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The Statistical Multifragmentation Model

A short presentation

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SMM Literature :

PART (I)
Copenhagen Group

(1) J. BONDORF et al
NPA 443 (1985) 321.
Formulation of model CANONICAL calcul.

(2) J. BONDORF et al.
NPA 444 (1985) 460
Microcanonical Calc.

(3) H. BARZ et al.
NPA 448 (1986) 753
Decay of primary fragments

(4) K. SNEPPEN et al. :
NPA 470 (1987) 213
Two-component system - partition (Biased branching)

(5) A. BOTVINA et al :
NPA 475 (1987) 663
SMM Code
Microcanonical / canonical + deex.

(6) A. BOTVINA et. al :
NPA 507 (1990) 649
Microcanonical ("OLD")
Nice SMM summary

(7) A. BOTVINA et al.
PRC 63 (2001) 061601
"New" SMM (Markov chain)
Metropolis partitioning

(8) A. BOTVINA
PRE 62 (2000) R 64
Partitioning Method
Metropolis scheme

(9) A. BOTVINA et al.
PRC 65 (2002) 044610
Isoscaling + SMM summary

(10) J. BONDORF (Review of SMM)
Phys. Rep. 257 (1995) 133.

SMM Literature : PART II Berlin Group

① D.H.E Gross
Rep. Prog. Phys. 53 (1990) 605

- Nice Review of { Evaporation, Sequential decay, Multifragmentation
- Ingredients of code MMMG

② D.H.E Gross, K. Sneppen
NPA 567 (1994) 317

- SMM - MMMC comparison

③ A.S. Botvina, DHE Gross
NPA 592 (1995) 257

- Effect of J (use of MMMC)

④ A.S. Botvina et al
PRC 59 (1999) 3444

- proximity effect treated with MMMG

⑤ D.H.E Gross "Microcanonical thermo"
Phys. Rep. 279 (1997) 119

Some Notes :

- Ordinary (macroscopic) thermodynamics :

  $N \sim N_A = 6.0 \cdot 10^{23}$

  Extensive (E, S ....)

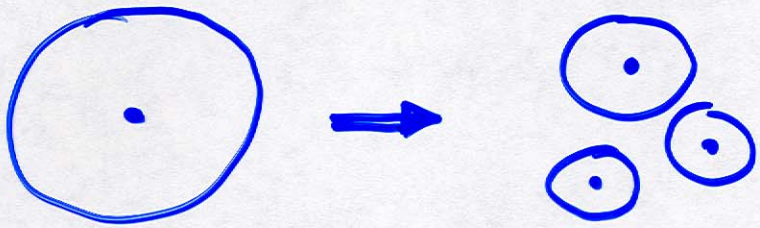
- Thermodynamics of finite systems (small)

  e.g. nuclei : $N \sim 100$

  clusters : $N \sim 10^2 - 10^3$

  - surface effects : $E_{surf} \sim 4\pi R^2 \sim A^{2/3}$
  - Coulomb effects : long range force

    $E_C \sim \frac{Z^2}{R}$



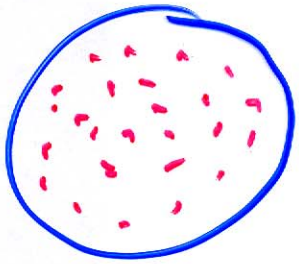
  - Non-extensive : careful treatment, size, partitioning

(coordinates + momenta) → Phase Space :

Nuclear Collisions :

Ⓘ

Ⓘ Dynamical Stage (fast)



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Ⓘ De-excitation Stage ("slowen")

Assume : thermodynamic Equilibrium

E → min

available phase space fully occupied

(determined by conservation laws + constraints)

Macroscopic state : Description :

a) E_0, A, Z ($\vec{P}$, $\vec{J}$ ...)

- Isolated System

(Microcanonical)

b) T, A, Z → $\langle E \rangle$

- in contact with "heat bath"
- mechanically isolated

(CANONICAL)

c) T, $\langle A \rangle$, $\langle Z \rangle$ → $\langle E \rangle$

in contact with

- heat bath
- "particle" bath

(Macro-canonical)

Macroscopic state : "weight" { How many different microscopic states can represent this "macro" state

(I) Microcanonical Ensemble : $S_i = k \cdot \ln W_i$

$$w_{micro} \sim \frac{e^{S_i}}{\sum_i e^{S_i}}$$

(II) Canonical Ensemble : "Helmoltz Free Energy"

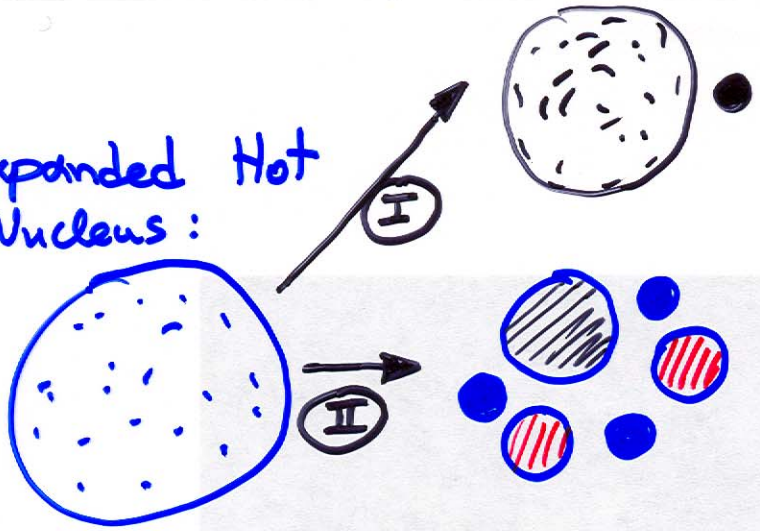
$$w_{can} \sim \frac{e^{-F_i/T}}{\sum_i e^{-F_i/T}} \qquad F_i = E_i - T \cdot S_i$$

(III) Macro canonical Ensemble:

$$w_{micro} \sim \frac{e^{-\frac{1}{T}(F_i - \mu_{A,i} A - \mu_{Z,i} Z)}}{\sum_i \left(e^{-\frac{1}{T}(F - \mu_{A,i} A - \mu_{Z,i} Z)} \right)}$$

$$\mu_{A,i} = \left(\frac{\partial F_i}{\partial A} \right)_{T,V,Z}$$ chemical potential

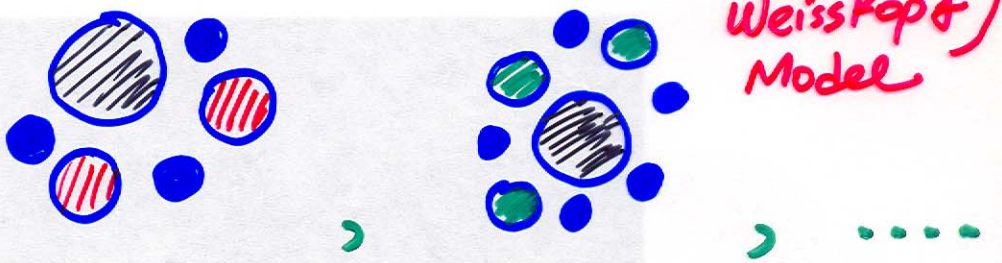
Expanded Hot Nucleus:



Evaporation

$$P \propto \frac{\rho(A-a)}{\rho(A)}$$

Weisskopf Model



A_0, Z_0, E_0

Break-up Partitions:

Combinations of all possible arrangements of the nucleons that can form A_0, Z_0

Break-up partition f:

$$\{ N_{AZ},\ 1 < A < A_0,\ 1 < Z < Z_0 \}$$

Microcanonical weight of f:

$$P \propto W_f \sim e^{S_f(E_0, A_0, Z_0)}$$

A, Z conservation:

$$\sum_{A,Z} N_{AZ} \cdot A = A_0$$

$$\sum_{A,Z} N_{AZ} \cdot Z = Z_0$$

How do we calculate S_f ?

Free Energy of the partition f:

$$F_f = E_f - T_f \cdot S_f \quad (1)$$

$$\underline{E_0} \equiv E_f = F_f + T_f \cdot S_f \quad (2) \qquad S_f = -\frac{\partial F_f}{\partial T_f} \quad (3)$$

"Recipe" to express the free energy:

based on liquid drop model

$$F_f = F_{f,coul} + \sum_{A,Z} N_{AZ} \cdot F_{AZ} \quad (3)$$

$$F_{AZ} = F_{AZ}^{vol} + F_{AZ}^{surf} + F_{AZ}^{Sym} + F_{AZ}^{Coul} + F_{AZ}^{trans} \quad (4)$$

$$F_{AZ}^{vol} = \left[-W_0 - \frac{T_f^2}{\varepsilon_0(A)} \right] \cdot A$$

$W_0 = 16$ MeV

$\varepsilon_0(A) = \underline{16}$ MeV

$$F_{AZ}^{surf} = \underbrace{\beta_0}_{\downarrow 4\pi\sigma_0} \left[\frac{T_c^2 - T_f^2}{T_c^2 + T_f^2} \right]^{5/4} \cdot A^{2/3}$$

$\beta_0 = 18$ MeV

$$F_{AZ}^{symm} = C_{sym} \cdot \frac{(N-Z)^2}{A}$$

$C_{sym} = 25$ MeV

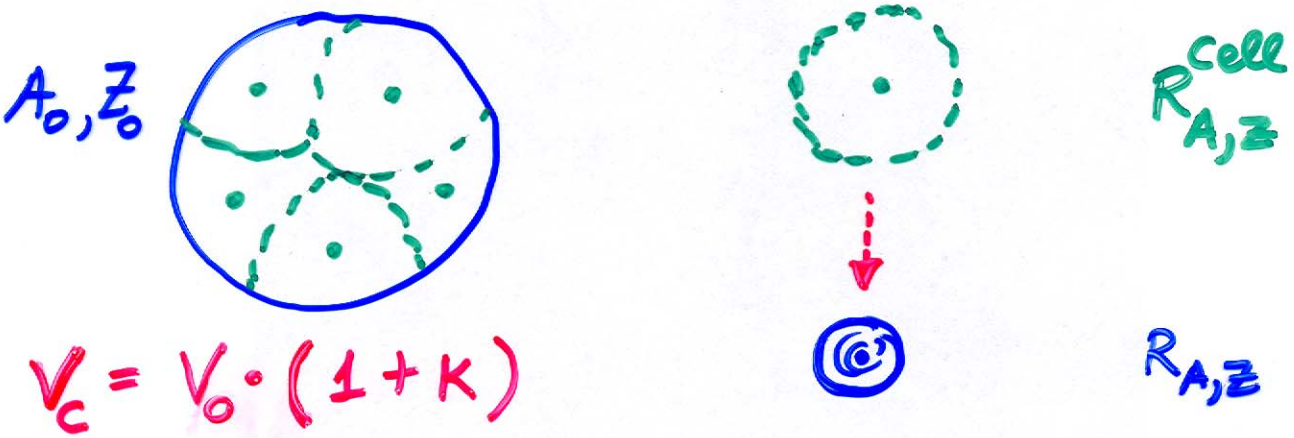
Coulomb Energy : (Wigner-Seitz approx.)

for a given partition : $\{N_{AZ}\}$

$$E_f^C = \frac{3}{5}\frac{e^2 Z_0^2}{R_C^2} + \sum_{A,Z} N_{AZ} \cdot \Delta E^C_{A,Z}$$

uniformly charged sphere : expanded

Additional Coulomb En. due to clusterization



$V_C = V_0 \cdot (1+K)$

Change of Energy due to the "collapse" of charge from radius $R^{cell}_{A,Z} \rightarrow R_{A,Z}$

$$\Delta E^C_{A,Z} = \frac{3}{5}\frac{e^2 Z^2}{R_{A,Z}} - \frac{3}{5}\frac{e^2 Z^2}{R^{cell}_{A,Z}}$$

radius of fragment at normal density

$$\Delta E^C_{A,Z} = \frac{3}{5} e^2 Z^2 \left[\frac{1}{R_{A,Z}} - \frac{1}{R^{cell}_{A,Z}}\right]$$

New SMM :
- Exact treatment of Coulomb Energy for each partition
- Exact treatment of J

Finding the partitions : $\{N_{AZ}\}$

I) Direct Method — find all possible partitions

e.g. for $A = 100$: $N_A \sim 10^8$

II) Choose a representative ensemble of all these partitions

a) Biased Branching Method ("OLD" SMM)

b) Metropolis Partitioning (Markov chain generation) (New SMM!)

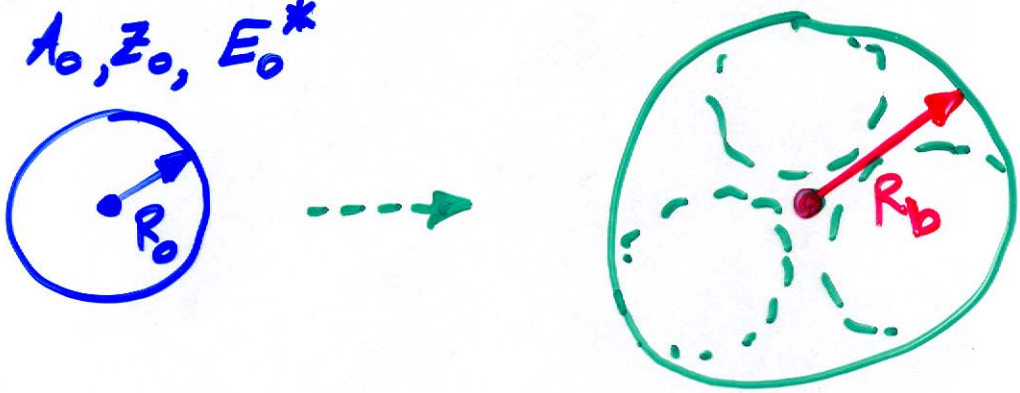
Fragment kinetic Energies : (Microcanonical Temperature of partition f) → T_f

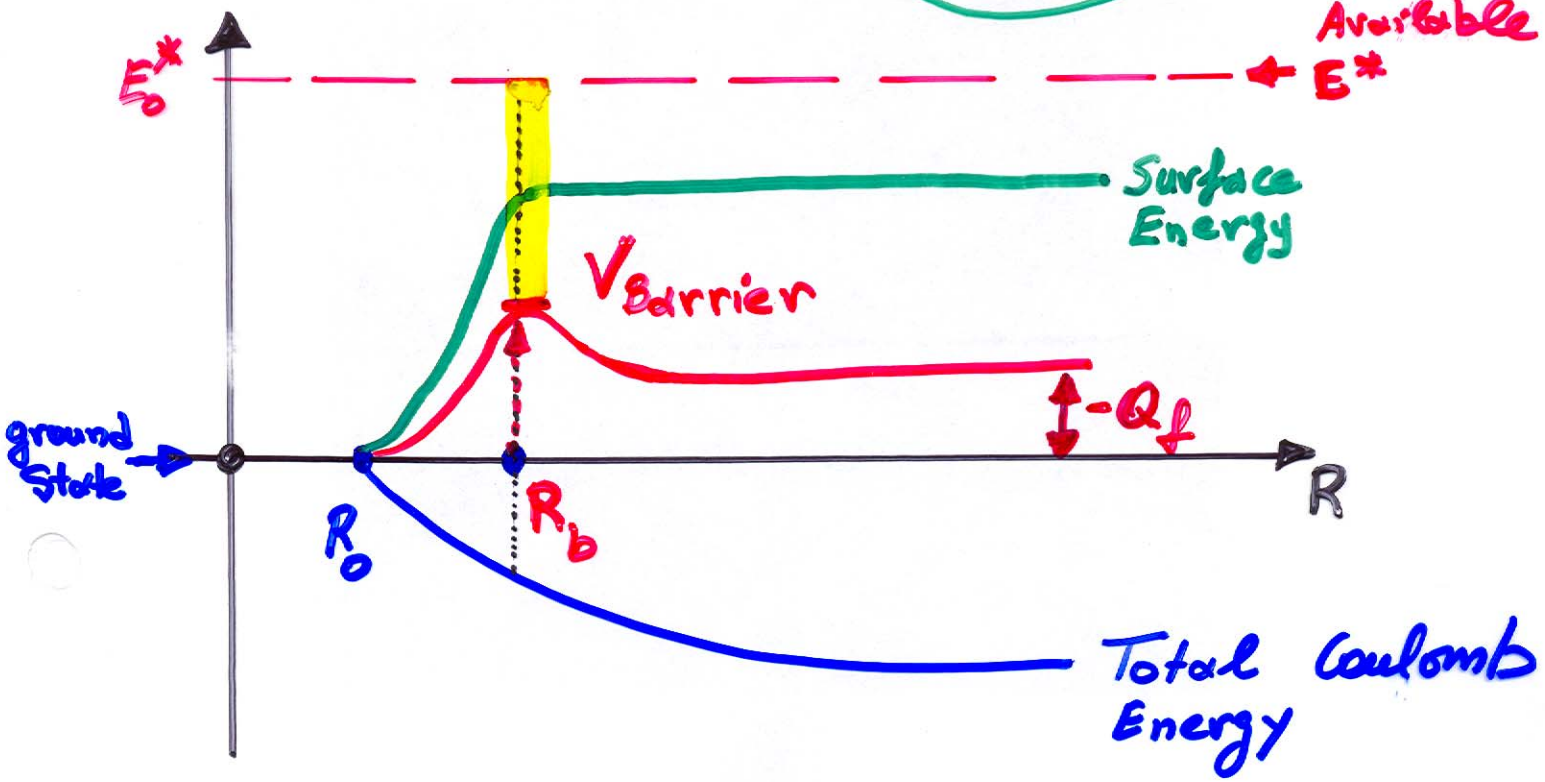
Maxwell - Boltzmann distributions at T_f :

$$\frac{dN_{A,Z}}{dE} \sim \sqrt{E} \cdot e^{-E/T_f}$$

Fragmentation "Barrier" for a partition:

A_0, Z_0, E_0^*





DIT / SMM2 (G.S. Feb 04)

① Run DIT Code → primary events

dit_25Kr086Sn124.dat ← input control file

E/A Proj tgt .outg ← primary events

② Prepare for SMM2 :

Code : smm2-input.f (Main source + info on second source)

.outg → .smm2-in

③ New SMM code (+ G.S. interface)

smm2gs.f ← smm2-param.in

← .dat

→ .smm2 , .smm2-hot

```
File: smm2_param.in  (input parameters for SMM2, G.S.) Kr+Sn      7-jan-04

  1       IMULF    0: only evap            1: full multifrag/(evap/fiss)
  1       IMETR    0: old sampling,        1: metropolis sampling imic2
  4       IMECH    2: A-space, METR-2      4: A,Z space, METR-4
  5       FKACOL   kappa:   V_total = (1.0 +kappa)*V_0
 16       EPSIL0   default=16, if 99999 -> 'cold' frag., can be A-dep.  see code
  1       ILIDR    1: liquid-drop mass,  0: exact mass formula
 0.5      E_A_ex_min,   min Eex/u to be treated. Lower Eex events thrown away.
 1.5      E_A_ex_mult, min Eex/u above which multifrag. is open (IMULF = 1)
 15.0     E_A_ex_max,   max Eex/u: limited value (above that do not treat this e
  3       z_min_gs,  Z min for writing to GS out file (to avoid light stuff)
  0       iprox,    0: no proximity effect,  1: proximity of 2nd source consider
  1       XRN0     for fragment overlap
  0       IQUAN    0: no,  1: quantum treatment of p,n, t, He
 25.0     Csym     symmetry energy coefficient
  0       IKAPPA,   0: Vfree mult depeendent(usual), 1: Vfree=Vcoul=const
  0       IMIC2     0: usual Metr. partition,  2: direct partition in 2 frags
```