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The Quark-Gluon Plasma: a Perfect Thermostat and a Perfect Particle Reservoir

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Abstract. A system $\mathscr{H}$ with a Hagedorn-like mass spectrum imparts its unique temperature $T_{\mathscr{H}}$ to any other system coupled to it. An $\mathscr{H}$ system radiates particles in preexisting physical and chemical equilibrium. These particles form a saturated vapor at temperature $T_{\mathscr{H}}$. This coexistence describes a first order phase transition. An $\mathscr{H}$ system is nearly indifferent to fragmentation into smaller $\mathscr{H}$ systems. A lower mass cut-off in the spectrum does not significantly alter the general picture. These properties of the Hagedorn thermostats naturally explain a single value of hadronization temperature observed in elementary particle collisions at high energies and lead to some experimental predictions.

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INTRODUCTION

A system A with energy E and degeneracy

$$\rho_A(E) \propto \exp(k_A E) \tag{1}$$

while seemingly having a partition function of the form

$$Z(T) = \int \rho_A(E) \exp(-E/T)\, dE \tag{2}$$

for all temperatures $T \leq 1/k_A$ in fact admits only one temperature $T = T_A = 1/k_A$ and it imparts that temperature to any system coupled to it.

The partition function of Eq. (2) implies that an external thermostat B which, by definition has $\rho_B(E) \propto \exp(-k_B E)$, can impart its temperature $T_B = 1/k_B$ to the system A. This is not so, as can be seen by considering the generating micro-canonical partition

$$\begin{aligned} P(x) &= \rho_A(E-x)\rho_B(x) = \exp(k_A[E-x])\exp(k_B x) \\ &= \exp\left[\frac{E-x}{T_A}\right]\exp\left[\frac{x}{T_B}\right]. \end{aligned} \tag{3}$$

The most probable partition is given by

$$\frac{\partial P(x)}{\partial x} = 0 = k_A - k_B = \frac{1}{T_A} - \frac{1}{T_B}. \tag{4}$$

But this is hardly possible since in general $T_A \neq T_B$: two thermostats can never be at equilibrium unless they are at the same temperature.

This preamble is motivated by the fact that the empirical hadronic mass spectra (Hagedorn spectra [1, 2]), the Statistical Bootstrap Model (SBM) [3, 4, 5] and the MIT bag model [6] have a degeneracy whose leading term is of the form of Eq. (1). It is the aim of this paper to explore in a pedagogical manner the implications of such a spectrum, making only passing references to the more complex physical situations occurring in particle-particle and nucleus-nucleus collisions.

Hagedorn noted that the hadronic mass spectrum (level density) has the asymptotic ($m \to \infty$) form

$$\rho_{\mathscr{H}}(m) \approx \exp(m/T_{\mathscr{H}})\,, \tag{5}$$

where m is the mass of the hadron in question and $T_{\mathscr{H}}$ is the temperature associated with the mass spectrum [1, 2]. The question of the mass range over which (5) is valid is still under discussion [4, 5].

The M.I.T. bag model [6] of partonic matter reproduces this behavior via a constant pressure B of a "bag" of partonic matter [7, 8]. The pressure p inside a bag at equilibrium without additional conserved quantities is

$$p = \frac{g\pi^2}{90} T_B^4 = B, \tag{6}$$

where g is the number of partonic degrees of freedom. The bag constant forces a constant temperature T_B on the bag. Similarly, the enthalpy density ε of the bag

$$\varepsilon = \frac{H}{V} = \frac{g\pi^2}{30} T_B^4 + B \tag{7}$$

is constant. Here H is the enthalpy and V is the volume of the bag. Thus, an injection of an arbitrary amount of energy leads to an isothermal, isobaric expansion of the bag and the bag entropy S is proportional to H:

$$S = \int \frac{\delta Q}{T} = \int_0^H \frac{dH}{T} = \frac{H}{T_B}, \tag{8}$$

where δQ is the change in heat of the bag. The bag's spectrum (level density) is then $\rho = \exp(S)$ given by Eq. (5) with $T_B = T_{\mathscr{H}}$ and $H \equiv m$.

Following our recent results [9, 10], we show here that a system $\mathscr{H}$ possessing a Hagedorn-like spectrum, characterized by an entropy of the form (8), not only has a unique microcanonical temperature

$$T_{\mathscr{H}} = \left(\frac{dS}{dE}\right)^{-1} = \left.\frac{\partial H}{\partial S}\right|_p = T_B, \tag{9}$$

but also imparts this same temperature to any other system to which $\mathscr{H}$ is coupled. In the language of standard thermodynamics: $\mathscr{H}$ is a perfect thermostat.

The property of a perfect thermostat is well known. For instance, it is indifferent to the transfer of any portion of its energy to any parcel within itself, no matter how small. In other words, it is at the limit of phase stability and its internal fluctuations of the energy density are maximal.

HARMONIC OSCILLATOR COUPLED TO $\mathscr{H}$

In order to demonstrate the thermostatic behavior of a Hagedorn system, let us begin by coupling $\mathscr{H}$ to a one dimensional harmonic oscillator and use a microcanonical treatment. The unnormalized probability $P(\varepsilon)$ for finding an excitation energy ε in the harmonic oscillator out of the system's total energy E is

$$\begin{aligned} P(\varepsilon) &\sim \rho_{\mathscr{H}}(E-\varepsilon)\,\rho_{\rm osc}(\varepsilon) \\ &= \exp\left(\frac{E-\varepsilon}{T_{\mathscr{H}}}\right) = \rho_{\mathscr{H}}(E)\exp\left(-\frac{\varepsilon}{T_{\mathscr{H}}}\right). \end{aligned} \tag{10}$$

Recall that for a one dimensional harmonic oscillator $\rho_{\rm osc}$ is a constant. The energy spectrum of the oscillator is canonical up to the upper limit $\varepsilon_{max} = E$ with an inverse slope (temperature) of $T_{\mathscr{H}}$ independent of E. The mean value of the energy of the oscillator is given by

$$\overline{\varepsilon} = T_{\mathscr{H}}\left[1 - \frac{E/T_{\mathscr{H}}}{\exp(E/T_{\mathscr{H}}) - 1}\right]. \tag{11}$$

Thus in the limit that $E \to \infty$: $\overline{\varepsilon} \to T_{\mathscr{H}}$, i.e. no temperature other that $T_{\mathscr{H}}$ is admitted. In the standard language of statistical mechanics this example means that a one dimensional harmonic oscillator can be used as an ideal thermometer.

AN IDEAL VAPOR COUPLED TO $\mathscr{H}$

For a more physically relevant example, let us consider a vapor of $N \gg 1$ non-interacting Boltzmann particles of mass m_B and degeneracy g_B coupled to $\mathscr{H}$. The microcanonical level density of the vapor with kinetic energy ε is

$$\rho_{\rm vapor}(\varepsilon) = \frac{V^N g_B^N}{N!\left(\frac{3}{2}N\right)!}\left(\frac{m_B\varepsilon}{2\pi}\right)^{\frac{3}{2}N}, \tag{12}$$

where V is is the volume. The microcanonical partition of the total system is

$$\begin{aligned}\rho_{\rm total}(E,\varepsilon) &= \rho_{\mathscr{H}}(E-\varepsilon)\rho_{\rm vapor}(\varepsilon)\\ &= \frac{V^N g_B^N}{N!\left(\frac{3}{2}N\right)!}\left(\frac{m_B\varepsilon}{2\pi}\right)^{\frac{3}{2}N} e^{\frac{E-m_BN-\varepsilon}{T_{\mathscr{H}}}}.\end{aligned} \tag{13}$$

Just as with the harmonic oscillator, the distribution of the vapor is exactly canonical up to $\varepsilon_{max} = E$, if the particles are independently present, or $\varepsilon_{max} = E - mN$, if the particles are generated by $\mathscr{H}$. In either case, the temperature of the vapor is always $T_{\mathscr{H}}$.

The maximum of $\rho_{\rm total}(E,\varepsilon)$ with respect to ε gives the most probable kinetic energy per particle as

$$\frac{\partial\rho_{\rm total}(E,\varepsilon)}{\partial\varepsilon} = \frac{3N}{2\varepsilon} - \frac{1}{T_{\mathscr{H}}} = 0 \quad\Rightarrow\quad \frac{\varepsilon}{N} = \frac{3}{2}T_{\mathscr{H}}, \tag{14}$$

provided that $E \geq m_BN + \frac{3}{2}NT_{\mathscr{H}}$. (For $m_BN < E < m_BN + \frac{3}{2}NT_{\mathscr{H}}$, the most probable value of the kinetic energy per particle is $\frac{\varepsilon}{N} = \frac{E}{N} - m_B < \frac{3}{2}T_{\mathscr{H}}$; for $E \leq m_BN$, $\frac{\varepsilon}{N} = 0$.) Again $T_{\mathscr{H}}$ is the sole temperature characterizing the distribution up to the microcanonical cut-off, which may be above or below the maximum of the distribution since the form of $\rho_{\rm total}(E,\varepsilon)$ is independent of E.

The maximum of $\rho_{\rm total}(E,\varepsilon)$ with respect to N at fixed V is given by

$$\frac{\partial\ln\rho_{\rm total}(E,\varepsilon)}{\partial N} = -\frac{m_B}{T_{\mathscr{H}}} + \ln\left[g_B\frac{V}{N}\left(\frac{m_BT_{\mathscr{H}}}{2\pi}\right)^{\frac{3}{2}}\right] = 0, \tag{15}$$

where Eq. (14) was used for ε. Thus the most probable particle density of the vapor is independent of V:

$$\frac{N}{V} = g_B\left(\frac{m_BT_{\mathscr{H}}}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{m_B}{T_{\mathscr{H}}}} \equiv n_{\mathscr{H}}. \tag{16}$$

Equation (16) demonstrates that not only is $\mathscr{H}$ a perfect thermostat but also a perfect particle reservoir. Particles of different mass m will be in chemical equilibrium with each other. At equilibrium, particles are emitted from $\mathscr{H}$ and form a saturated vapor at coexistence with $\mathscr{H}$ at temperature $T_{\mathscr{H}}$. This describes a first order phase transition (hadronic to partonic). Coexistence occurs at a single temperature fixed by the bag pressure.

These results explain the common value of: the hadronization temperatures obtained within the statistical hadronization model [11]; the inverse slopes of the transverse mass spectra of hadrons observed in high energy elementary particle collisions [12, 13]; and the transition temperature from lattice QCD calculations for low baryonic density [14]. For further discussion see [10].

$\mathscr{H}$ AS A RADIANT BAG

Let us assume that $\mathscr{H}$ is a bag thick enough to absorb any given particle of the vapor striking it. Then, detailed balance requires that on average $\mathscr{H}$ radiates back the same particle. Under these conditions particles can be considered to be effectively emitted from the surface of $\mathscr{H}$. Thus the relevant fluxes do not depend in any way upon the inner structure of $\mathscr{H}$.

In fact, the results given in equations (14) and (16) show that the saturated vapor concentration depends only upon m_B and $T_{\mathscr{H}}$ as long as $\mathscr{H}$ is present. A decrease in the volume V does not increase the vapor concentration, but

induces a condensation of the corresponding amount of energy out of the vapor and into $\mathscr{H}$. An increase in V keeps the vapor concentration constant via evaporation of the corresponding amount of energy out of $\mathscr{H}$ and into the vapor. This is reminiscent of liquid-vapor equilibrium at fixed temperature, except that here coexistence occurs at a single temperature $T_{\mathscr{H}}$, rather than over a range of temperatures as in ordinary fluids.

The bag wall is Janus faced: one side faces the partonic world, and, aside from conserved charges, radiates a partonic black body radiation responsible for balancing the bag pressure; the other side faces the hadronic world and radiates a hadronic black body radiation, mostly pions. Both sides of the bag wall are at the temperature $T_{\mathscr{H}}$. It is tempting to attribute most, if not all, of the hadronic and partonic properties to the wall itself, possibly even the capability to enforce conservation laws globally (quantum number conductivity). Despite the fact that this wall is an insurmountable horizon, with hadronic measurements such as bag size and total radiance we can infer some properties of the partonic world, e.g. the number of degrees of freedom [12].

We can estimate an upper limit for the emission time using the outward energy flux of particles radiated from the bag. At equilibrium the in-going and out-going fluxes must be the same, thus the outward flux of particles in the nonrelativistic approximation using Eq. (16) is

$$\varphi_{n_{\mathscr{H}}} \simeq \frac{n_{\mathscr{H}}}{4}\left(\frac{m_B}{m_B+2T_{\mathscr{H}}}\right)\sqrt{8\frac{T_{\mathscr{H}}}{\pi m_B}}. \tag{17}$$

Using the technique developed in [15, 16], one finds the energy flux $\varphi_{E_{\mathscr{H}}}$ and momentum flux $p_{\rm rad}$ as

$$\varphi_{E_{\mathscr{H}}} \simeq (m_B+2T_{\mathscr{H}})\,\varphi_{n_{\mathscr{H}}}\,, \quad p_{\rm rad}=\frac{1}{2}n_{\mathscr{H}}T_{\mathscr{H}}\,. \tag{18}$$

The pressure $p_{\rm rad}$ exerted on the bag by its radiation can be compared to the intrinsic bag pressure in Eq. (6): for pions $p_{\rm rad}\sim 0.02B$. The time τ for the bag to dissolve into its own radiation is approximately

$$\tau \simeq \frac{3\pi\,\exp\left(\frac{m_B}{T_{\mathscr{H}}}\right)E_0}{g_B\left(m_B^2T_{\mathscr{H}}^2\right)R_0^2}\,, \tag{19}$$

where R_0 is the initial bag radius and E_0 is the initial bag total energy.

The fluxes written in Eqs. (17) and (18) (particle or energy per unit surface area) are integrated over an assumed spherical bag to give the result in Eq. (19). However, because of the lack of surface tension, the bag's maximum entropy corresponds to either an elongated (cylinder) or a flattened shape (disc). Thus, Eq. (19) should be interpreted as an upper limit. More detailed studies of hadron emission from bags concerning hydrodynamic shock waves and freeze out shocks can be found elsewhere [15, 16, 17, 18].

The decoupling between the vapor concentration and m_B and $T_{\mathscr{H}}$ occurs when $\mathscr{H}$ has completely evaporated (i.e. when $E-m_BN-\frac{3}{2}NT_{\mathscr{H}}=0$) at a volume of

$$V_d \simeq \frac{1}{n_{\mathscr{H}}}\frac{E}{\left[m_B+\frac{3}{2}T_{\mathscr{H}}\right]}. \tag{20}$$

The disappearance of $\mathscr{H}$ allows the vapor concentration to decrease inversely proportionally to V as

$$\frac{N}{V}=\frac{n_{\mathscr{H}}V_d}{V}. \tag{21}$$

The temperature, however, remains fixed at $T_{\mathscr{H}}$ because of conservation of energy and particle number above V_d. Solid curves in Fig. 1 show this schematically.

The discussion above assumes that the Hagedorn spectrum extends down to $m=0$. However, experimentally there appears to be a lower cut off of the spectrum at m_0. This modifies the above results as follows (for a detailed analysis see the section "Generalization to a Complete Hagedorn Spectrum").

For energies $E-m_BN-\varepsilon\gg m_0$ and $V<V_d$ the above results hold as written. However, if we increase the volume well beyond V_d at which the Hagedorn spectrum is truncated at m_0, the situation is slightly different. $\mathscr{H}$ evaporates until its mass is m_0. If the entire mass of $\mathscr{H}$ is fully transformed into vapor particles as the volume is increased further, then the excess particles temporarily increase the concentration and permanently decrease the temperature. As the volume increases further, the concentration changes inversely proportional to V

$$\frac{N}{V}=\frac{n_{\mathscr{H}}V_d+\frac{m_0}{m_B}}{V}, \tag{22}$$

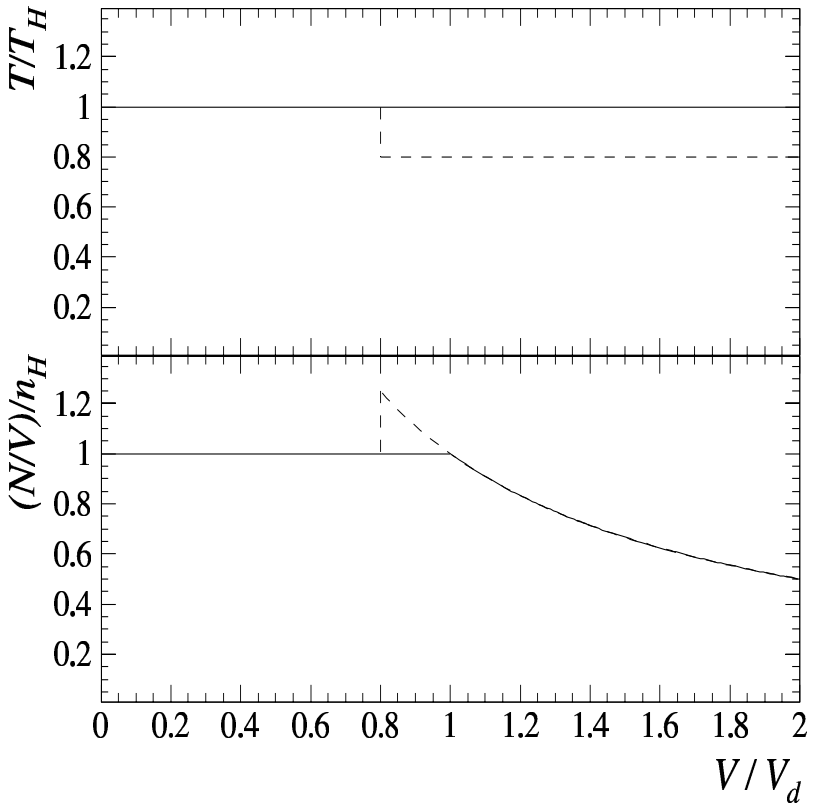

FIGURE 1. Typical behavior of the entire system's temperature T and concentration N/V as the function of the system's volume V in the absence of restrictions (solid curve) and for a finite cut off at m_0 of the Hagedorn spectrum (dashed curve).

while the temperature remains constant at

$$T = \frac{n_{\mathscr{H}} V_d}{n_{\mathscr{H}} V_d + \frac{m_0}{m_B}} T_{\mathscr{H}}. \tag{23}$$

Dashed curves in Fig. 1 show this schematically.

FRAGMENTATION OF $\mathscr{H}$

A question of interest is the stability of $\mathscr{H}$ against fragmentation. If the translational degrees of freedom are neglected, $\mathscr{H}$ is indifferent to fragmentation into an arbitrary number of particles of arbitrary mass (within the constraints of mass/energy conservation).

Let us now consider the case in which the mass of the vapor particle m_B is allowed to be free. The system's level density $\rho_{\text{total}}(E,\varepsilon)$ is still given by Eq. (13). Using Eqs. (14) and (16), one finds the most probable value of the system's level density as $\rho^*_{\text{total}}(E,\varepsilon) \approx \exp[S^*]$, where the entropy is $S^* = E/T_{\mathscr{H}} + N$. Differentiating $\rho^*_{\text{total}}(E,\varepsilon)$ with respect to m_B and applying Eq. (16) gives

$$\frac{\partial \ln \rho^*_{\text{total}}(E,\varepsilon)}{\partial m_B} = N\left[\frac{3}{2m_B} - \frac{1}{T_{\mathscr{H}}}\right] \;\Rightarrow\; m_B = \frac{3}{2} T_{\mathscr{H}}\,, \tag{24}$$

i.e. the last equality provides the maximum of level density for $N \neq 0$. Since all the intrinsic statistical weights in $\rho^*_{\text{total}}(E,\varepsilon)$ are factored into a single $\mathscr{H}$, the system breaks into fragments with $m_B = \frac{3}{2} T_{\mathscr{H}}$ except for one whose mass is determined by mass/energy conservation.

Substituting the most probable value of ε and m_B into the most probable value of N one obtains the vapor concentration

$$\frac{N}{V} = g_B \left(\frac{3}{4\pi e} \right)^{\frac{3}{2}} T_{\mathscr{H}}^3 . \tag{25}$$

The density of the vapor of nonrelativistic particles acquires the form typical of the ultrarelativistic limit.

If the value of mass given by Eq. (24) does not exist, then the most probable value of level density $\rho^*_{\text{total}}(E,\varepsilon)$ corresponds to the mass m^* which is nearest to $\frac{3}{2}T_{\mathscr{H}}$ and $N(m^*)$ given by Eq. (16). In terms of hadron spectroscopy the value of m^* that maximizes the level density $\rho^*_{\text{total}}(E,\varepsilon)$ is the pion mass.

If $\mathscr{H}$ is required to fragment totally into a number of equal fragments of mass m_H all endowed with their translational degrees of freedom, then (for $g_B = 1$)

$$\rho_T = \frac{e^{\frac{E-\varepsilon}{T_{\mathscr{H}}}} V^N}{N!\left(\frac{3}{2}N\right)!} \left[\frac{m_H \varepsilon}{2\pi} \right]^{\frac{3}{2}N} = \frac{e^{\frac{E}{T_{\mathscr{H}}}} V^N}{N!} \left[\frac{m_H T_{\mathscr{H}}}{2\pi} \right]^{\frac{3}{2}N} , \tag{26}$$

where in the last step we substituted the most probable value of the kinetic energy (14) and used the Stirling formula for $\left(\frac{3}{2}N\right)!$. From Eq. (26) it is seen that all the Hagedorn factors collapse into a single one with the m-independent argument E. Maximization of (26) with respect to m_H leads to

$$\frac{\partial \ln \rho_T}{\partial m_H} = \frac{3N}{2m_H} = 0 , \tag{27}$$

which is consistent with $N = 1$ and $m_H = E$, namely a single Hagedorn particle with all the available mass.

This again illustrates the indifference of $\mathscr{H}$ toward fragmentation. Of course Eq. (14) gives directly the mass distribution of the Hagedorn fragments under the two conditions discussed above. These results justify the assumption of the canonical formulation of the statistical hadronization model that smaller clusters appear from a single large cluster [19].

INTERMEDIATE CONCLUSIONS

A system $\mathscr{H}$, with a Hagedorn-like mass spectrum, is a perfect thermostat and a perfect particle reservoir. Consequently, any system coupled to $\mathscr{H}$ can have only the temperature of $\mathscr{H}$: $T_{\mathscr{H}}$. This behavior may explain the common value of: the hadronization temperatures obtained within statistical models; the transition temperature from lattice QCD calculations for low baryonic density; and the inverse slopes of the transverse mass spectra of hadrons (temperature) observed in high energy elementary particle collisions and high energy nucleus-nucleus collisions (for details see [10]). The common temperature of the experimental spectra suggest that the observed particles originate from an $\mathscr{H}$-like system.

The hadronic side of $\mathscr{H}$ radiates particles in preexisting physical and chemical equilibrium just as a black body radiates photons in physical and chemical equilibirum (compare to Ref. [20]). Particles emitted from $\mathscr{H}$ form a saturated vapor that coexists with $\mathscr{H}$. This coexistence describes a first order phase transition (hadronic to partonic) and occurs at a single temperature fixed by the bag pressure. An $\mathscr{H}$ system is nearly indifferent to fragmentation into smaller $\mathscr{H}$ systems. A lower cut-off in the mass spectrum does not alter our results [10].

GENERALIZATION TO A COMPLETE HAGEDORN SPECTRUM

To have a more realistic model we should consider a more complicated Hagedorn mass spectrum $g_H(m_H) = \exp[m_H/T_{\mathscr{H}}](m_o/m_H)^a$ for the resonance masses m_H above the lower cut-off $m_o \gg T_{\mathscr{H}}$ (a is a parameter discussed below). Let us study the microcanonical ensemble of N_B Boltzmann point-like particles of mass m_B and degeneracy g_B, and N_H hadronic point-like resonances of mass m_H with a mass spectrum $g_H(m_H)$ assuming that $m_o > m_B$. A recent analysis [21] suggests that the Hagedorn mass spectrum can be established for $m_o < 2$ GeV.

In the Statistical Bootstrap Model (SBM) [22] and the MIT bag model [7] it was found that for $m_H \to \infty$ the parameter $a \leq 3$. For finite resonance masses the value of a is unknown, so it will be considered as a fixed parameter.

The microcanonical partition of the system, with volume V, total energy E and zero total momentum, can be written as follows

$$\Omega = \frac{V^{N_H}}{N_H!}\left[\prod_{k=1}^{N_H} g_H(m_H)\int\frac{d^3Q_k}{(2\pi)^3}\right]\frac{V^{N_B}}{N_B!}\left[\prod_{l=1}^{N_B} g_B\int\frac{d^3p_l}{(2\pi)^3}\right]\delta\Big(E-\sum_{i=1}^{N_H}\varepsilon_i^H-\sum_{j=1}^{N_B}\varepsilon_j^B\Big), \tag{28}$$

where the quantity $\varepsilon_i^H = \varepsilon(m_H,Q_i)$ $\Big(\varepsilon_j^B=\varepsilon(m_B,p_j)$ and $\varepsilon(M,P)\equiv\sqrt{M^2+P^2}\Big)$ denotes the energy of the Hagedorn (Boltzmann) particle with the 3-momentum $\vec{Q}_i$ ($\vec{p}_j$). In order to simplify the presentation of our idea, Eq. (28) accounts for energy conservation only and neglects momentum conservation.

The microcanonical partition (28) can be evaluated by the Laplace transform in total energy E [23]. Then the momentum integrals in (28) are factorized and can be performed analytically. The inverse Laplace transform in the conjugate variable λ can be done analytically for the nonrelativistic and ultrarelativistic approximations of the one-particle momentum distribution function

$$\int_0^\infty\frac{d^3p\,e^{-\lambda\varepsilon(M,p)}}{(2\pi)^3}\approx\begin{cases}\left[\frac{2M}{\lambda}\right]^{\frac{3}{2}}I_{\frac{1}{2}}e^{-M\lambda}\,, & MRe(\lambda)\gg 1\,,\\ \frac{2}{\lambda^3}\,I_2\,e^{-M\lambda}\,, & MRe(\lambda)\ll 1\,,\end{cases} \tag{29}$$

where the auxiliary integral is denoted as

$$I_b \equiv \int_0^\infty\frac{d\xi}{(2\pi)^2}\,\xi^b\,e^{-\xi}\,. \tag{30}$$

Since the formal steps of further evaluation are similar for both cases, we discuss in detail the nonrelativistic limit only, and later present the results for the other case. The nonrelativistic approximation ($MRe(\lambda)\gg 1$) for Eq. (28) is as follows

$$\Omega_{nr} = \frac{\Big[Vg_H(m_H)\,[2m_H]^{\frac{3}{2}}\,I_{\frac{1}{2}}\Big]^{N_H}}{N_H!}\;\frac{\Big[Vg_B\,[2m_B]^{\frac{3}{2}}\;I_{\frac{1}{2}}\Big]^{N_B}}{N_B!}\;\frac{E_{kin}^{\frac{3}{2}(N_H+N_B)-1}}{\big(\frac{3}{2}(N_H+N_B)-1\big)!}\,, \tag{31}$$

where $E_{kin}=E-m_HN_H-m_BN_B$ is the kinetic energy of the system.

As shown below, the most realistic case corresponds to the nonrelativistic treatment of the Hagedorn resonances because the resulting temperature is much smaller than their masses. Therefore, it is sufficient to consider the ultra-relativistic limit for the Boltzmann particles only. In this case ($MRe(\lambda)\ll 1$) the equation (28) can be approximated as

$$\Omega_{ur} = \frac{\Big[Vg_H(m_H)\,[2m_H]^{\frac{3}{2}}\;I_{\frac{1}{2}}\Big]^{N_H}}{N_H!}\;\frac{[Vg_B\,2\,I_2]^{N_B}}{N_B!}\;\frac{E_{kin}^{\frac{3}{2}(N_H+2N_B)-1}}{\big(\frac{3}{2}(N_H+2N_B)-1\big)!}\,, \tag{32}$$

where the kinetic energy does not include the rest energy of the Boltzmann particles, i.e. $E_{kin}=E-m_HN_H$.

Within our assumptions the above results are general and can be used for any number of particles, provided $N_H+N_B\geq 2$. It is instructive to consider first the simplest case $N_H=1$. This formulation of the model, in which a Hagedorn thermostat is always present, allows us to study the problem rigorously and provides us with a qualitative picture for $N_H>1$. For $N_H=1$ and $N_B\gg 1$ we treat the mass of Hagedorn thermostat m_H as a free parameter and determine the value which maximizes the entropy of the system. The solution $m_H^*>0$ of the extremum condition

$$\frac{\delta\ln\Omega_{nr}(N_H=1)}{\delta\,m_H}\,\frac{1}{T_{\mathscr{H}}}+\big(\tfrac{3}{2}-a\big)\,\frac{1}{m_H^*}-\frac{3(N_B+1)}{2\,E_{kin}}=0 \tag{33}$$

provides the maximum of the system's entropy, if for $m_H=m_H^*$ the second derivative is negative

$$\frac{\delta^2\ln\Omega_{nr}(N_H=1)}{\delta\,m_H^2}-\big(\tfrac{3}{2}-a\big)\,\frac{1}{m_H^{*2}}-\frac{3(N_B+1)}{2\,E_{kin}^2}<0\,. \tag{34}$$

The inequality (34) is a necessary condition of the maximum of the microcanonical partition. Postponing the analysis of (34) till the next section, where we study it in more details, let us assume for a moment that the inequality (34) is

satisfied. Then the extremum condition (33) defines the temperature of the system of (N_B+1) nonrelativistic particles

$$T^*(m_H^*) \equiv \frac{2\,E_{kin}}{3(N_B+1)} = \frac{T_{\mathscr{H}}}{1\,+\,\left(\frac{3}{2}\,-\,a\right)\frac{T_{\mathscr{H}}}{m_H^*}}\,. \tag{35}$$

Thus, as $m_H^* \to \infty$ it follows that $T^*(m_H^*) \to T_{\mathscr{H}}$, while for finite $m_H^* \gg T_{\mathscr{H}}$ and $a > \frac{3}{2}$ ($a < \frac{3}{2}$) the temperature of the system is slightly larger (smaller) than the Hagedorn temperature, i.e. $T^* > T_{\mathscr{H}}$ ($T^* < T_{\mathscr{H}}$). Formally, the temperature of the system in equation (35) may differ essentially from $T_{\mathscr{H}}$ for a light thermostat, i.e. for $m_H^* \leq T_{\mathscr{H}}$. However, it is assumed that the Hagedorn mass spectrum exists above the cut-off mass $m_{\rm o} \gg T_{\mathscr{H}}$, thus $m^* \gg T_{\mathscr{H}}$.

THE ROLE OF THE MASS CUT-OFF

Now we study the effect of the mass cut-off of the Hagedorn spectrum on the inequality (34) in more detail. For $a \leq \frac{3}{2}$ the condition (34) is satisfied. For $a > \frac{3}{2}$ the inequality (34) is equivalent to the following inequality

$$\frac{m_H^{*2}}{\left(a-\frac{3}{2}\right)\,T^*(m_H^*)} \;>\; \frac{3}{2}\,(N_B+1)\,T^*(m_H^*)\,, \tag{36}$$

which means that a Hagedorn thermostat should be massive compared to the kinetic energy of the system.

A more careful analysis shows that for a negative value of the determinant D_{nr} ($\tilde{N} \equiv N_B - \frac{2}{3}a$)

$$\begin{aligned} D_{nr} \;\;\equiv\;\; & \left(E - m_B N_B - \tfrac{3}{2}\,T_{\mathscr{H}}\,\tilde{N}\right)^2 - \\ & 4\left(a-\tfrac{3}{2}\right)\,T_{\mathscr{H}}\,\left(E - m_B N_B\right) \;<\; 0\,, \end{aligned} \tag{37}$$

equation (33) has two complex solutions, while for $D_{nr} = 0$ there exists a single real solution of (33). Solving (37) for $(E - m_B N_B)$, shows that for $\tilde{N} > \frac{2}{3}a - 1$, i.e. for $N_B > \frac{4}{3}, a - 1$ the inequality (37) does not hold and $D_{nr} > 0$. Therefore, in what follows we will assume that $N_B > \frac{4}{3}a - 1$ and only analyze the case $D_{nr} > 0$. For this case equation (33) has two real solutions

$$m_H^{\pm} = \tfrac{1}{2}\left[E - m_B N_B - \tfrac{3}{2}\,T_{\mathscr{H}}\,\tilde{N} \;\pm\; \sqrt{D_{nr}}\right]\,. \tag{38}$$

For $a \leq \frac{3}{2}$ only m_H^+ solution is positive and corresponds to a maximum of the microcanonical partition Ω_{nr}.

For $a > \frac{3}{2}$ both solutions of (33) are positive, but only m_H^+ is a maximum. From the two limiting cases:

$$\frac{\delta \ln \Omega_{nr}(N_H = 1)}{\delta\, m_H} \;\;\approx\;\; \left(\tfrac{3}{2} - a\right)\tfrac{1}{m_H} \quad \text{for} \quad m_H \approx 0\,, \tag{39}$$

$$\frac{\delta \ln \Omega_{nr}(N_H = 1)}{\delta\, m_H} \;\;\approx\;\; \tfrac{3(N_B+1)}{2\,E_{kin}} \quad \text{for} \quad E_{kin} \approx 0\,, \tag{40}$$

and the fact that $m_H^{\pm}$ obey the inequalities

$$0 \;<\; m_H^- \;\leq\; m_H^+ \;<\; E - m_B N_B\,, \tag{41}$$

it is clear that $m_H^* = m_H^-$ is a local minimum of the microcanonical partition Ω_{nr}, while $m_H^* = m_H^+$ is a local maximum of the partition Ω_{nr}.

Using Eq. (38) for m_H^+, it is clear that for any value of a the constraint $m_H^+ \geq m_{\rm o}$ is equivalent to the inequality

$$N_B \;\leq\; N_B^{kin} \equiv \frac{E \;-\; [\tfrac{m_{\rm o}}{T_{\mathscr{H}}} \;-\; a]\;T^*(m_{\rm o})}{m_B + \tfrac{3}{2}\,T^*(m_{\rm o})}\,. \tag{42}$$

Thus, at fixed energy E for all $N_B \leq N_B^{kin}$ at $m_H^* = m_H^+$ there is a local maximum of the microcanonical partition Ω_{nr} with the temperature $T = T^*(m_H^+)$. For $N_B > N_B^{kin}$ the maximum of the partition Ω_{nr} cannot be reached due to the cut-off constraint and, consequently, the most probable state corresponds to $m_H = m_{\rm o}$ with $T \leq T^*(m_{\rm o})$ from Eq. (35). In other words, for $N_B > N_B^{kin}$ the amount of energy E is insufficient for the mass of the Hagedorn thermostat to be above the cut-off $m_{\rm o}$ and simultaneously maintain the temperature of the Boltzmann particles according to Eq. (35). By assumption there is a single Hagedorn thermostat in the system, therefore, as N_B grows the temperature of the

system decreases from $T^*(m_o)$ value. Thus, the equality (42) defines the kinematical limit for reaching the maximum of the microcanonical partition.

To prove that the maximum of the microcanonical partition at $m_H = m_H^+$ is global it is sufficient to show that the constraint $m_H^+ \geq m_o$ is not consistent with the condition $m_H^- > m_o$. For $a \leq \frac{3}{2}$ the maximum is global because for $0 < m_H < m_H^+$ ($m_H > m_H^+$) the partition $\Omega_{nr}(N_H = 1, m_H)$ monotonically increases (decreases) with m_H. For $a > \frac{3}{2}$ it is clear that the maximum at $m_H = m_H^+$ is local, if the state with mass $m_H = m_o$ is more probable, i.e. $\Omega_{nr}(N_H = 1, m_o) > \Omega_{nr}(N_H = 1, m_H^+)$. Due to (41) this can occur, if $m_H^- > m_o$. Substituting Eq. (38) into the last inequality, shows that this inequality reduces to the condition $N_B > N_B^{kin}$. This contradicts the constraint $m_H^+ \geq m_o$ in the form of Eq. (42). Thus, the maximum of the microcanonical partition is global.

To complete our consideration of the nonrelativistic case let us express the partition () in terms of the temperature (35). Applying the Stirling approximation to the factorial $(\frac{3}{2}(N_B+1)-1)!$ for $N_B^{kin} > N_B \gg 1$ and reversing the integral representations (29) and (30) for $\lambda = 1/T^*(m_H^+)$, one finds

$$\Omega_{nr}(N_H = 1) \;=\; \frac{V\, g_H(m_H^+)}{T^*(m_H^+)} \left[\int \frac{d^3Q}{(2\pi)^3}\, e^{-\frac{\sqrt{m_H^{+2}+Q^2}}{T^*(m_H^+)}} \right] \frac{e^{\frac{E}{T^*(m_H^+)}}}{N_B!} \left[V\, g_B \int \frac{d^3p}{(2\pi)^3}\, e^{-\frac{\sqrt{m_B^2+p^2}}{T^*(m_H^+)}} \right]^{N_B} . \qquad (43)$$

This is just the grand canonical partition of (N_B+1) Boltzmann particles with temperature $T^*(m_H^+)$. If $N_B > N_B^{kin} \gg 1$, then $T^*(m_H^+)$ in (43) should be replaced by $T_o(N_B) \equiv \frac{2(E-m_B N_B - m_o)}{3(N_B+1)}$.

Fig. 1 shows that for $a > \frac{3}{2}$ the system's temperature $T = T^*(m_H^+)$ as a function of N_B remains almost constant for $N_B < N_B^{kin}$, reaches a maximum at N_B^{kin} and rapidly decreases like $T = T_o(N_B)$ for $N_B > N_B^{kin}$. For $a < \frac{3}{2}$ the temperature has a plateau $T = T^*(m_H^+)$ for $N_B < N_B^{kin}$, and rapidly decreases for $N_B > N_B^{kin}$ according to $T_o(N_B)$.

The same results are valid for the ultrarelativistic treatment of Boltzmann particles. Comparing the nonrelativistic and ultrarelativistics expressions for the microcanonical partition, i.e. equations () and (32), respectively, one finds that the derivation of the ultrarelativistic limit requires only the substitution $N_B \to 2N_B$ and $m_B/T_{\mathscr{H}} \to 0$ in equations (33 – 43). Note that this substitution does not alter the expression for the temperature of the system, i.e. the right hand side of (35).

Finally, we show that for a heavy Hagedorn thermostat ($m_H^+ \gg m_o$) these results remain valid for a single Hagedorn thermostat split into N_H pieces of the same mass. Substituting $m_H \to m_H N_H$ in the nonrelativistic expressions () and minimizing it with respect to m_H, the temperature of the system in the form of equation (35) is $T^*(m_H^* N_H)$, where the mass of N_H Hagedorn thermostats m_H^* is related to the solution m_H^+ of equation (38) as $m_H^* = m_H^+/N_H$. Since the original single thermostat of mass m_H^+ was assumed to be heavy, it follows $T^*(m_H^* N_H) = T^*(m_H^+) \to T_{\mathscr{H}}$. A more careful study (see also [9]) using an exact expression for the microcanonical partition of N_H Hagedorn thermostats of the same mass m_H gives the same result, if $m_H \gg m_o$. A generalization of these statements to the case of N_H heavy Hagedorn thermostats of different masses also leads to the same result. Thus, splitting a single heavy Hagedorn thermostat into an arbitrary number of heavy resonances (heavier than m_o) does not change the temperature of the system.

THE BAG SURFACE

The bag expressions reported above contain only volume terms. Given the finite size of the bags that are typically considered (resonances), it may be of interest to consider finite size effects and their role in the description of the bags properties. The simplest generalization, assuming that the bags are leptodermous (which is supported by the short range of hadron-hadron interaction and by the saturating properties implicit in Eqs. (6) and (7)), is the introduction of surface energy. This can be done phenomenologically by introducing a $V^{\frac{2}{3}}$ term in the free energy. Then the pressure of a spherical bag can be written as

$$p = \frac{\sigma}{3}T^4 \;-\; B \;-\; \frac{2}{3}a_s(T)V^{-\frac{1}{3}} = \frac{\sigma}{3}T^4 \;-\; B \;-\; \frac{2}{3}\frac{a_s(T)}{\alpha R}\,, \qquad (44)$$

where $a_s(T)$ is the temperature dependent surface energy coefficient, R is the bag radius and $\alpha \equiv \left[\frac{4\pi}{3}\right]^{\frac{1}{3}}$. Using the thermodynamic identities for the free energy F and entropy S

$$p = -\left(\frac{\partial F}{\partial V}\right)_T , \quad \text{and} \quad S = -\left(\frac{\partial F}{\partial T}\right)_V , \qquad (45)$$

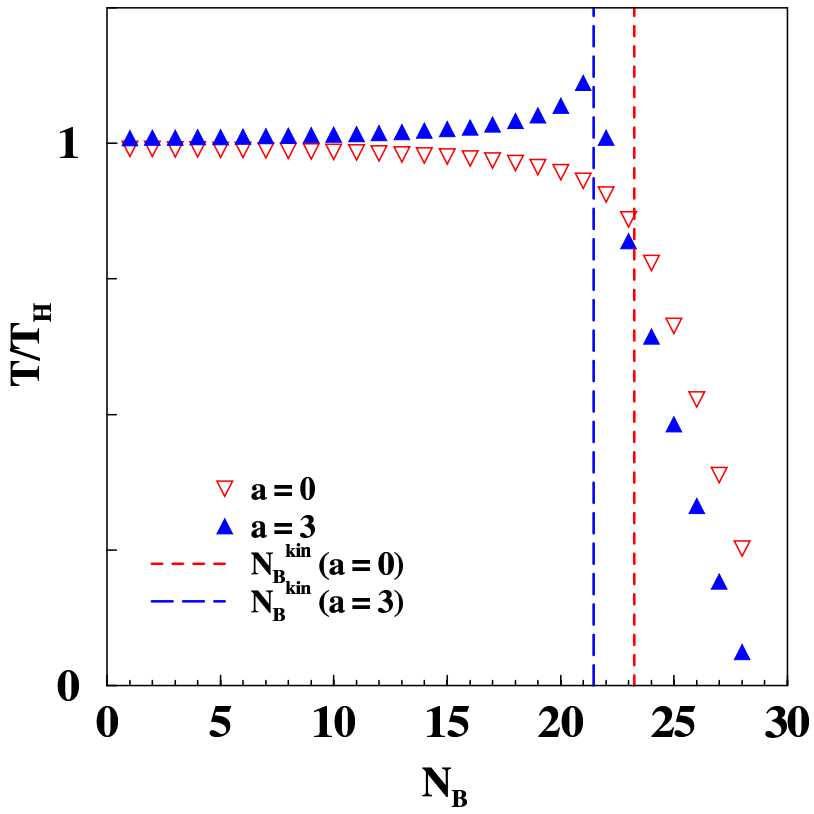

FIGURE 2. A typical behavior of the system's temperature as the function of the number of Boltzmann particles N_B for $a = 3$ and $a = 0$ for the same value of the total energy $E = 30\,m_B$. Due to the thermostatic properties of a Hagedorn resonance the system's temperature is nearly constant up to the kinematically allowed value N_B^{kin} given by (42).

one can find all thermodynamic functions as follows

$$F = -\left[\frac{\sigma}{3}T^4 - B\right]V + a_s(T)\,V^{\frac{2}{3}}, \tag{46}$$

$$S = \frac{4\sigma}{3}T^3V - \frac{da_s(T)}{d\ T}\,V^{\frac{2}{3}}, \tag{47}$$

$$E \equiv \varepsilon V = \left[\sigma T^4 + B\right]V + \left[a_s(T) - \frac{da_s(T)}{d\ T}\right]V^{\frac{2}{3}}. \tag{48}$$

In evaluating the expression (46) we fixed the integration constant (an arbitrary function of T) to zero because the free energy should vanish for the bag of zero volume.

While the magnitude of $a_s(T)$ is unknown, there are surprising consequences for $a_s(T) > 0$. In Eq. (44) the surface term appears as an additional pressure to the bag pressure. Therefore, for a bag in a vacuum the total pressure should be zero, i.e. $p = 0$, and, consequently, the bag temperature acquires volume dependence:

$$T(R) = \left[\frac{3}{\sigma}\left(B + \frac{2\,a_s(T)}{3\,\alpha R}\right)\right]^{\frac{1}{4}}. \tag{49}$$

When R is large we recover the previous bag temperature and the associated physics. When R becomes small, however, the bag temperature increases! The implications of this dependence are strange indeed. The first is the peculiar behavior of the bag's heat capacity. The second is the stability of the gas of bags (or lack thereof). The third is the signature of a bag's decay.

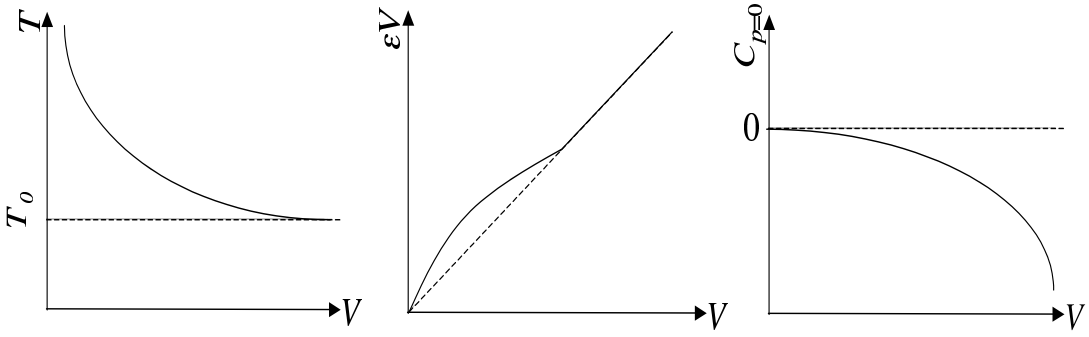

FIGURE 3. A schematic volume dependence of the bag temperature (left panel), bag energy (middle panel) and its heat capacity (right panel) for the temperature independent surface tension $a_s(T) = a_o > 0$. The left and right panels show the volume dependence of the right hand side of Eqs. (49) and (48), respectively. The resulting heat capacity of the bag is negative (right panel). The parameter T_o is defined by the bag constant as follows: $T_o = [3B/\sigma]^{1/4}$.

HEAT CAPACITY

In the standard bag model the heat capacity is infinite: no matter how much energy is fed to the bag, its temperature remains constant [9, 10]. The only effect is to make the bag larger. This is completely consistent with what we observe in isobaric phase transitions in ordinary matter. Here the isobaric condition is produced by the bag constant, and the phase transition is from hadronic to partonic phase.

Including surface effects, shows that the more energy is put into the bag, the lower its temperature becomes: i.e. the bag's heat capacity is negative. To illustrate how the negative heat capacity of the bag appears, let us consider a temperature independent surface tension: $a_s(T) = a_o > 0$. For this case, Eq. (49) shows that the bag temperature is decreasing function of its volume, whereas, according to Eq. (48), the energy of the bag is an increasing function of the bag volume. Therefore, the bag's heat capacity, defined as $\partial E/\partial T$, is negative. This is shown schematically in Fig. 3.

For a formal analysis of the heat capacity of the bag it is necessary to use Eqs. (45) and (47). From these equations one can find the heat capacity of the bag at constant pressure C_p and at constant volume C_V as:

$$C_p \equiv T\left(\frac{\partial S}{\partial T}\right)_p = C_V \; - \; \frac{3TV^{\frac{4}{3}}}{2a_s(T)}\left[4\sigma T^3 - \frac{2}{V^{\frac{1}{3}}}\frac{d\,a_s}{d\,T}\right]^2, \tag{50}$$

$$C_V \equiv T\left(\frac{\partial S}{\partial T}\right)_V = 4\sigma T^3 V \; - \; TV^{\frac{2}{3}}\frac{d^2\,a_s}{d\,T^2}\,. \tag{51}$$

In evaluating the expression for C_p we used an explicit form of the derivative

$$\left(\frac{\partial V}{\partial T}\right)_p \equiv -\left(\frac{\partial p}{\partial T}\right)_V\left(\frac{\partial p}{\partial V}\right)_T^{-1} = - \frac{3V^{\frac{4}{3}}}{2a_s(T)}\left[4\sigma T^3 - \frac{2}{V^{\frac{1}{3}}}\frac{d\,a_s}{d\,T}\right]. \tag{52}$$

From Eqs. (50) and (51) it is clearly seen that for any T where $a_s(T) \geq 0$ there may exist a range of parameters for which the heat capacity C_p, corresponding to the bag equilibrium in vacuum, is negative. This leads to a "convex intruder " in the entropy or an unusual behavior of its second derivative:

$$\left(\frac{\partial^2 S}{\partial E^2}\right)_{p=0} = \; - \; \frac{1}{T^2 C_p}\,, \tag{53}$$

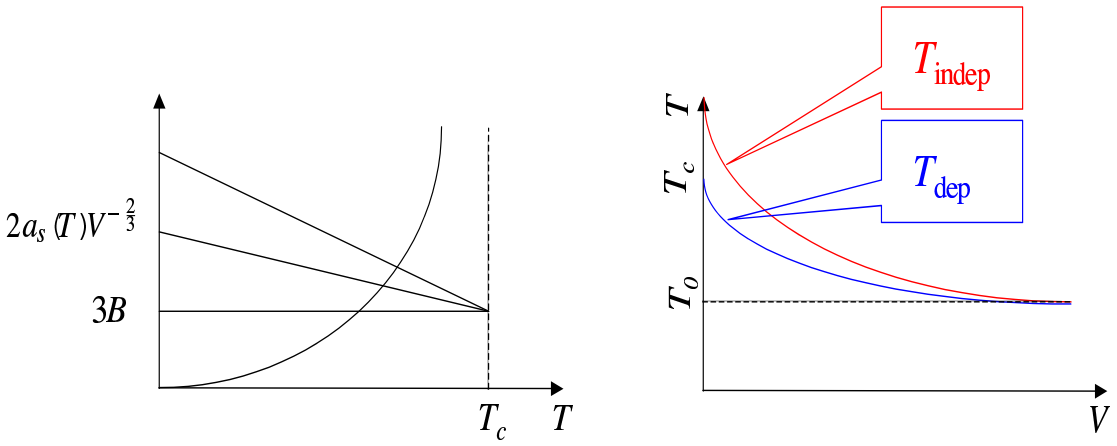

FIGURE 4. Left panel: The bag temperature T_H as a graphical solution of Eq. (55) for the linear T dependence of the bag surface tension. The left hand side of Eq. (55) is shown by a bi-quadratic parabola σT_H^4 and its right hand side is depicted by the straight lines for different values of the bag volume V. The solution of Eq. (55) is found as an intersection point between the parabola and the straight line.
Right panel: Shows schematically the range of available temperatures of the bag for T independent (red curve) and for the linear T dependent (blue curve) surface tension of the bag. See text for the details.

which becomes positive for this range of parameters.

In the literature on this subject it is argued [39, 40, 41] that all small systems (comparable in size with the range of the prevailing force) should show this effect. However, we stress that a convex intruder in the bag model with surface tension exists not for small systems, but for large ones and does not disappear in thermodynamic limit. This behavior can be verified by examining the decay products of heavy resonances: heavier resonances should decay into light hadrons of a lower temperature (but never lower than $T(R=\infty)$).

Let us now demonstrate the appearance of a convex intruder in a few simple cases. First we consider the case of constant surface tension, i.e. $a_s(T) = a_o > 0$, in more detail. Substituting a_o into Eq. (52), one obtains that $\left(\frac{\partial V}{\partial T}\right)_p < 0$. Since the heat capacity at $p=0$ is defined as $C_p \equiv \left(\frac{\partial E}{\partial V}\right)_{p=0}\left(\frac{\partial V}{\partial T}\right)_{p=0}$, its sign is opposite to the sign of the derivative $\left(\frac{\partial E}{\partial V}\right)_{p=0}$, which can be found from the expression for the energy of the bag:

$$E\Big|_{p=0} = 4BV + 3a_o V^{\frac{2}{3}} \quad \Rightarrow \quad \left(\frac{\partial E}{\partial V}\right)_{p=0} = 4B + \frac{2a_o}{V^{\frac{1}{3}}} > 0. \tag{54}$$

Thus, in the case of a constant surface tension the heat capacity at $p=0$ is negative which corresponds to a convex intruder.

Now we consider a surface tension with a linear T dependence in a spirit of the Fisher droplet model [33] or using a more elaborate approach of the recently solved "Hills and Dales Model" for surface deformations [34, 35]: $a_s(T) = c_o\frac{(T_c-T)}{T_c}$, which is defined for the temperatures not above the critical temperature T_c.

Introducing the notation $B \equiv \frac{\sigma}{3}T_o^4$, one can rewrite the equilibrium condition of the bag $p(T_H)=0$ as follows:

$$\sigma T_H^4 = \sigma T_o^4 + 2c_o\frac{T_c - T_H}{T_c V^{\frac{1}{3}}}, \tag{55}$$

which should be solved for the bag temperature $T_H(V)$.

For positive values of T_H the left hand side of Eq. (55) is a monotonically increasing function of T_H, whereas its right hand side is a monotonically decreasing function of T_H (see the left panel of Fig. 4). Therefore, there can exist a single intersection point of these two functions for any positive value of bag volume V. Using Eq. (55), one can show that the inequality $T_H \leq T_c$ is always fulfilled, if $T_c > T_o$. Moreover, one can also show that the allowed interval of

the bag temperatures is between T_o and T_c with limiting cases $T_H(V \to 0) \to T_c$ and $T_H(V \to \infty) \to T_o$ (see the right panel of Fig. 4). Similarly from Eq. (52) one finds that the bag temperature decreases, while bag volume grows, i.e. $\left(\frac{\partial V}{\partial T}\right)_p < 0$ for any V and any $T_H \leq T_c$. Since the range of allowed bag temperatures is bound between T_o and T_c, then from Eq. (50) one can immediately see that for any $T_H \leq T_c$ the heat capacity of the bag at $p = 0$ is negative for large volumes. Thus, in the case of a linear T dependence of the surface tension of the bag the convex intruder exists for large volumes of the bag. In fact, this proves the following statement: *if the surface tension $a_s(T) \geq 0$ is a regular function of T that $\frac{d\, a_s}{d\, T} \leq 0$ and $\left|\frac{d^2\, a_s}{d\, T^2}\right|$ is finite provided that the solution $T(V)$ of Eq. (49) does not vanish in the limit $V \to \infty$, then in this limit the heat capacity at constant $p = 0$ is negative and sign $C_p = sign\left(\frac{\partial V}{\partial T}\right)_p < 0$.*

STABILITY OF A GAS OF BAGS

A gas of resonances (bags) is frequently considered either in equilibrium or in transport problems. In our previous papers [9, 10] (see also preceding sections) we have shown that an ordinary bag (no surface energy) is nearly indifferent to fragmentation into smaller bags. In fact, under rather general conditions it appears that there is a mild tendency for a gas of bags to collapse into a single one. We show now that the introduction of the surface leads to an even stranger tendency for a gas of bags toward collapse.

Let us assume an arbitrary mass distribution in a gas of bags, and for simplicity, let us assume that the gas is confined in a fixed volume with its decay products (say pions). The gas cannot be isothermal since the smaller bags have larger temperature than the big ones. Thus the smaller bags evaporate first and their evaporation products are absorbed by the larger bags until only one remains. It may be argued that isothermicity can be achieved by having all the bags to be of the same size. But this situation is clearly unstable. Any small perturbation in size will lead to a catastrophic collapse of all bags into a single one.

DECAY OF A BAG

A hot bag, unless constrained by conserved quantities, must decay. As it decays, the instantaneous spectrum of the decay products indicates the bag's instantaneous temperature. Without surface effects the bag temperature is constant and the overall spectrum and the instantaneous spectrum is the same.

With the surface effects, as the bag decays and becomes smaller, its temperature increases. Therefore the overall spectrum integrated over the overall decay must differ from the instantaneous spectrum associated with each temperature. The shape deviation of the overall spectrum from that of an instantaneous spectrum at fixed temperature may be an interesting observable to characterize both the effect and the magnitude of the surface energy and its temperature dependence. It is amusing to notice the similarities with a black hole and its temperature as it decays through the Hawking radiation.

CONCLUSIONS

In Refs. [9, 10] we generalized the SBM results [22] to systems of finite energy by showing explicitly that even a single resonance with the Hagedorn mass spectrum degeneracy, i.e. *a Hagedorn thermostat,* keeps an almost constant temperature close to $T_{\mathscr{H}}$ for any number of Boltzmann particles $3 < N_B \leq N_B^{kin}$. For the high energy limit $E \gg m_o$ this means that a single Hagedorn resonance defines the temperature of the system to be only slightly different from $T_{\mathscr{H}}$ until the energy of the Hagedorn thermostat is almost negligible compared to E. In contrast to the grand canonical formulation of the original SBM [22], in the presence of a Hagedorn thermostat the temperature $T_{\mathscr{H}}$ can be reached at any energy density.

The thermostatic nature of a Hagedorn system obviously explains the ubiquity of both the inverse slopes of measured transverse mass spectra [13] and hadronization temperature found in numerical simulations of hadrons created in elementary particle collisions at high energies [19, 11, 24]. By a direct evaluation of the microcanonical partition we showed that in the presence of a single Hagedorn thermostat the energy spectra of particles become exponential with no additional assumptions, e.g. *phase space dominance* [25] or *string tension fluctuations* [26]. Also the limiting temperature found in the URQMD calculations made in a finite box [27] can be explained by the effect of the Hagedorn thermostat. We expect that, if the string parametrization of the URQMD in a box [27] was done microcanonically instead of grand canonically, the same behavior would be found.

The Hagedorn thermostat model generalizes the statistical hadronization model which successfully describes the particle multiplicities in nucleus-nucleus and elementary collisions [19, 11, 24]. The statistical hadronization model

accounts for the decay of heavy resonances (clusters in terms of Refs. [19, 11, 24]) only and does not consider the additional particles, e.g. light hadrons, free quarks and gluons, or other heavy resonances. As we showed, the splitting of a single heavy Hagedorn resonance into several does not change the temperature of the system. This finding justifies the main assumption of the canonical formulation of the statistical hadronization model [19] that smaller clusters may be reduced to a single large cluster. Also our approach naturally explains why a sophisticated transport model [28], which treats the hadronic reactions microscopically, leads to the thermal equilibration at the Hagedorn temperature $T_{\mathscr{H}}$ and to a chemical composition of hadrons given by the equilibrium values of particle concentrations. Thus, recalling the MIT Bag model interpretation of the Hagedorn mass spectrum [7, 8], we conclude that quark gluon matter confined in heavy resonances (hadronic bags) controls the temperature of surrounding particles close to $T_{\mathscr{H}}$, and, therefore, this temperature can be considered as a coexistence temperature for confined color matter and hadrons. Moreover, as we showed, the emergence of a coexistence temperature does not require the actual deconfinement of the color degrees of freedom, which, in terms of the Gas of Bags Model [29], is equivalent to the formation of the infinitely large and infinitely heavy hadronic bag.

Within the framework of the Hagedorn thermostat model we found that even for a single Hagedorn thermostat and $a > \frac{3}{2}$ the system's temperature $T = T^*(m_H^+)$ as a function of N_B remains almost constant for $N_B < N_B^{kin}$, reaches a maximum at N_B^{kin} and rapidly decreases for $N_B > N_B^{kin}$ (see Fig. 1). For $a < \frac{3}{2}$ the temperature has a plateau $T = T^*(m_H^+)$ for $N_B < N_B^{kin}$, and rapidly decreases for $N_B > N_B^{kin}$. If such characteristic behavior of the hadronization temperature or the hadronic inverse slopes can be measured as a function of event multiplicity, it may be possible to experimentally determine the value of a. For quantitative predictions it is necessary to include more hadronic species into the model, but this will not change our result.

If we apply the Hagedorn thermostat model to elementary particle collisions at high energy, then, as shown above, the temperature of created particles will be defined by the most probable mass of the Hagedorn thermostat. If the most probable resonance mass grows with the energy of collision, then the hadronization temperature should decrease (increase) to $T_{\mathscr{H}}$ for $a > \frac{3}{2}$ ($a < \frac{3}{2}$). Such a decrease is observed in reactions of elementary particles at high energies, see Table 1 in Ref. [24].

Further we discussed the effects of the surface energy on the properties of a bag (heavy resonance) in vacuum. We showed that in the presence of non-zero surface tension the temperature of the bag (and any system in thermal contact it) acquires a volume dependence, so that smaller bags are hotter. The temperature of large bags cannot be smaller than the Hagedron temperature. Under not too restrictive conditions we found that the heat capacity of large bags at zero pressure is negative, i.e. such bags have abnormal behavior of the second derivative of entropy with respect to energy. These unusual properties lead to an instability of any number of bags other than one. We argued that the temperature of the decay products of the evaporating bag should grow during the evaporation process, which, hopefully, can be observed.

In order to apply these results in a more physical fashion to the quark gluon plasma formation in relativistic nucleus-nucleus collisions (where the excluded volume effects are known to be important [30, 29, 31, 32, 36, 37] for all hadrons) the eigen volumes of all particles should be incorporated into the model. For pions this should be done in relativistic fashion [38]. Also the effect of finite width of Hagedorn resonances may be important [5] and should be studied.

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