3.1 *Simple Harmonic Motion* (cf. Exercise 3.1 in the textbook)  
(2 pts.)

The simple pendulum is defined by the differential equation

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l} \theta.$$  

(1)

(use $g=9.8 \text{ m/s}^2$, $l=1 \text{ m}$).

(a) Write a FORTRAN program to numerically describe the corresponding motion of a simple pendulum using the Euler-Cromer Method. Plot the result and compare to the exact solution.

(b) Verify energy conservation by calculating total, kinetic and potential energy as a function of time and plot over 5 periods.

3.2 *Pendulum Motion and Chaos* (cf. Ex. 3.13, 3.14 in the textbook)  
(8 pts.)

Consider a damped, driven, nonlinear pendulum defined by the differential equation

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l} \sin(\theta) - 2\gamma \frac{d\theta}{dt} + \alpha_D \sin(\Omega_D t).$$  

(2)

(use $g=9.8 \text{ m/s}^2$, $l=9.8 \text{ m}$, $\gamma=0.25/\text{s}$, $\Omega_D=\frac{2}{3} \text{ rad/s}$, $\alpha_D=1.2 \text{ rad/s}^2$).

(a) Write a FORTRAN program to calculate $|\Delta\theta(t)|$ for several trajectories with slightly different initial angle (0.001 rad or so). Plot the results and estimate the Lyapunov exponent $\lambda$ of the system.

(b) Investigate the change of $\lambda$ under (moderate) variations of $\gamma$.

(c) Calculate the Poincaré section in phase space, $\omega(\theta)$, for the system in part (a) for times $t = n(2\pi)/\Omega_D$ (with integer $n=1, 2, \ldots$). Use a sufficiently long running time to map out and plot the strange attractors.