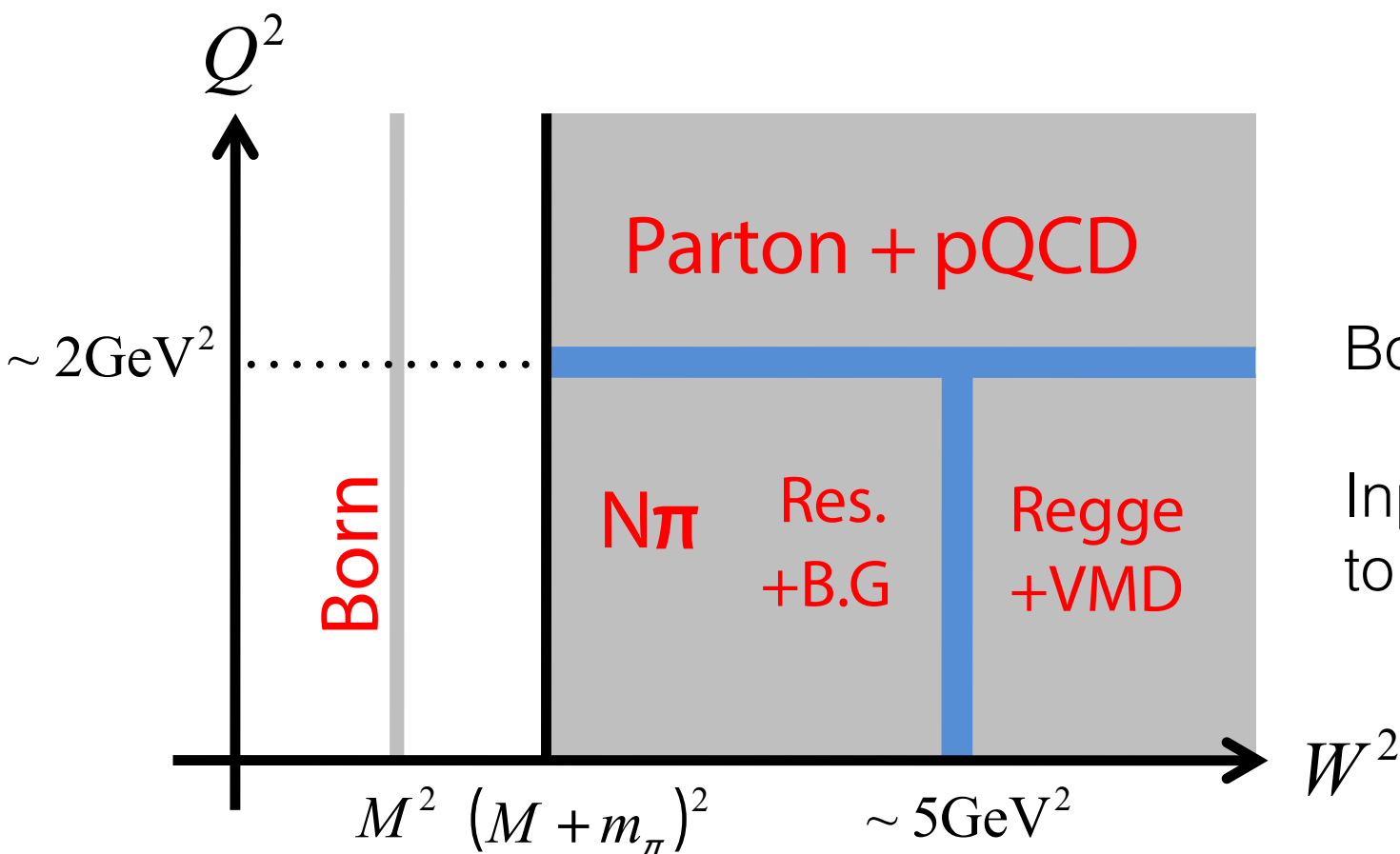
$$\square_{\gamma W}^{VA(0)} = \frac{3\alpha}{2\pi} \int_0^\infty \frac{dQ^2 M_W^2}{M_W^2 + Q^2} M_3^{(0)}(1, Q^2)$$

(Nachtmann moment = Mellin moment + kinematical higher twist)

Input into dispersion integral

Dispersion in energy: $W^2 = M^2 + 2M\nu - Q^2$
scanning hadronic intermediate states

Dispersion in Q^2 :
scanning dominant physics pictures



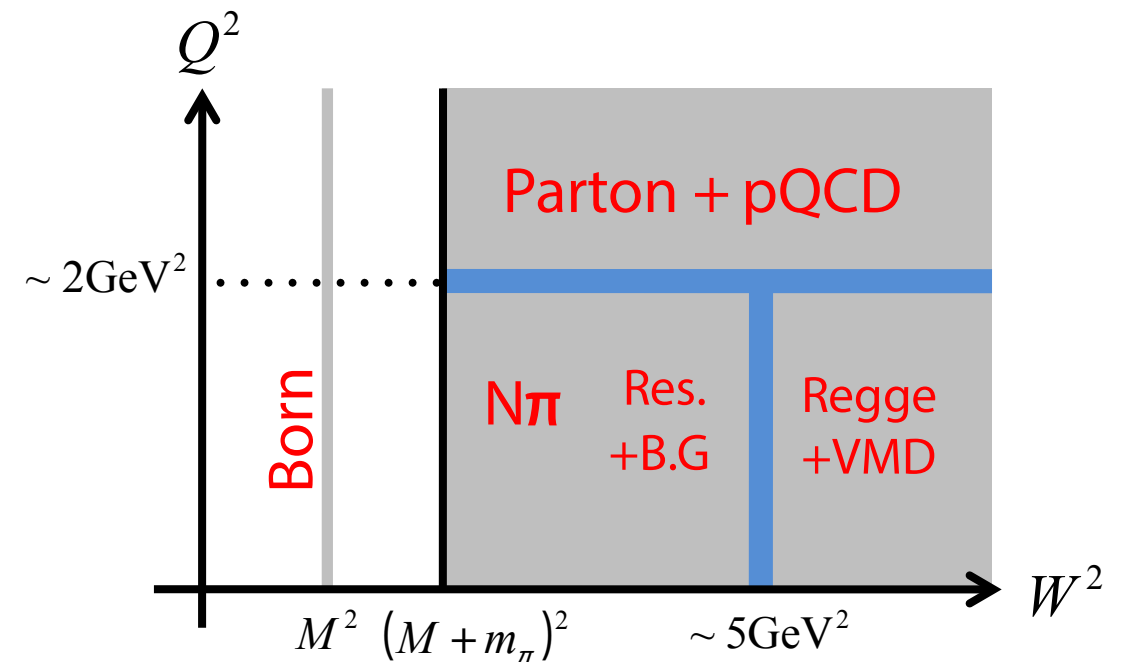
Boundaries between regions - approximate

Input in DR related (directly or indirectly)
to experimentally accessible data

Input into dispersion integral

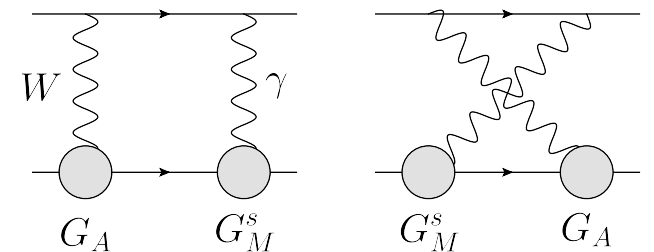
Our parametrization of the needed SF follows from this diagram

$$F_3^{(0)} = F_{\text{Born}} + \begin{cases} F_{\text{pQCD}}, & Q^2 \gtrsim 2 \text{ GeV}^2 \\ F_{\pi N} + F_{\text{res}} + F_{\mathbb{R}}, & Q^2 \lesssim 2 \text{ GeV}^2 \end{cases}$$



Born: elastic FF from e^- , ν scattering data

$$\square_{\gamma W}^{VA, \text{Born}} = -\frac{\alpha}{\pi} \int_0^\infty dQ \frac{2\sqrt{4M^2 + Q^2} + Q}{\left(\sqrt{4M^2 + Q^2} + Q\right)^2} G_A(Q^2) G_M^S(Q^2)$$



πN :

relativistic ChPT calculation plus nucleon FF

Resonances:

axial excitation from PCAC (Lalakulich et al 2006) - neutrino scattering

isoscalar photo-excitation from MAID and PDG - electron and γ inelastic scattering

Above resonance region:

multiparticle continuum economically described by Regge exchanges

Input into dispersion integral

Unfortunately, no data can be obtained for $F_3^{\gamma W(0)}$

Data exist for the pure CC processes

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dxdy} = \frac{G_F^2 ME}{\pi} \left[xy^2 F_1 + \left(1 - y - \frac{Mxy}{2E}\right) F_2 \pm x \left(y - \frac{y^2}{2}\right) F_3 \right]$$

$$\sigma^{\nu p} - \sigma^{\bar{\nu} p} \sim F_3^{\nu p} + F_3^{\bar{\nu} p} = u_v^p(x) + d_v^p(x)$$

Gross-Llewellyn-Smith sum rule $\int_0^1 dx (u_v^p(x) + d_v^p(x)) = 3$

Validate the model for CC process; apply an isospin rotation to obtain γW

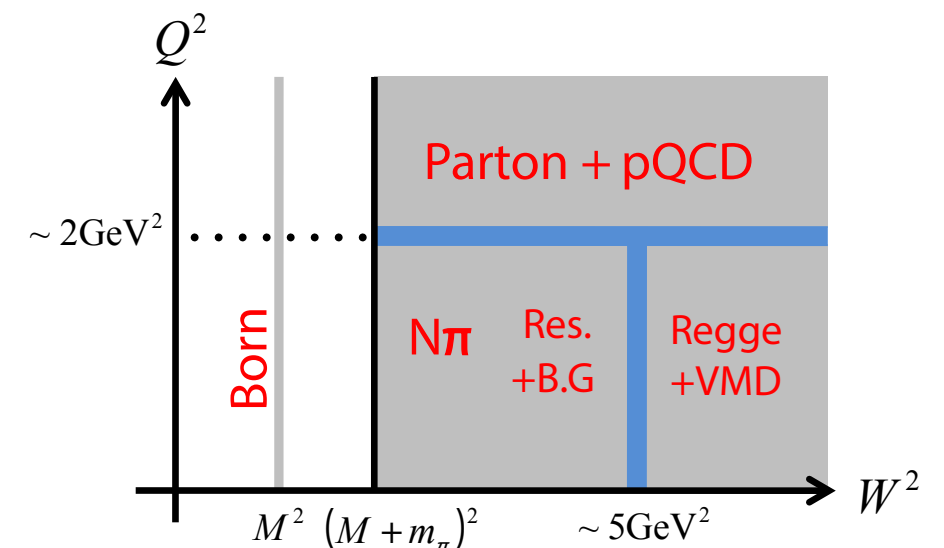
$$F_{3, \text{low-}Q^2}^{\nu p + \bar{\nu} p} = F_{3, \text{el.}}^{\nu p + \bar{\nu} p} + F_{3, \pi N}^{\nu p + \bar{\nu} p} + F_{3, R}^{\nu p + \bar{\nu} p} + F_{3, \text{Regge}}^{\nu p + \bar{\nu} p}$$

Low-W part of spectrum:

neutrino data from MiniBooNE, Minerva, ...

- axial FF, resonance contributions, pi-N continuum

High-W: Regge behavior $F_3 \sim q^{\nu} \sim x^{-\alpha}$, $\alpha \sim 0.5-0.7$



Parameters of the Regge model from neutrino scattering

Low $Q^2 < 0.1 \text{ GeV}^2$: Born + $\Delta(1232)$ dominate
 Not fitted: modern data more precise but
 cover only limited energy range
 Fit driven by 4 data points between 0.2 and 2 GeV^2

Model & Uncertainty fully specified
 - compare M&S vs This work

$$M_3^{WW}(1, Q^2)$$



Isospin symmetry

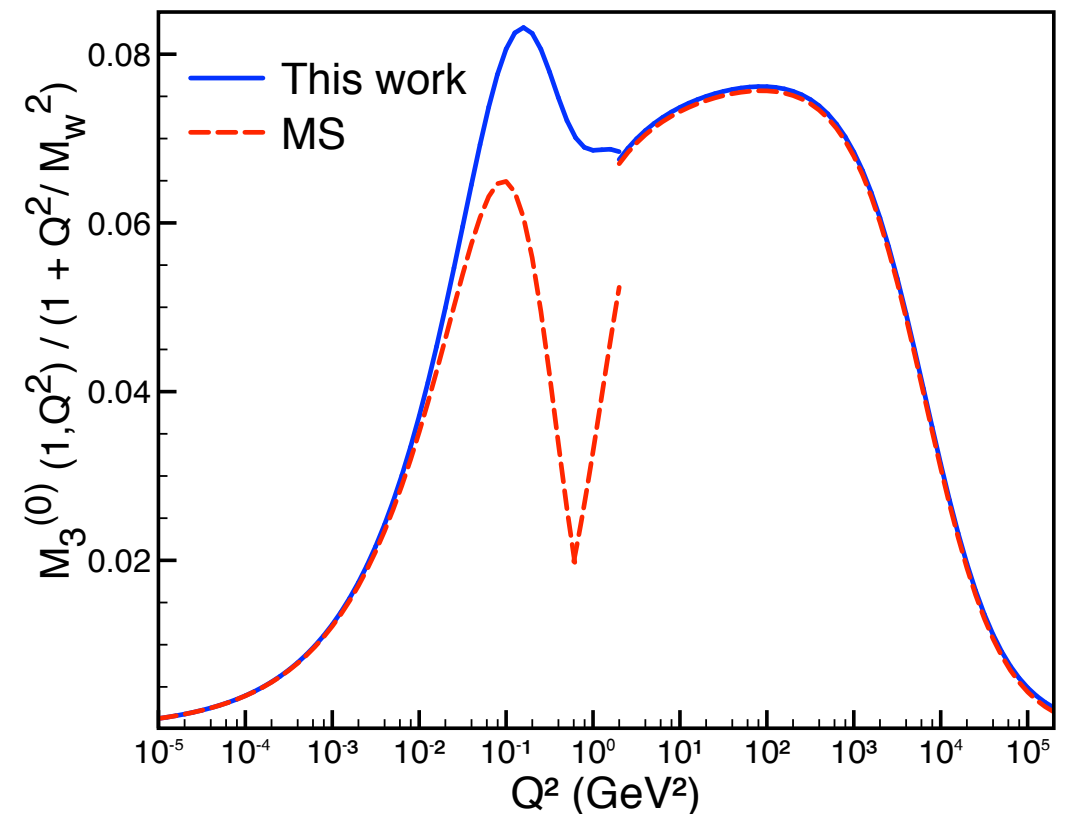
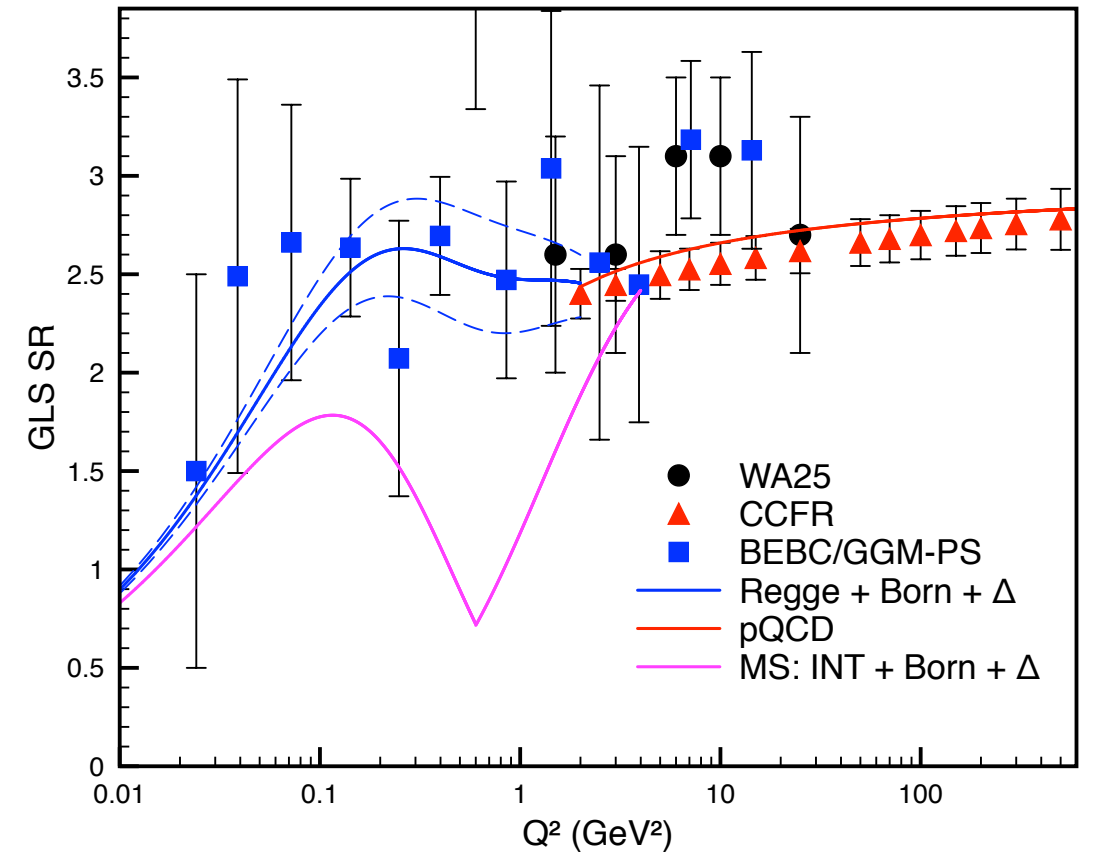
$$M_3^{\gamma W}(1, Q^2)$$

Log scale for x-axis: integral = surface under the curve

MS Total : $\square_{\gamma W}^{(0)} = 0.00324 \pm 0.00018$

New Total : $\square_{\gamma W}^{(0)} = 0.00379 \pm 0.00010$

Uncertainty reduced by almost factor 2;
 ~ 3-5 sigma shift from the old value



Universal γW -box

$$|V_{ud}|^2 = \frac{2984.432(3) \text{ s}}{\mathcal{F}t(1 + \Delta_R^V)}$$

Marciano & Sirlin 2006

Dispersion relations

$$\Delta_R^V = 0.02361(38)$$

$$\Delta_R^V = 0.02467(22)$$

$$|V_{ud}| = 0.97420(10)_{Ft(18)} \Delta_R^V$$

$$|V_{ud}| = 0.97370(10)_{Ft(10)} \Delta_R^V$$

DR allowed to reduce the uncertainty in Δ_R^V by almost factor of 2
due to the use of neutrino data

But the shift is more significant than anticipated from the uncertainty estimate by MS

Tension with CKM unitarity

Before

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994 \pm 0.0005$$

After

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9984 \pm 0.0004$$

2. Radiative corrections to nuclear decays: Nuclear structure modification of the free-n RC

C-Y Seng, MG, M J Ramsey-Musolf, arXiv: 1812.03352

Splitting the γW -box into Universal and Nuclear Parts

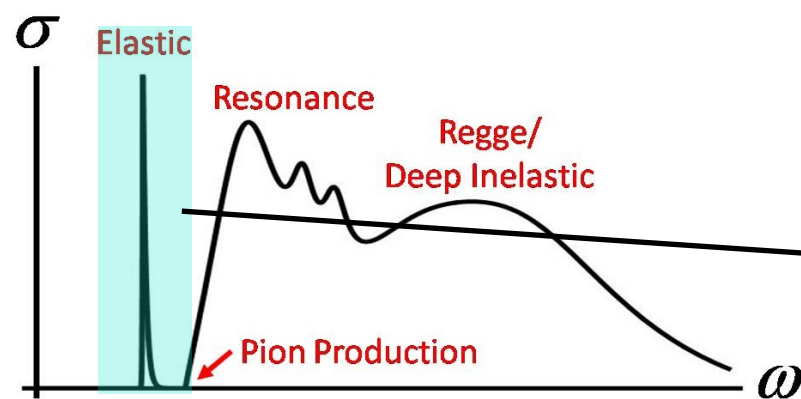
General structure of RC for nuclear decay

$$ft(1 + RC) = Ft(1 + \delta'_R)(1 - \delta_C + \delta_{NS})(1 + \Delta_R^V)$$

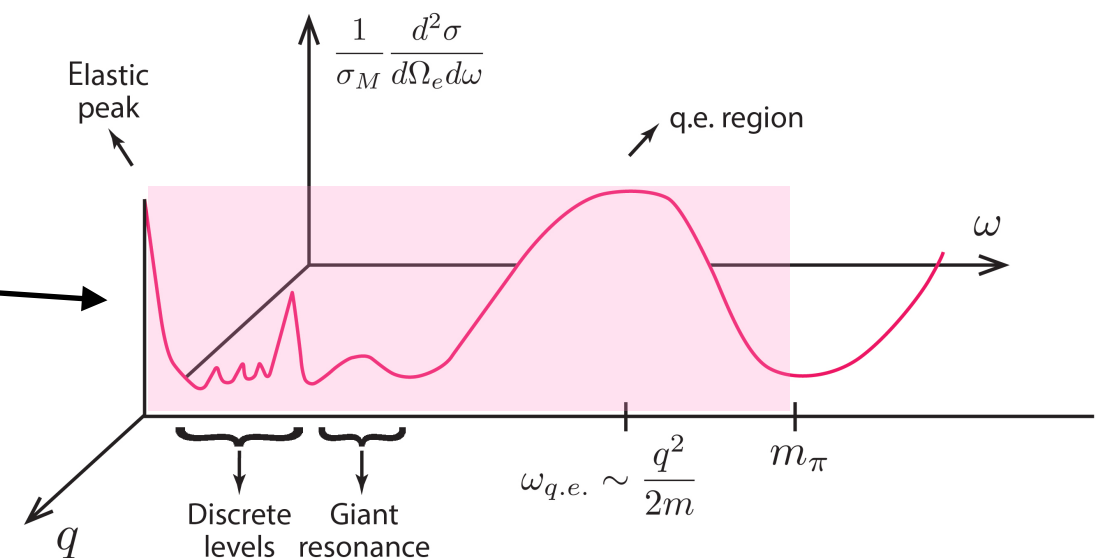
$\uparrow \qquad \qquad \qquad \uparrow$
 $\delta_{NS} \qquad \Delta_R^V$

$$\square_{\gamma W}^{\text{VA, Nucl.}} = \square_{\gamma W}^{\text{VA, free n}} + \left[\square_{\gamma W}^{\text{VA, Nucl.}} - \square_{\gamma W}^{\text{VA, free n}} \right]$$

Input in the DR for the universal RC



Input in the DR for the RC on a nucleus



Splitting the full RC into “universal” and “nuclear structure” is natural if the two pieces are treated differently.

In the dispersion framework this splitting is unnatural: just calculate the RC on a nucleus

Towner 1994: elastic box is quenched in nuclei;
size of quenching - from quenching of 1-body spin operators

$$\delta_{NS}^{\text{quenched Born}} \sim (q_M^S q_A - 1) C_B$$

Modification of C_B in a nucleus - QE

Integral is peaked at low ν , Q^2

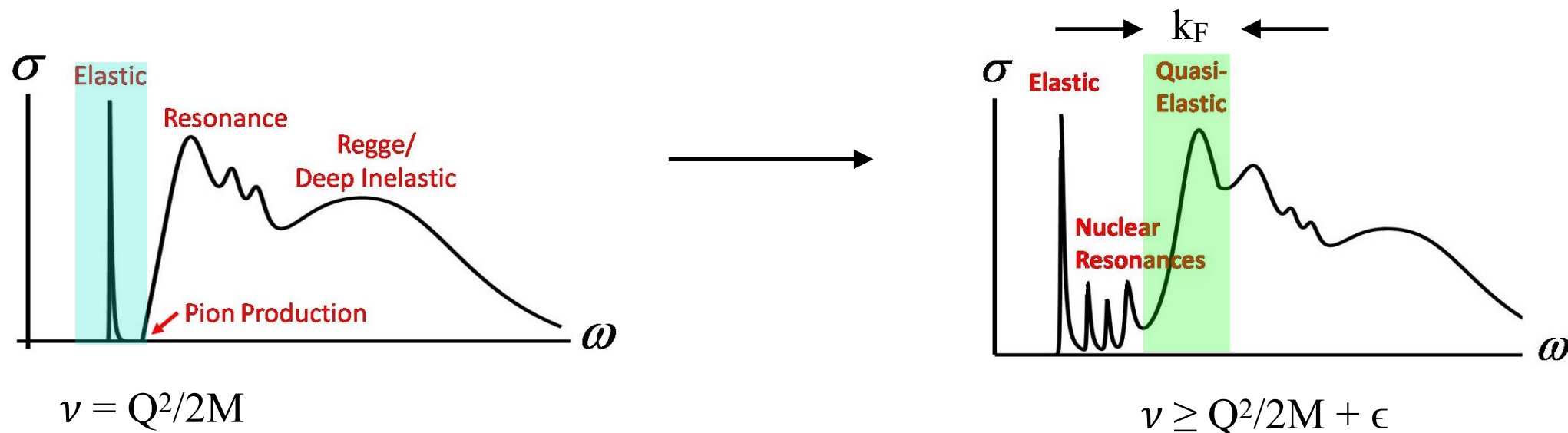
$$\square_{\gamma W}^{VA, Nucl.} = \frac{\alpha}{N\pi M} \int_0^\infty \frac{dQ^2 M_W^2}{M_W^2 + Q^2} \int_0^\infty d\nu \frac{(\nu + 2q)}{\nu(\nu + q)^2} F_{3, \gamma W}^{(0), Nucl.}(\nu, Q^2)$$

Born on free n:

$$F_3^{(0), B} = -\frac{Q^2}{4} G_A G_M^S \delta(2M\nu - Q^2)$$

Reduction for QE:

finite threshold ϵ (binding energy) + Fermi momentum k_F



Exploratory QE calculation in free Fermi gas model with Pauli blocking;

Disregard fine detail; estimate the bulk effect averaged over all superallowed decays

C-Y Seng, MG, M J Ramsey-Musolf, arXiv: 1812.03352

New $\langle \delta_{NS}^{QE} \rangle \sim -0.10(1)\%$ to be compared to the
H&T quenched result averaged over 20 decays $\langle \delta_{NS}^{quenched} \rangle \sim -0.055(5)\%$

QE calculation - effect on Ft values and V_{ud}

Adopting a new estimate of the in-nucleus modification of the free-nucleon Born

Shifts the Ft value according to $\overline{\mathcal{F}t} \rightarrow \overline{\mathcal{F}t}(1 + \delta_{NS}^{new} - \delta_{NS}^{old})$

Numerically: $\mathcal{F}t = 3072.07(63)s \rightarrow [\mathcal{F}t]^{new} = 3070.65(63)(28)s$

Will affect the extracted V_{ud} $|V_{ud}|^2 = \frac{2984.432(3)s}{\mathcal{F}t(1 + \Delta_R^V)}$

Compensates for a part of the shift due to a new evaluation of Δ_R^V

$$V_{ud}^{old} = 0.97420(21) \rightarrow V_{ud}^{new} = 0.97370(14) \rightarrow V_{ud}^{new, QE} = 0.97392(14)(04)$$

Brings the first row a little closer to the unitarity ($4\sigma \rightarrow 3\sigma$)

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9984 \pm 0.0004 \rightarrow |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9988 \pm 0.0004$$

Important message:

Dispersion relations as a unified tool for treating hadronic and nuclear parts of RC

Further work to make the model adequate for the needed precision

3.Splitting of the RC into inner and outer

MG, arXiv: 1812.04229

Splitting the RC into “inner” and “outer”

Radiative corrections $\sim \alpha / 2\pi \sim 10^{-3}$

Precision goal: $\sim 10^{-4}$

When does energy dependence matter?

Correction $\sim E_e/\Lambda$, with $\Lambda \sim$ relevant mass (m_e ; M_p ; M_A)

Maximal E_e ranges from 1 MeV to 10.5 MeV

Electron mass regularizes the IR divergent parts - (E_e/m_e important) - “outer” correction

If Λ of hadronic origin (at least m_π) $\rightarrow E_e/\Lambda$ small, correction $\sim 10^{-5} \rightarrow$ negligible

- certainly true for the neutron decay
- hadronic contributions do not distort the spectrum, may only shift it as a whole

However, in nuclei binding energies \sim few MeV — similar to Q-values

A scenario is possible when $RC \sim (\alpha / 2\pi) \times (E_e/\Lambda^{\text{Nucl}}) \sim 10^{-3}$

Nuclear structure may distort the electron spectrum

With dispersion relations can be checked straightforwardly!

Nuclear structure distorts the β -spectrum!

With DR: can include energy dependence explicitly

Even and odd powers of energy - leading terms

$$\text{Re } \square_{\gamma W}^{\text{even}} = \frac{\alpha}{\pi N} \int_0^\infty dQ^2 \int_{\nu_{thr}}^\infty d\nu \frac{F_3^{(0)}}{M\nu} \frac{\nu + 2q}{(\nu + q)^2} + O(E^2)$$

$$\text{Re } \square_{\gamma W}^{\text{odd}}(E) = \frac{8\alpha E}{3\pi N M} \int_0^\infty dQ^2 \int_{\nu_{thr}}^\infty \frac{d\nu}{(\nu + q)^3} \left[\mp F_1^{(0)} \mp \left(\frac{3\nu(\nu + q)}{2Q^2} + 1 \right) \frac{M}{\nu} F_2^{(0)} + \frac{\nu + 3q}{4\nu} F_3^{(-)} \right] + O(E^3)$$

E-dependent correction from dimensional analysis: nuclear radii and polarizabilities

Nuclear excitations live at few MeV \rightarrow large nuclear polarizabilities

$$\alpha_E = \frac{2\alpha_{em}}{M} \int \frac{d\omega}{\omega^3} F_1(\omega, Q^2 = 0) = 2\alpha_{em} \int \frac{d\omega}{\omega^2} \frac{\partial}{\partial Q^2} F_2(\omega, Q^2 = 0)$$

Assume Q^2 dependence to follow that of charge form factor $\sim \text{Exp}[-R_{Ch}^2 Q^2/6]$

New energy scale:
polarizability/radius²

$$\text{Re } \square_{\gamma W}^{\text{odd}} \sim \mp \frac{2E\alpha_E}{\pi N R_{Ch}^2}$$

$$R_{Ch} \sim 1.2 \text{ fm } A^{1/3}$$

$$\alpha_E \sim (2.2 \times 10^{-3} \text{ fm}) A^{5/3}$$

Expect $\text{Re } \square_{\gamma W}^{\text{odd}} \sim \mp 1 \times 10^{-5} \frac{E}{\text{MeV}} \frac{A}{N}$

Nuclear structure distorts the β -spectrum!

E-dependent correction from free Fermi gas model (A-V piece)

- same model as was used for the E-independent piece

$$\text{Re } \square_{\gamma W}^{odd} = (1.4 \pm 0.2) \times 10^{-4} \frac{E}{\text{MeV}}$$

Almost an o.o.m. larger than the estimate with polarizabilities
(large isovector magnetic moment)

How reliable? Strong dependence on fine details of nuclear structure

Rough estimate (bound) on the E-dependent correction to diff. spectrum

$$\Delta_R^{NS}(E) = 2\text{Re } \square_{\gamma W}^{odd} = (1.6 \pm 1.6) \times 10^{-4} \frac{E}{\text{MeV}}$$

Correction to Ft values: integrate over spectrum (only total rate measured)

$$\Delta_E^{NS} = \frac{\int_{m_e}^{E_m} dE E p(Q - E)^2 \Delta_R(E)}{\int_{m_e}^{E_m} dE E p(Q - E)^2} \longrightarrow \tilde{F}t = ft(1 + \delta'_R)(1 - \delta_C + \delta_{NS} + \Delta_E^{NS})$$

Nuclear structure distorts the β -spectrum!

$$\tilde{\mathcal{F}}t = ft(1 + \delta'_R)(1 - \delta_C + \delta_{NS} + \Delta_E^{NS})$$

Absolute shift in Ft values $\delta\mathcal{F}t = \mathcal{F}t \times \Delta_E^{NS}$

Decay	Q (MeV)	$\Delta_E^{NS}(10^{-4})$	$\delta\mathcal{F}t(s)$	$\mathcal{F}t(s)$ [3]
^{10}C	1.91	1.5	0.5	3078.0(4.5)
^{14}O	2.83	2.3	0.7	3071.4(3.2)
^{22}Mg	4.12	3.3	1.0	3077.9(7.3)
^{34}Ar	6.06	4.8	1.5	3065.6(8.4)
^{38}Ca	6.61	5.3	1.6	3076.4(7.2)
^{26m}Al	4.23	3.4	1.0	3072.9(1.0)
^{34}Cl	5.49	4.4	1.4	$3070.7^{+1.7}_{-1.8}$
^{38m}K	6.04	4.8	1.5	3071.6(2.0)
^{42}Sc	6.43	5.1	1.6	3072.4(2.3)
^{46}V	7.05	5.6	1.7	3074.1(2.0)
^{50}Mn	7.63	6.1	1.9	3071.2(2.1)
^{54}Co	8.24	6.6	2.0	$3069.8^{+2.4}_{-2.6}$
^{62}Ga	9.18	7.3	2.2	3071.5(6.7)
^{74}Rb	10.42	8.3	2.6	3076(11)

Shift due to Δ_E^{NS} : comparable to precision of 7 best-known decays

$$\overline{\mathcal{F}}t = 3072.07(63)\text{s} \rightarrow \overline{\mathcal{F}}t = 3073.6(0.6)(1.5)\text{s}$$

Decay electron polarizes the daughter nucleus

As a result the spectrum is slightly distorted towards the upper end

Positive-definite correction to Ft $\sim 0.05\%$

Previously found: E-independent piece lowers the Ft value by about the same amount

$$\mathcal{F}t = 3072.07(63)\text{s} \rightarrow [\mathcal{F}t]^{\text{new}} = 3070.65(63)(28)\text{s}$$

No evidence of a **net shift** of the Ft value when combined together; uncertainty?

CKM first-row unitarity constraint is low.
Solutions: SM or beyond?

Discrepancy - BSM?

BSM explanation: non-standard CC interactions \longrightarrow new V,A,S(PS),T(PT) terms

$$H_{S+V} = (\bar{\psi}_p \psi_n)(C_S \bar{\phi}_e \phi_{\bar{\nu}_e} + C'_S \bar{\phi}_e \gamma_5 \phi_{\bar{\nu}_e}) + (\bar{\psi}_p \gamma_\mu \psi_n) [C_V \bar{\phi}_e \gamma_\mu (1 + \gamma_5) \phi_{\bar{\nu}_e}]$$

Scalar and Tensor interactions: distort the beta decay spectra

Complementarity to LHC searches!

Exp. high precision measurement of ${}^6\text{He}$ spectrum (O. Naviliat-Cuncic, A. Garcia, ...)

$$N(E)dE = p_e E (E_m - E)^2 \left[1 + C_1 E + b \frac{m_e}{E} \right]$$

$C_1 = 0.00650$ (7) MeV^{-1} - effect of weak magnetism - positive slope

$b \sim \pm 0.001$ - negative slope

Energy-dep. polarizability correction $\longrightarrow C'_1 \sim 0.00020$ (20) MeV^{-1} — at the level 3σ of C_1

Conclusions & Outlook

- The γW -box was evaluated in a new dispersion relation framework
- Confirmed dominant features of previous calculations but corrected subdominant ones
- Related the model-dependent contribution to neutrino data - systematically improvable!
- Hadronic and nuclear corrections in a unified framework
- Nuclear structure leaks in the outer correction, distorts the beta decay spectrum
- Nuclear uncertainties shift the emphasis on free neutron decay
- Tensions with CKM unitarity: $\sum_{i=d,s,b} |V_{ui}|^2 - 1 = -0.0016(4-6)$

Hadronic correction Δ_R^V

Neutrino data at low Q^2 used in this analysis are not precise
DUNE@Fermilab will provide better data for F_3 - direct check
Moments $M_3^{(0)}(N, Q^2)$ at $\sim 1 \text{ GeV}^2$ on the lattice - doable!

Nuclear correction δ_{Ns}

DR allow to address hadronic and nuclear parts of the calculation on the same footing
Better calculations than free Fermi gas are needed
The full nuclear correction should be calculated (not just QE)

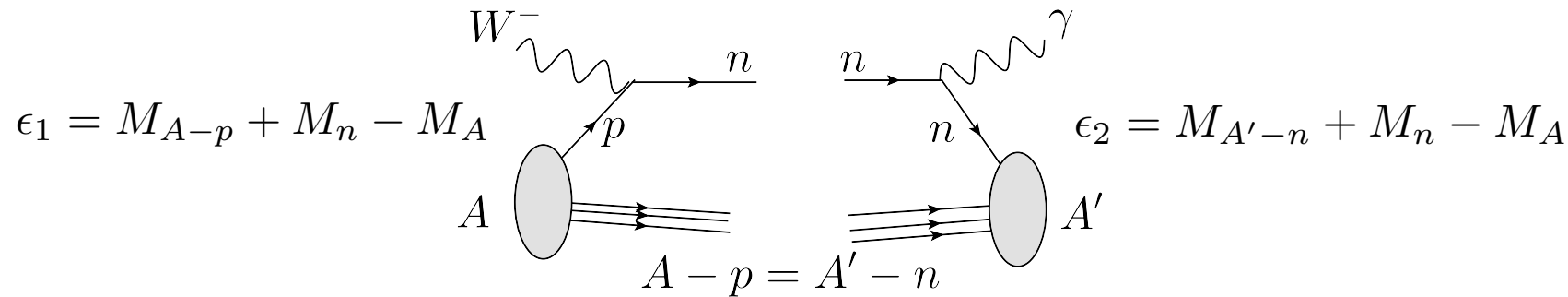
Decay spectra and nuclear polarizabilities

This novel effect needs a confirmation in more sophisticated models
Can contaminate the extraction of Fierz interference from precise spectra!

QE contribution to the γW -box

Bulk nuclear properties: Fermi momentum and break-up threshold

20 decays: $^{10}\text{C} \rightarrow ^{10}\text{B}$ through $^{74}\text{Rb} \rightarrow ^{74}\text{Kr}$



$$\bar{\epsilon} = \sqrt{\epsilon_1 \epsilon_2}$$

Effective removal energies - all in a small range

$$\bar{\epsilon} = 7.5 \pm 1.5 \text{ MeV}$$

Fermi momentum also not too different for all A

$$k_F(A = 10) = 228 \text{ MeV}, \quad k_F(A = 74) = 245 \text{ MeV}$$

Can define a universal correction that correctly represents bulk nuclear effect!

Further ingredients:

Free Fermi gas model (or superscaling)

+ Pauli blocking

Decay	ϵ_2 (MeV)	ϵ_1 (MeV)	$\bar{\epsilon}$ (MeV)
$^{10}\text{C} \rightarrow ^{10}\text{B}$	8.44	4.79	6.36
$^{14}\text{O} \rightarrow ^{14}\text{N}$	10.55	5.41	7.55
$^{18}\text{Ne} \rightarrow ^{18}\text{F}$	9.15	4.71	6.56
$^{22}\text{Mg} \rightarrow ^{22}\text{Na}$	11.07	6.28	8.34
$^{26}\text{Si} \rightarrow ^{26}\text{Al}$	11.36	6.30	8.46
$^{30}\text{S} \rightarrow ^{30}\text{P}$	11.32	5.18	7.66
$^{34}\text{Ar} \rightarrow ^{34}\text{Cl}$	11.51	5.44	7.91
$^{38}\text{Ca} \rightarrow ^{38}\text{K}$	12.07	5.33	8.02
$^{42}\text{Ti} \rightarrow ^{42}\text{Sc}$	11.55	4.55	7.25
$^{26m}\text{Al} \rightarrow ^{26}\text{Mg}$	11.09	6.86	8.72
$^{34}\text{Cl} \rightarrow ^{34}\text{S}$	11.42	5.92	8.22
$^{38m}\text{K} \rightarrow ^{38}\text{Ar}$	11.84	5.79	8.28
$^{42}\text{Sc} \rightarrow ^{42}\text{Ca}$	11.48	5.05	7.61
$^{46}\text{Va} \rightarrow ^{46}\text{Ti}$	13.19	6.14	9.00
$^{50}\text{Mn} \rightarrow ^{50}\text{Cr}$	13.00	5.37	8.35
$^{54}\text{Co} \rightarrow ^{54}\text{Fe}$	13.38	5.13	8.28
$^{62}\text{Ga} \rightarrow ^{62}\text{Zn}$	12.90	3.72	6.94
$^{66}\text{As} \rightarrow ^{66}\text{Ge}$	13.29	3.16	6.48
$^{70}\text{Br} \rightarrow ^{70}\text{Se}$	13.82	3.20	6.65
$^{74}\text{Rb} \rightarrow ^{74}\text{Kr}$	13.85	3.44	6.90