1.1 Cesium Isotope Decay (10 pts.)

The $^{137}\text{Cs}$ isotope $\beta$-decays with a half-life of $T_{1/2} = 30$ years (recall that the "lifetime" $\tau = 1/\Gamma$ as introduced in class is related to the half-life by $\tau = T_{1/2}/\ln 2$; why?). The initial size of the sample consists of 2 kg of $^{137}\text{Cs}$ (recall that 1 mol of a substance, which has a mass corresponding to the atomic mass-number in grams, contains $N_A=6.022 \cdot 10^{23}$ particles).

(a) Write a FORTRAN code to numerically calculate the activity of the sample, defined as $R_{Cs}(t) = -dN_{Cs}/dt$, over the first 20 years. Use a time-step width of 0.5 years. Plot the result in appropriate units using GNUPLOT together with the exact (analytical) solution in the same graph. Attach both the plot and your fortran source code.

(b) Increase the time-step width to 5 years and replot. Is the accuracy of the numerical solution still acceptable?