For problems 1, 2, and 3, do your work in the space provided, and write your final answer in the blank. Points will be deducted for the wrong units or wrong number of significant digits. Other than that, no partial credit will be awarded for incorrect answers.

1. (5 points) A dog in an open field runs 11.0 m west and then 29.0 m in a direction 57.0° east of north. In what direction and how far must the dog then run to end up 10.0 m south of her original starting point?

Direction \[ 27.3° \text{ W of S} \]

Distance \[ 29.0 \text{ m} \]

\[
\vec{d}_1 = (-11.0 \text{ m}) \hat{i} \\
\vec{d}_2 = (29.0 \text{ m}) (\sin 57.0° \hat{i} + \cos 57.0° \hat{j}) = (24.3 \text{ m}) \hat{i} + (15.8 \text{ m}) \hat{j}
\]

\[
\vec{d}_{\text{tot}} = (-10.0 \text{ m}) \hat{j} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 \Rightarrow \vec{d}_3 = \vec{d}_{\text{tot}} - \vec{d}_1 - \vec{d}_2
\]

\[
\vec{d}_3 = \left[ -13.3 \text{ m} \right] \hat{i} + \left[ -25.8 \text{ m} \right] \hat{j}
\]

\[
\theta = \tan^{-1} \left( \frac{-13.3 \text{ m}}{-25.8 \text{ m}} \right) = 27.3°
\]

\[
\text{Dist} = \sqrt{(-13.3 \text{ m})^2 + (-25.8 \text{ m})^2} = 29.0 \text{ m}
\]

2. (5 points) An antelope moving with constant acceleration covers the distance between two points 60.0 m apart in 6.80 s. Its speed as it passes the second point is 13.0 m/s. What is its acceleration?

Acceleration \[ 1.23 \text{ m/s}^2 \]

\[
\Delta x = \frac{v_0 + v(t)}{2} \times t \Rightarrow 60.0 \text{ m} = \frac{v_0 + 13.0 \text{ m/s}}{2} (6.80 \text{ s}) \Rightarrow v_0 = 4.65 \text{ m/s}
\]

\[
\Rightarrow a = \frac{13.0 \text{ m/s} - 4.65 \text{ m/s}}{6.80 \text{ s}} = 1.23 \text{ m/s}^2
\]

3. (5 points) An airplane pilot wishes to fly due west. The airspeed of the plane (speed in still air) is 270. km/hr. A wind of 130. km/hr is blowing toward the south. In what direction should the pilot head?

Direction \[ 28.8° \text{ N of W} \]

\[
\Theta = \sin^{-1} \left( \frac{130 \text{ km/hr}}{270 \text{ km/hr}} \right) = 28.8°
\]
For problems 4, 5, 6, and 7, do your work in the space provided, and write your final answer in the blank. For these problems, partial credit will be awarded where appropriate, based on the work that you show.

4. (15 points) At the instant the traffic light turns green, a car that has been waiting at an intersection starts ahead with a constant acceleration of 3.10 m/s². At the same instant a truck, traveling with a constant speed of 22.0 m/s, overtakes and passes the car.

(a) How far beyond its starting point does the car overtake the truck?
(b) How fast is the car traveling when it overtakes the truck?
(c) Sketch an x-t graph of the motions of the car and truck on the axes below. Take x = 0 at the intersection. On your graph, be sure to label clearly which is the car and which is the truck.

Distance \[ \frac{312 \text{ m}}{44.0 \text{ m/s}} \]

\[ X_{\text{car}} = \frac{1}{2} a_c t^2 = 1.55 \frac{\text{m}}{\text{s}^2} t^2 \]

\[ X_{\text{truck}} = \left(22.0 \frac{\text{m}}{\text{s}}\right) t \]

\[ \left(22.0 \frac{\text{m}}{\text{s}}\right) t = 1.55 \frac{\text{m}}{\text{s}^2} t^2 \Rightarrow t = \frac{22.0 \frac{\text{m}}{\text{s}}}{1.55 \frac{\text{m}}{\text{s}^2}} \]

\[ = 14.2 \text{ s} \]

\[ \Rightarrow X_{\text{car}} = 312 \text{ m} \]

\[ v_{\text{car}} = a_c t = (3.10 \frac{\text{m}}{\text{s}^2})(14.2 \text{ s}) = 44.0 \frac{\text{m}}{\text{s}} \]
5. (15 points) A model of a helicopter rotor has four blades, each 3.40 m long from the central shaft up to the blade tip. The model is rotated in a wind tunnel at 525 rev/min.

(a) What is the speed of the blade tip, in m/s?

(b) What is the acceleration of the blade tip, expressed as a multiple of $g$?

**Speed** $\frac{187 \text{ m/s}}{}$

**Acceleration** $\frac{1050 g}{\text{(last } \phi \text{ not significant)}}$

\[
\omega = \frac{\theta}{t} = \frac{2\pi \times (525)}{60 \text{ s}} = 187 \frac{\text{m}}{\text{s}}
\]

\[
a = \frac{v^2}{R} = \frac{(187 \text{ m/s})^2}{3.40 \text{ m}} = 10,280 \frac{\text{m}}{\text{s}^2} \times \frac{1 \text{ g}}{9.80 \frac{\text{m}}{\text{s}^2}}
\]

\[= 1050 g\]
6. The position of the front bumper of a test car under microprocessor control is given by:

\[ x(t) = 2.27 \text{ m} + (3.90 \text{ m/s}^2)t^2 - (0.105 \text{ m/s}^3)t^3. \]

(a) Find the non-zero time \( t \) when the car has zero velocity.

(b) What is the position and acceleration of the car at the instant in part (a)?

(c) What is the car's maximum velocity when it is moving in the +x direction?

<table>
<thead>
<tr>
<th>Time</th>
<th>1.88 s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position</td>
<td>11.4 m</td>
</tr>
<tr>
<td>Acceleration</td>
<td>-31.2 ( \text{m/s}^2 )</td>
</tr>
<tr>
<td>Maximum velocity</td>
<td>7.83 ( \text{m/s} )</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\omega(t) &= \frac{dx}{dt} = 2(3.90 \frac{\text{m}}{\text{s}^2})t - 6(0.105 \frac{\text{m}}{\text{s}^3})t^2 \\
&= (7.80 \frac{\text{m}}{\text{s}^2})t - (0.630 \frac{\text{m}}{\text{s}^3})t^2 = 0 \\
\Rightarrow\ t^2 &= \frac{7.80 \frac{\text{m}}{\text{s}^2}}{0.630 \frac{\text{m}}{\text{s}^3}} = 12.4 \text{ s}^2 \Rightarrow t = 1.88 \text{ s} \\
x(1.88 \text{ s}) &= 2.27 \text{ m} + (3.90 \frac{\text{m}}{\text{s}^2})(1.88 \text{ s})^2 - (0.105 \frac{\text{m}}{\text{s}^3})(1.88 \text{ s})^3 \\
&= 2.27 \text{ m} + 13.7 \text{ m} - 4.57 \text{ m} = 11.4 \text{ m} \\
a(t) &= \frac{d\omega}{dt} = 7.80 \frac{\text{m}}{\text{s}^2} - 5(0.630 \frac{\text{m}}{\text{s}^3})t^2 \\
&= 7.80 \frac{\text{m}}{\text{s}^2} - 3.15 \frac{\text{m}}{\text{s}^2}t^2 \\
a(1.88 \text{ s}) &= 7.80 \frac{\text{m}}{\text{s}^2} - (3.15 \frac{\text{m}}{\text{s}^2})(1.88 \text{ s})^2 \\
&= 7.80 \frac{\text{m}}{\text{s}^2} - 39.0 \frac{\text{m}}{\text{s}^2} = -31.2 \frac{\text{m}}{\text{s}^2} \\
\omega &= \text{MAX} \Rightarrow \text{not increasing or decreasing} \Rightarrow a < 0 \\
\Rightarrow\ t^2 &= \frac{7.80 \frac{\text{m}}{\text{s}^2}}{3.15 \frac{\text{m}}{\text{s}^2}} = 2.48 \text{ s}^2 \Rightarrow t = 1.25 \text{ s} \\
u_{\text{MAX}} &= (7.80 \frac{\text{m}}{\text{s}^2})(1.25 \text{ s}) - (0.630 \frac{\text{m}}{\text{s}^3})(1.25 \text{ s})^3 \\
&= 9.78 \frac{\text{m}}{\text{s}} - 1.96 \frac{\text{m}}{\text{s}} = 7.83 \frac{\text{m}}{\text{s}}
\end{align*}
\]
7. A movie stuntwoman drops from a helicopter that is 13.0 m above the ground and moving with a constant velocity whose components are 10.0 m/s upward and 12.0 m/s horizontal and toward the south. You can ignore air resistance. Where on the ground (relative to the position of the helicopter when she drops) should the stuntwoman have placed the foam mats that break her fall?

Location \(35.3\text{ m South}\)

\[
x(t) = \left(12.0 \frac{\text{m}}{\text{s}}\right)t
\]

\[
y(t) = 13.0 + \left(10.0 \frac{\text{m}}{\text{s}}\right)t - 4.90 \frac{\text{m}}{\text{s}^2}t^2
\]

\[= 0\]

\[
\Rightarrow 4.90 \frac{\text{m}}{\text{s}^2}t^2 - \left(10.0 \frac{\text{m}}{\text{s}}\right)t - 13.0\text{ m} = 0
\]

\[
\Rightarrow t = \frac{\left(10.0 \frac{\text{m}}{\text{s}}\right) \pm \sqrt{100 \frac{\text{m}^2}{\text{s}^2} + 4 \left(4.90 \frac{\text{m}}{\text{s}^2}\right)(13.0\text{ m})}}{9.80 \frac{\text{m}}{\text{s}^2}}
\]

\[= \pm 2.94\text{ s}\]

\[
x_0 = \left(12.0 \frac{\text{m}}{\text{s}}\right)(2.94\text{ s})
\]

\[\approx 35.3\text{ m}\]
For problems 1, 2, and 3, do your work in the space provided, and write your final answer in the blank. Points will be deducted for the wrong units or wrong number of significant digits. Other than that, no partial credit will be awarded for incorrect answers.

1. (5 points) A dog in an open field runs 13.0 m west and then 32.0 m in a direction 53.0° east of north. In what direction and how far must the dog then run to end up 10.0 m south of her original starting point?

   Direction  \[ \text{WNW} \]
   Distance  \[ 31.8 \text{ m} \]

   \[
   \overrightarrow{d_{\text{tot}}} = \overrightarrow{d_1} + \overrightarrow{d_2} + \overrightarrow{d_3}
   \]

   \[
   \overrightarrow{d_1} = (-13.0 \text{ m}) \hat{i}
   \overrightarrow{d_2} = (32.0 \text{ m}) \left( \sin 53.0° \hat{i} + \cos 53.0° \hat{j} \right)
   \]

   \[
   \overrightarrow{d_{\text{tot}}} = \left( 25.6 \text{ m} \right) \hat{i} + \left( 9.3 \text{ m} \right) \hat{j}
   \]

   \[
   \overrightarrow{d_3} = \overrightarrow{d_{\text{tot}}} - \overrightarrow{d_1} - \overrightarrow{d_2} = (-12.6 \text{ m}) \hat{i} + (-29.3 \text{ m}) \hat{j}
   \]

   \[
   \theta = \tan^{-1} \left( \frac{-12.6 \text{ m}}{-29.3 \text{ m}} \right) = 23.2°
   \]

   \[
   \text{Dist} = \sqrt{(-12.6 \text{ m})^2 + (-29.3 \text{ m})^2} = 31.8 \text{ m}
   \]

2. (5 points) An antelope moving with constant acceleration covers the distance between two points 60.0 m apart in 6.20 s. Its speed as it passes the second point is 13.0 m/s. What is its acceleration?

   Acceleration  \[ 1.07 \text{ m/s}^2 \]

   \[
   \Delta x = \frac{v_0 + v(t)}{2} \Rightarrow (60.0 \text{ m}) = \frac{v_0 + 13.0 \text{ m/s}}{2} 6.20 \text{ s}
   \]

   \[
   \Rightarrow v_0 = 6.35 \text{ m/s}
   \]

   \[
   \Rightarrow a = \frac{13.0 \text{ m/s} - 6.35 \text{ m/s}}{6.20 \text{ s}} = 1.07 \text{ m/s}^2
   \]

3. (5 points) An airplane pilot wishes to fly due west. The airspeed of the plane (speed in still air) is 260. km/hr. A wind of 140. km/hr is blowing toward the south. In what direction should the pilot head?

   Direction  \[ \text{WNW} \]

   \[
   \theta = \sin^{-1} \left( \frac{140 \text{ km/hr}}{260 \text{ km/hr}} \right) = 32.6°
   \]

   \[
   \overrightarrow{v_{\text{plane/air}}}
   \]

   \[
   \overrightarrow{v_{\text{wind}}} \quad \overrightarrow{v_{\text{plane/ground}}}
   \]

   \[
   \theta = \sin^{-1} \left( \frac{140 \text{ km/hr}}{260 \text{ km/hr}} \right) = 32.6°
   \]
For problems 4, 5, 6, and 7, do your work in the space provided, and write your final answer in the blank. For these problems, partial credit will be awarded where appropriate, based on the work that you show.

4. (15 points) At the instant the traffic light turns green, a car that has been waiting at an intersection starts ahead with a constant acceleration of 3.30 m/s². At the same instant a truck, traveling with a constant speed of 18.0 m/s, overtakes and passes the car.

(a) How far beyond its starting point does the car overtake the truck?
(b) How fast is the car traveling when it overtakes the truck?
(c) Sketch an x-t graph of the motions of the car and truck on the axes below. Take x = 0 at the intersection. On your graph, be sure to label clearly which is the car and which is the truck.

Distance 196 m
Speed 36.0 m/s

\[ x_{\text{car}}(t) = \frac{1}{2} a_c t^2 = 1.65 \frac{m}{s^2} t^2 \]
\[ x_{\text{truck}}(t) = u_0 t = (18.0 \frac{m}{s}) t \]
\[ x_{\text{car}} = x_{\text{truck}} \Rightarrow (1.65 \frac{m}{s^2}) t^2 = (18.0 \frac{m}{s}) t \]
\[ \Rightarrow t = \frac{18.0}{1.65} \approx 10.9 \text{ s} \]

\[ x_{\text{car}} = (1.65 \frac{m}{s^2})(10.9 \text{ s})^2 = 196 \text{ m} \]
\[ x_{\text{truck}} = (18.0 \frac{m}{s})(10.9 \text{ s}) = 196 \text{ m} \]
\[ v_{\text{car}} = a_c t = (3.30 \frac{m}{s^2})(10.9 \text{ s}) = 36.0 \frac{m}{s} \]
5. (15 points) A model of a helicopter rotor has four blades, each 3.40 m long from the central shaft up to the blade tip. The model is rotated in a wind tunnel at 575 rev/min.

(a) What is the speed of the blade tip, in m/s?

(b) What is the acceleration of the blade tip, expressed as a multiple of g?

\[
\text{Speed} \quad \frac{205 \text{ m}}{s} \\
\text{Acceleration} \quad \frac{1260 g}{(\text{ trailing zero not significant})}
\]

\[
\begin{align*}
\omega &= \frac{D}{t} = \frac{2\pi (3.40 \text{ m})}{575} \frac{575}{60 \text{ s}} = \frac{205 \text{ m}}{s} \\
\alpha &= \frac{\omega^2}{R} = \frac{(205 \frac{\text{m}}{s})^2}{3.40 \text{ m}} = \left(12,330 \frac{\text{m}^2}{s^2}\right) \frac{1 \text{ g}}{(9.80 \frac{\text{m}}{s^2})} = 1260 \text{ g}
\end{align*}
\]
6. The position of the front bumper of a test car under microprocessor control is given by:

\[ x(t) = 2.07 \text{ m} + (3.60 \text{ m/s}^2)t - (0.115 \text{ m/s}^3)t^2. \]

(a) Find the non-zero time \( t \) when the car has zero velocity.
(b) What are the position and acceleration of the car at the instant in part (a)?
(c) What is the car's maximum velocity when it is moving in the +x direction?

\[
\begin{align*}
\text{Time} & : 1.80 \text{ s} \\
\text{Position} & : 9.82 \text{ m} \\
\text{Acceleration} & : -28.8 \text{ m/s}^2 \\
\text{Maximum velocity} & : 6.92 \text{ m/s}
\end{align*}
\]

\[
\begin{align*}
\dot{x}(t) &= \frac{dx}{dt} = 2(3.60 \text{ m/s})t - 6(0.115 \text{ m/s}^3)t^2 \\
&= 7.20 \frac{\text{m}}{\text{s}}t - 0.690 \frac{\text{m}}{\text{s}^2}t^2 = 0
\end{align*}
\]

\[\Rightarrow t^* = \frac{7.20 \frac{\text{m}}{\text{s}}}{0.690 \frac{\text{m}}{\text{s}^2}} = 10.4 \text{ s} \Rightarrow t = 1.80 \text{ s}\]

\[x(1.80 \text{ s}) = 2.07 \text{ m} + (3.60 \frac{\text{m}}{\text{s}})(1.80 \text{ s})^2 - (0.115 \frac{\text{m}}{\text{s}^2})(1.80 \text{ s})^3
\]

\[= 2.07 \text{ m} + 11.6 \text{ m} - 3.88 \text{ m} = 9.82 \text{ m}\]

\[a(t) = \frac{dv}{dt} = 7.20 \frac{\text{m}}{\text{s}^2} - 5(0.690 \frac{\text{m}}{\text{s}^2})t^* = 7.20 \frac{\text{m}}{\text{s}^2} - (3.45 \frac{\text{m}}{\text{s}^3})t^*
\]

\[a(1.80 \text{ s}) = 7.20 \frac{\text{m}}{\text{s}^2} - (3.45 \frac{\text{m}}{\text{s}^3})(1.80 \text{ s}) = -28.8 \frac{\text{m}}{\text{s}^2}
\]

\( v = \text{MAX} \Rightarrow \text{not increasing or decreasing} \Rightarrow a = 0
\]

\[\Rightarrow t^* = \frac{7.20 \frac{\text{m}}{\text{s}}}{3.45 \frac{\text{m}}{\text{s}^3}} = 2.09 \text{ s} \Rightarrow t = 1.20 \text{ s}
\]

\[\Rightarrow v_{\text{MAX}} = (7.20 \frac{\text{m}}{\text{s}})(1.20 \text{ s}) - (0.690 \frac{\text{m}}{\text{s}^2})(1.20 \text{ s})^3
\]

\[= 8.65 \frac{\text{m}}{\text{s}} - 1.73 \frac{\text{m}}{\text{s}} = 6.92 \frac{\text{m}}{\text{s}}\]
7. A movie stuntwoman drops from a helicopter that is 17.0 m above the ground and moving with a constant velocity whose components are 10.0 m/s upward and 18.0 m/s horizontal and toward the south. You can ignore air resistance. Where on the ground (relative to the position of the helicopter when she drops) should the stuntwoman have placed the foam mats that break her fall?

Location 56.6 m South

\[
\begin{align*}
    x(t) &= v_x t = \left(18.0 \frac{m}{s}\right) t \\
    y(t) &= y_0 + v_0 t + \frac{1}{2} a t^2 = (17.0 \text{ m}) + (10.0 \frac{m}{s}) t - (4.90 \frac{m}{s^2}) t^2 \\
        &= 0
\end{align*}
\]

\[
\Rightarrow \left(4.90 \frac{m}{s^2}\right) t^2 - (10.0 \frac{m}{s}) t - 17.0 \text{ m} = 0
\]

\[
\Rightarrow t = \frac{\left(10.0 \frac{m}{s}\right) \pm \sqrt{100 \frac{m^2}{s^2} + 4 \left(4.90 \frac{m}{s^2}\right) (17.0 \text{ m})}}{9.80 \frac{m}{s^2}}
\]

\[
= \begin{cases} 
    +3.14 \text{ s} & \Rightarrow \text{Distance} = \left(18.0 \frac{m}{s}\right)(3.14 \text{ s}) \\
    -146.2 \text{ s} & \Rightarrow \text{Distance} = -56.6 \text{ m}
\end{cases}
\]

\[
= 56.6 \text{ m}
\]