

Isotopic trends in the production  
of superheavy nuclei

G.G.Adamian, N.V.Antonenko,

A.N.Bezbakh, V.V.Sargsyan.

W.Scheid, T.M.Shneidman

BLTP JINR, Dubna, Giessen Uni

## Experimental data on structure of SHE (Z=112-118):

**The found experimental trend of the nuclear properties [Q(alpha)-values and half-lives] of the SHE indicates**

*the importance of N=184 shell*

**but**

*a small influence of the proton shell at Z=114.*

**No discontinuity is observed when the proton number 114 is crossed at N=172 - 176!**

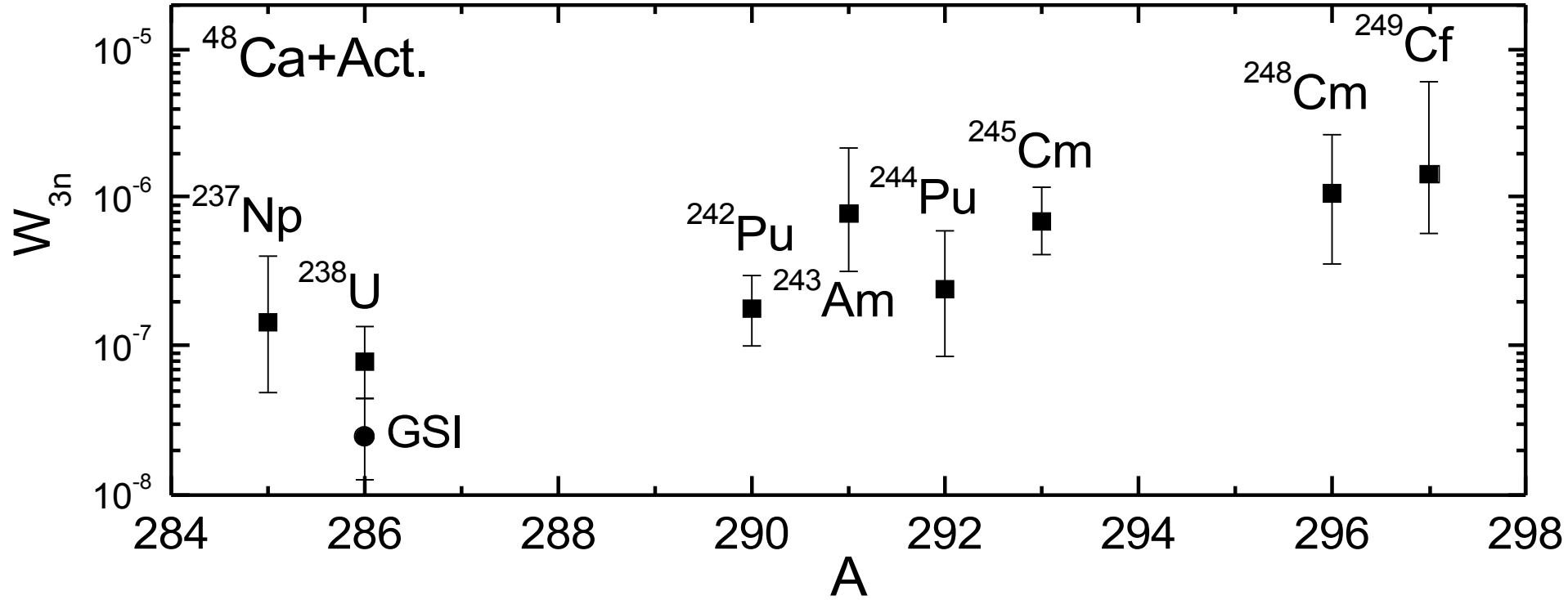
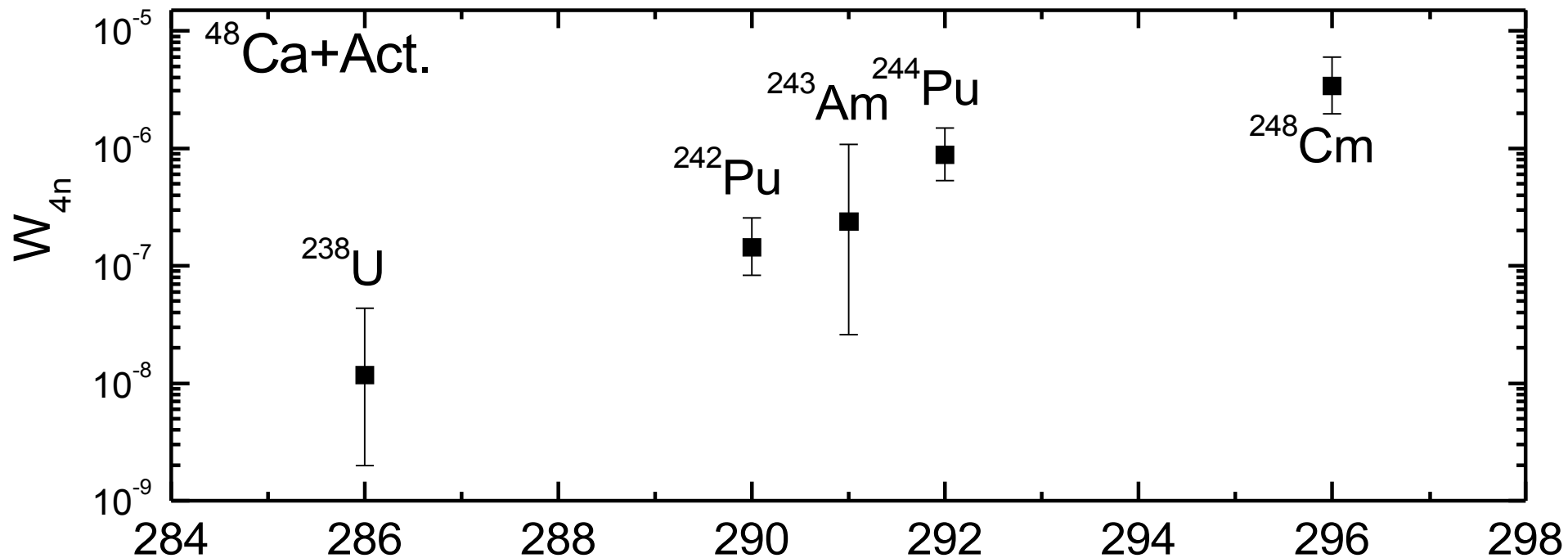
**Magic numbers Z=114 & N=184 were predicted with microscopic-macroscopic models.**

## Experimental data on SHE production cross sections:

The measured production cross sections in reactions  $^{48}\text{Ca} + \text{Actinide}$  do not depend strongly on atomic number  $Z$  of SHE and are on the picobarn level.

The fusion cross section strongly decreases with increasing  $Z_1 \times Z_2$  !

Production cross section =  
= (Fusion cross section)  $\times$  (Survival probability)



**Thus, the present experimental data point out that magic proton shell is located at  $Z > 118$  !**

**This observation is an accordance with predictions of mean field models where island of stability is near nucleus with  $Z=120 - 126$  and  $N=184$ .**

**All microscopic-macroscopic approaches based on various shape parametrizations provide the closed proton shell  $Z=114$  ?**

**Microscopic-macroscopic approach based on the two-center shell model (TCSM) [J.Maruhn and W.Greiner] was used.**

**Parameters were set so to describe spins & parities of g.-s. of known heavy nuclei.**

# Comparison with other calculations

$^{270}\text{Hs}$  gs.:

$$\lambda=1.14, \beta=1.06 \rightarrow \beta_2=0.25, \beta_4=-0.03$$

P.Möller et al.:  $\beta_2=0.231, \beta_4=-0.086$

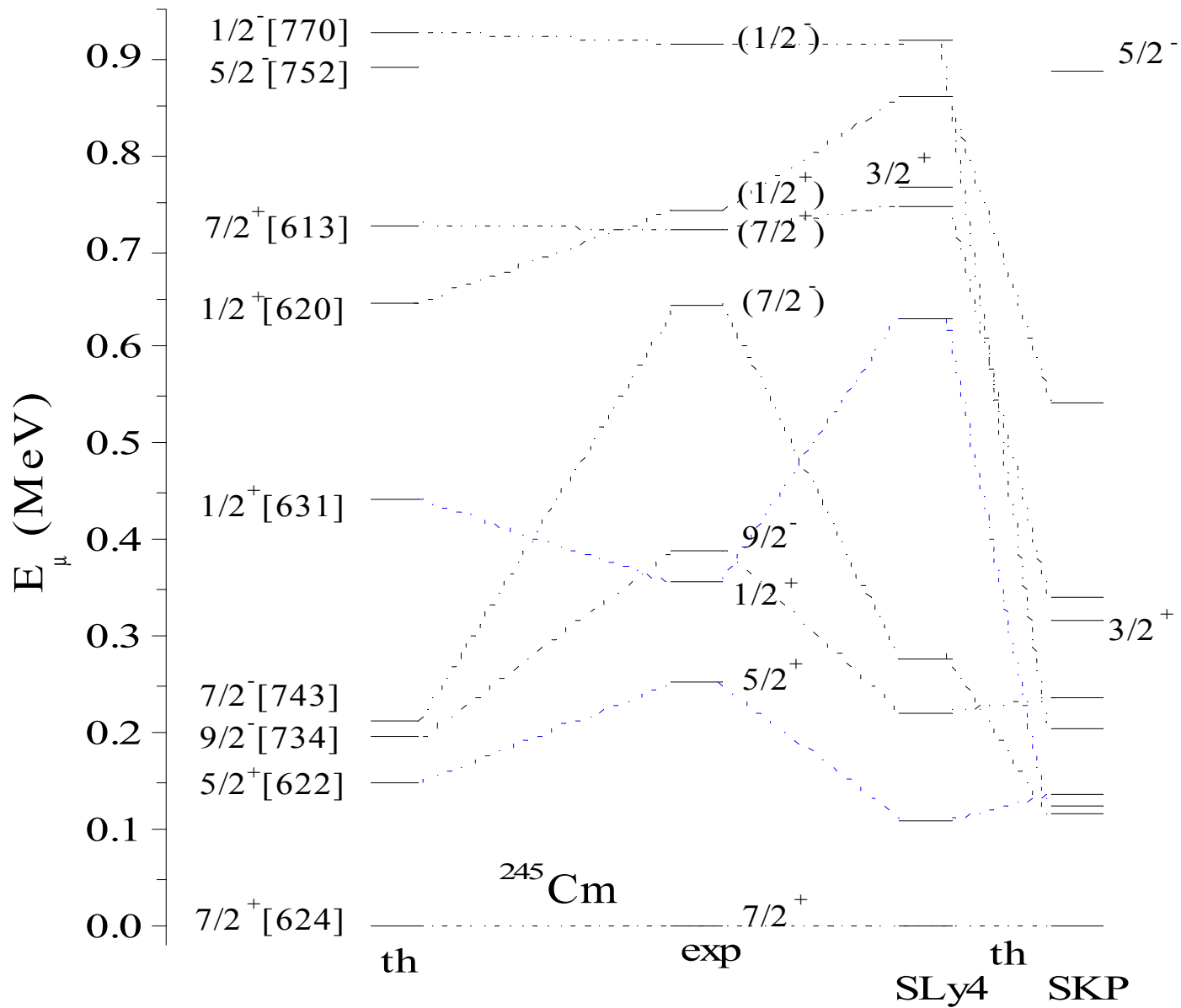
For  $^{268,269,270,271}\text{Hs}$ , the microscopic corrections are

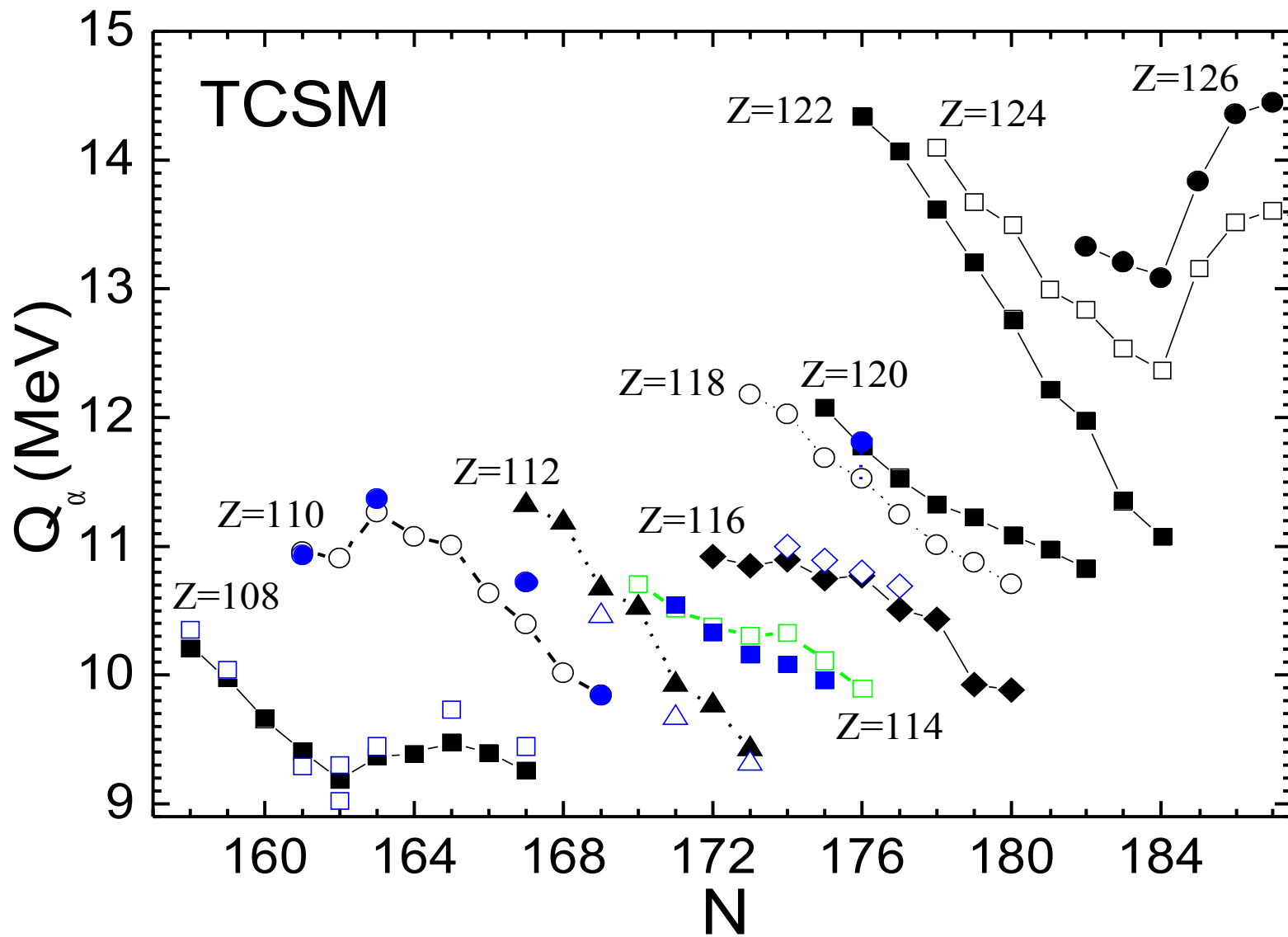
$$-5.95, -6.38, -6.54, -6.64 \text{ MeV}$$

P.Möller et al.:  $-5.94, -6.37, -5.95, -5.86 \text{ MeV}$

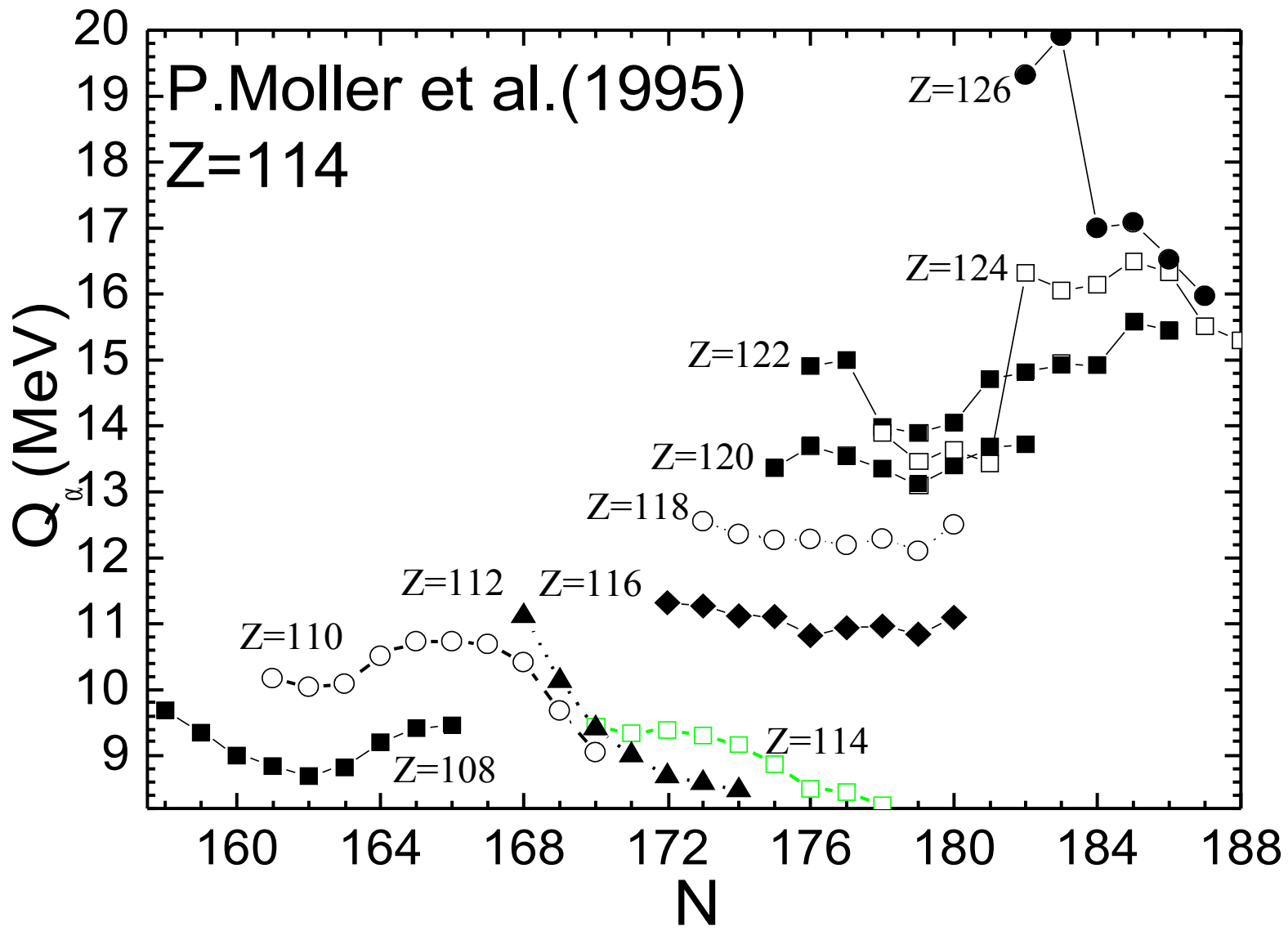
A.Kuzmina et al., Eur. Phys. J. A 47 (2011) 145

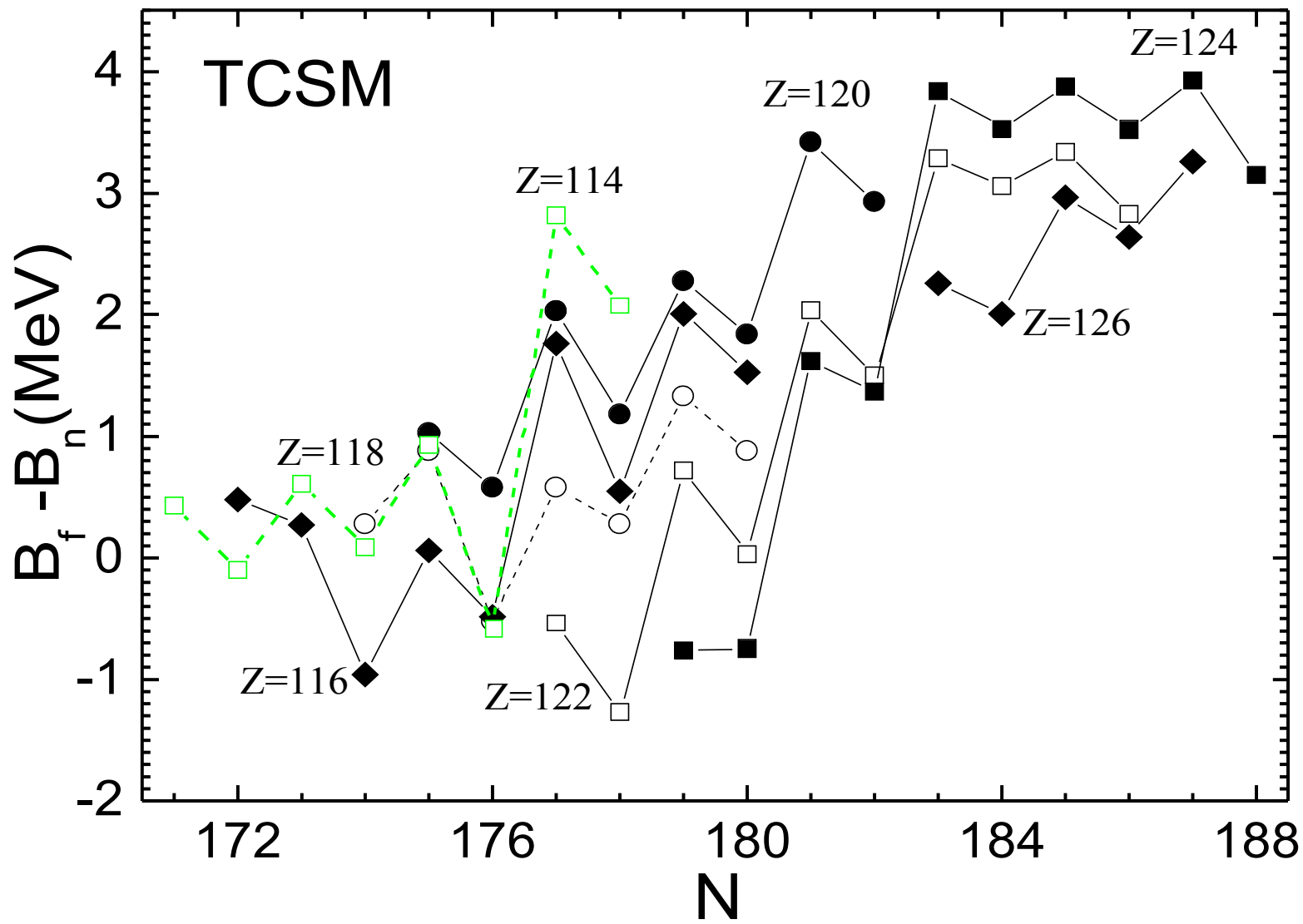
Phys. Rev. C 85 (2012) 014319; 017302

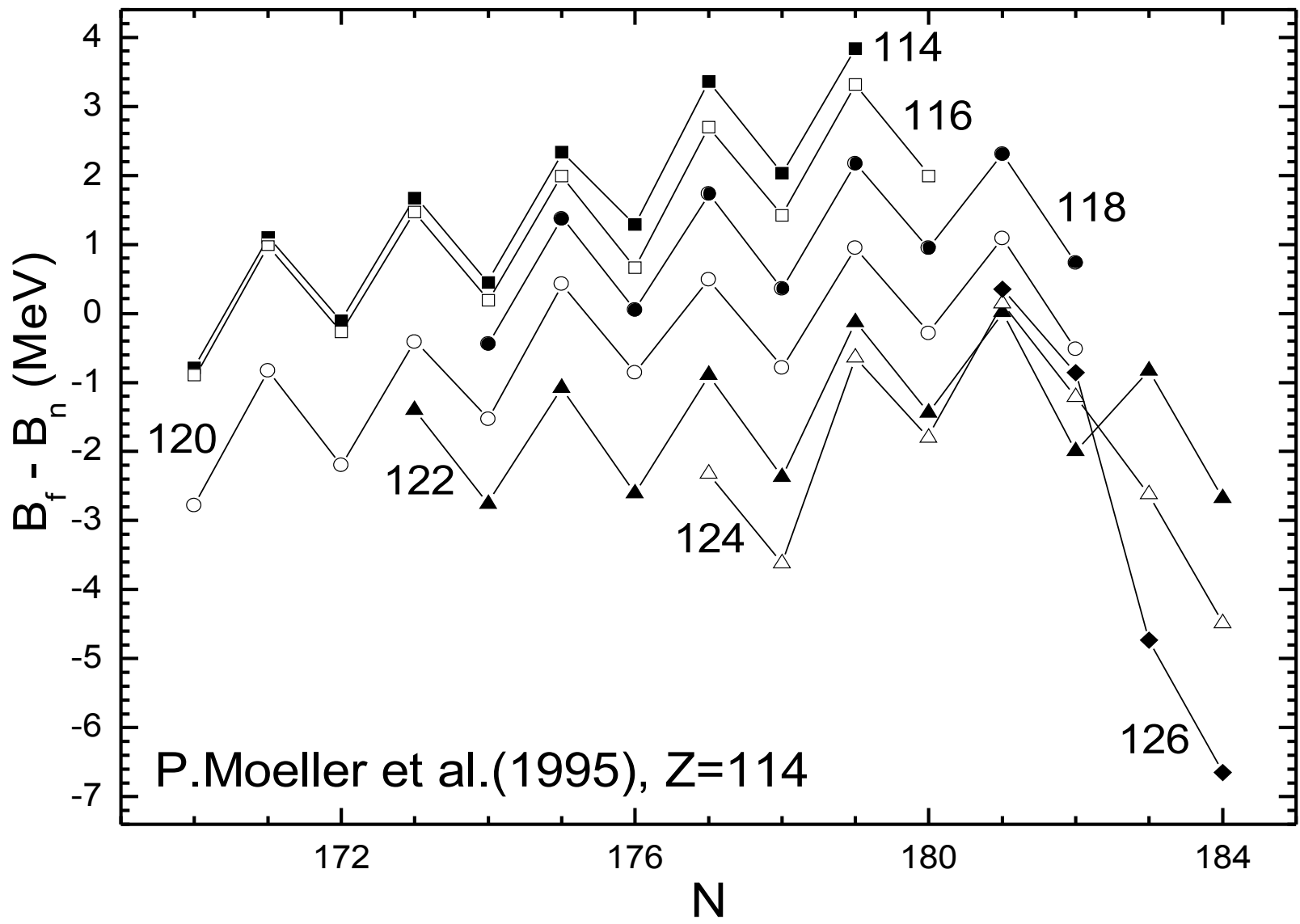


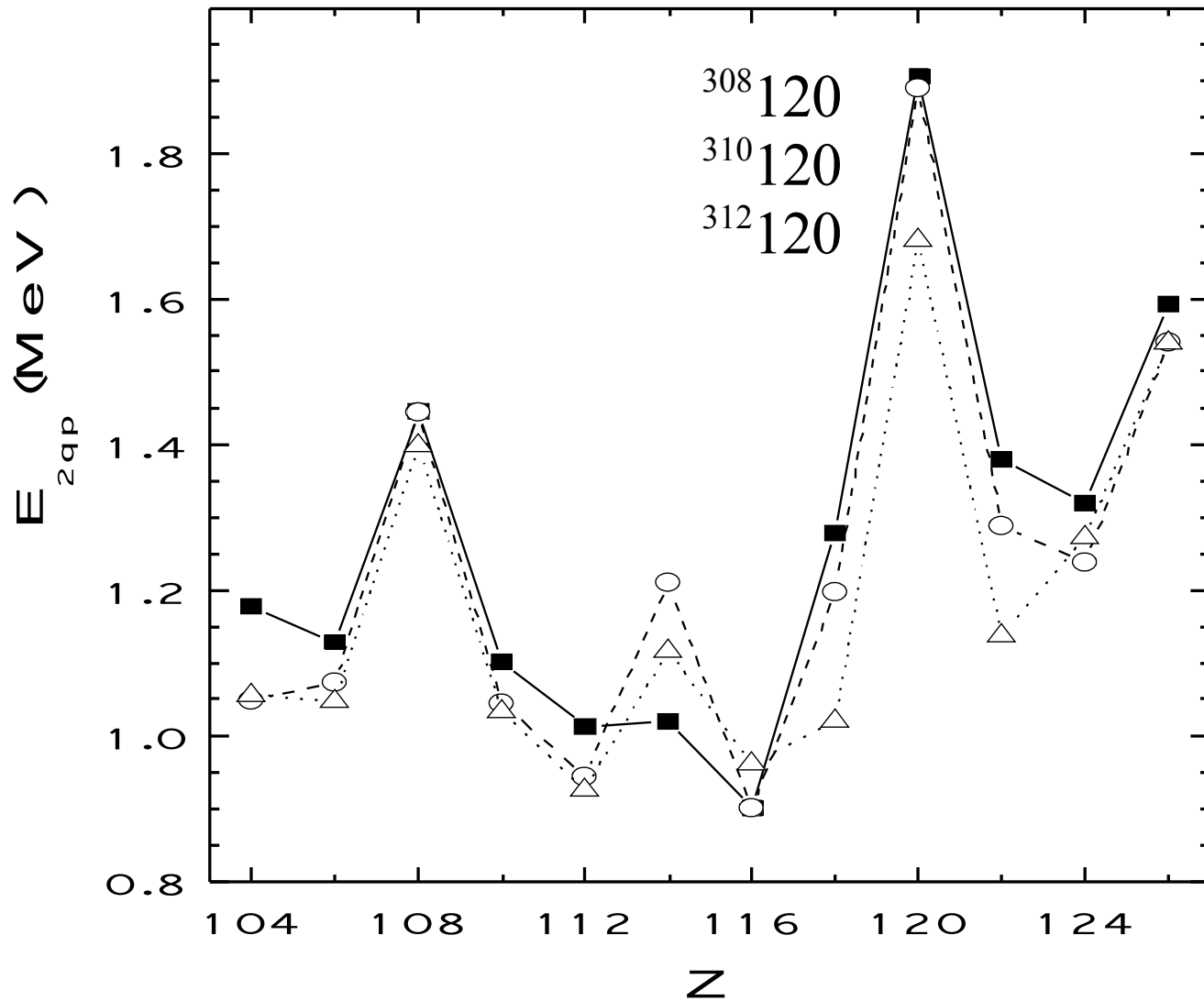






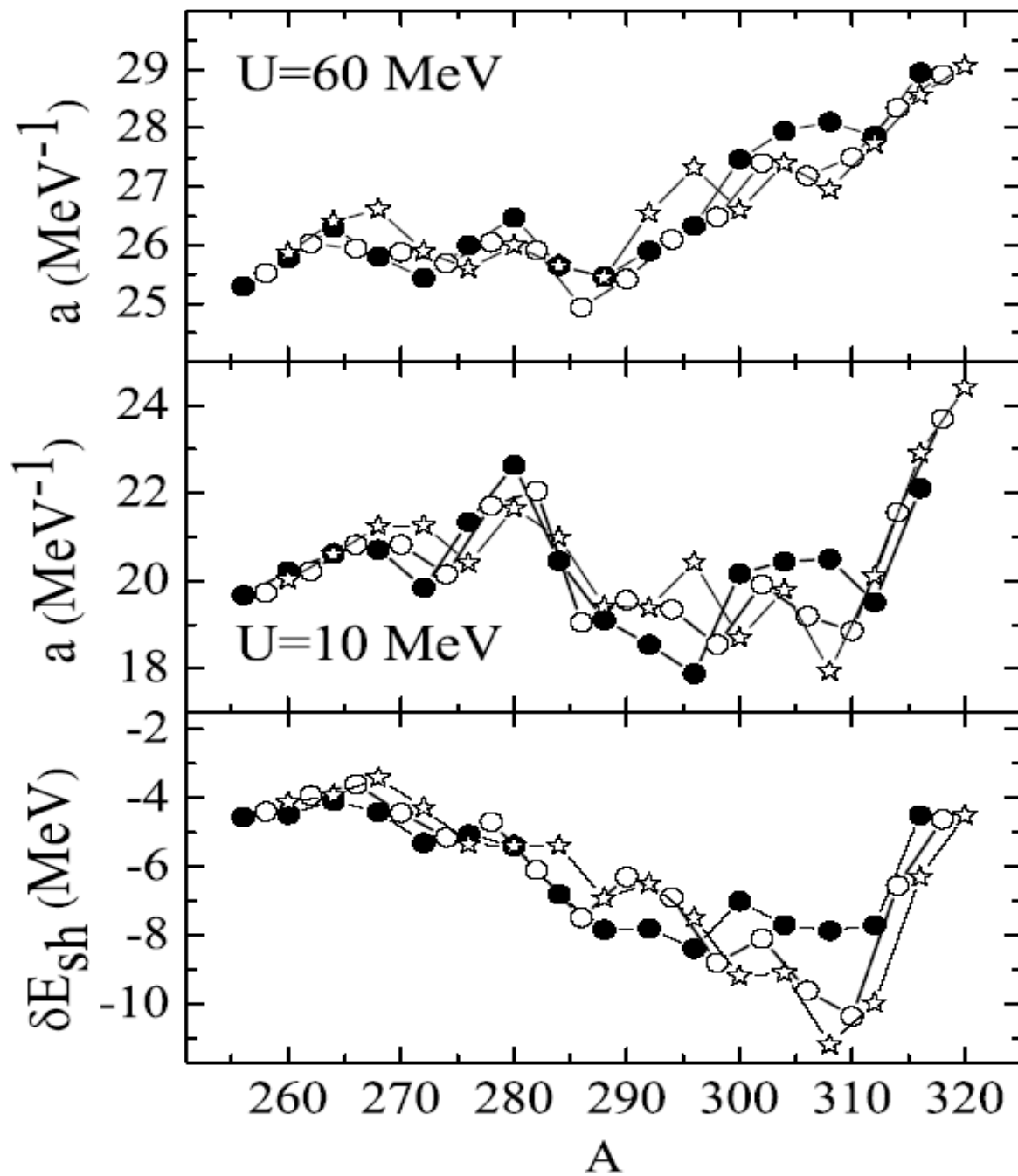


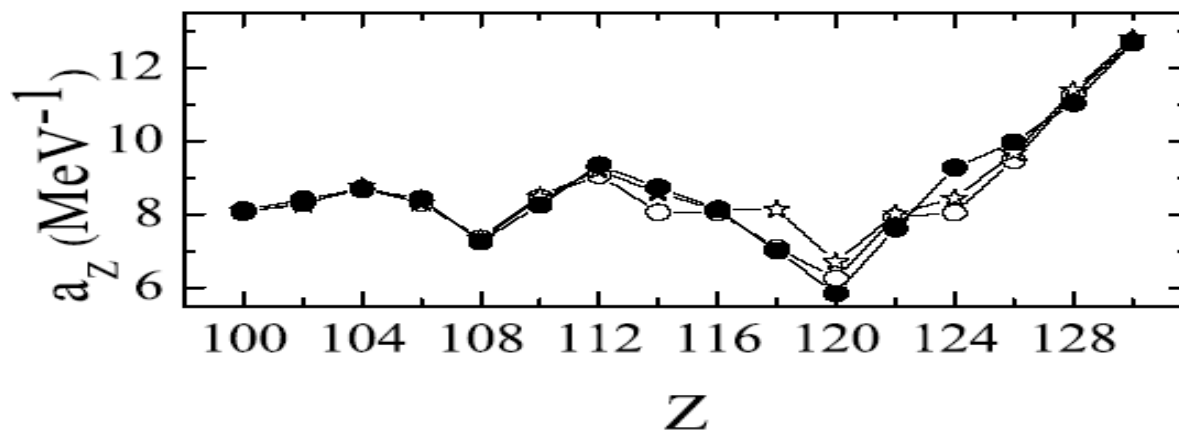
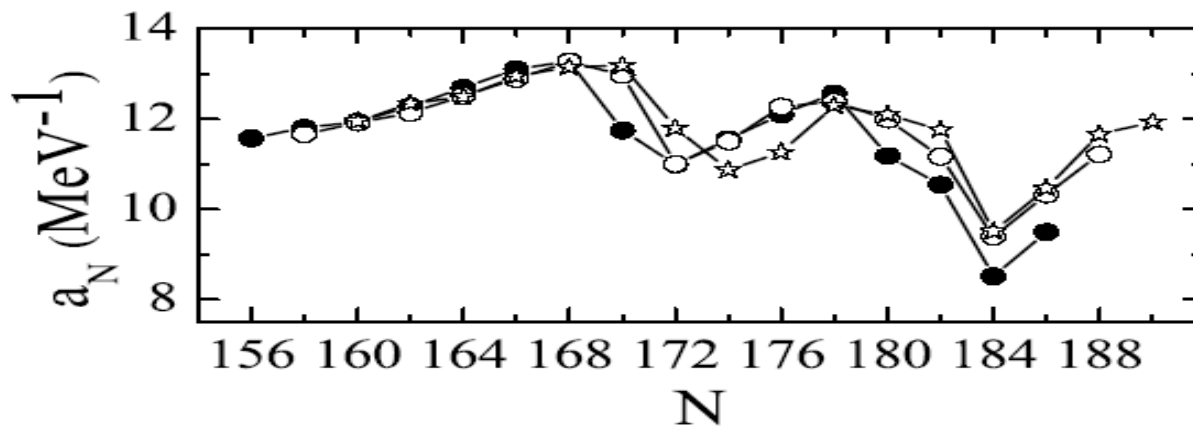




Energies of two-quasi-proton states in alpha-decay chains containing indicated nuclei with  $Z=120$

296,298,300 120





$$a(A, U) = \tilde{a}(A) \left[ 1 + \frac{1 - \exp\{-E^*/E'_D\}}{E^*} \delta E_{sh} \right]$$

$$\tilde{a}(A) = \alpha A + \beta A^2$$

$$E'_D = 27 \text{ MeV}$$

$$\alpha = 0.118 \text{ MeV}^{-1}$$

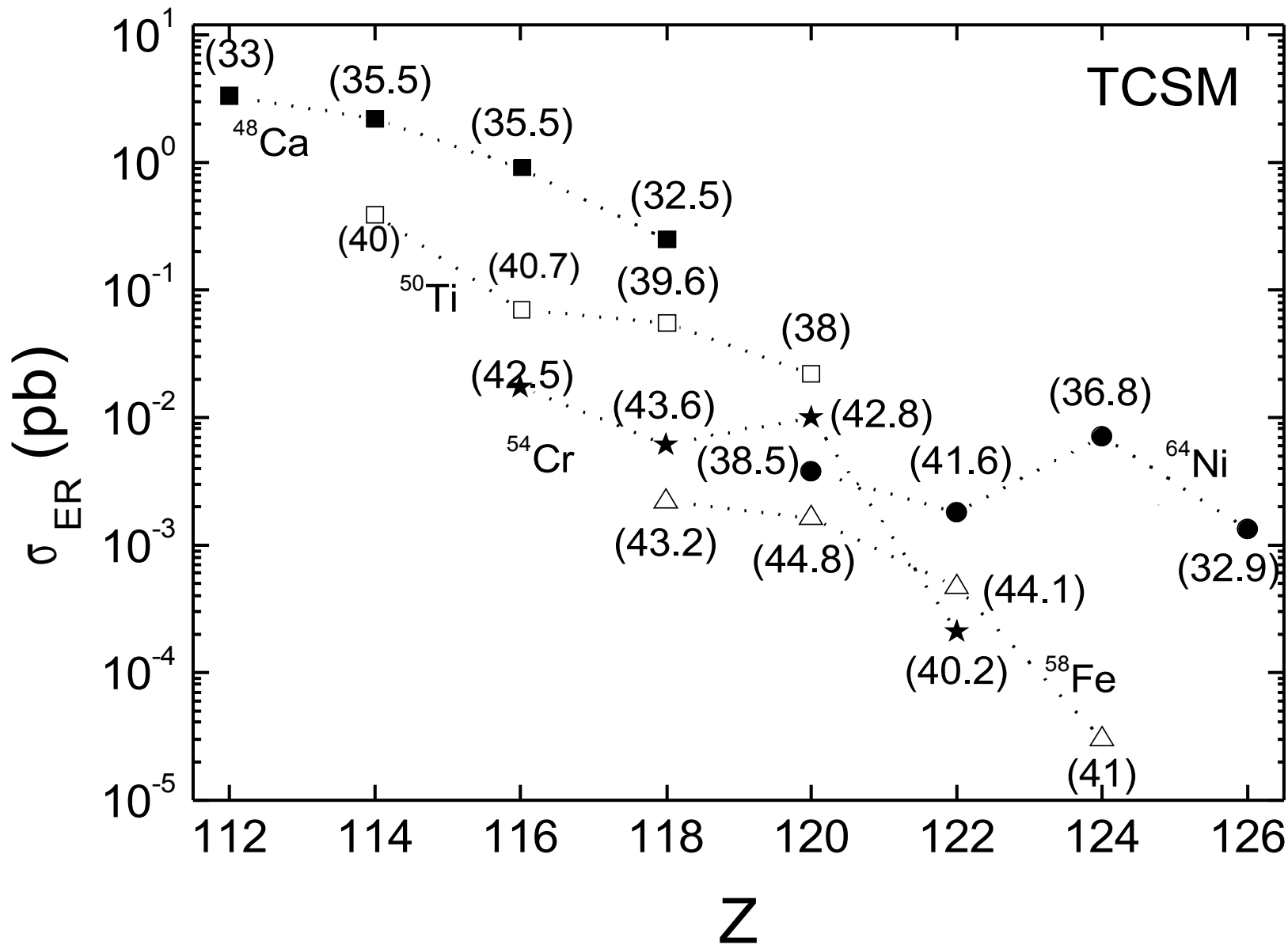
$$\beta = -0.53 \times 10^{-4} \text{ MeV}^{-1}$$

# Dependence of the calculated evaporation residue cross sections on the predicted shell structure and magic numbers of SHE

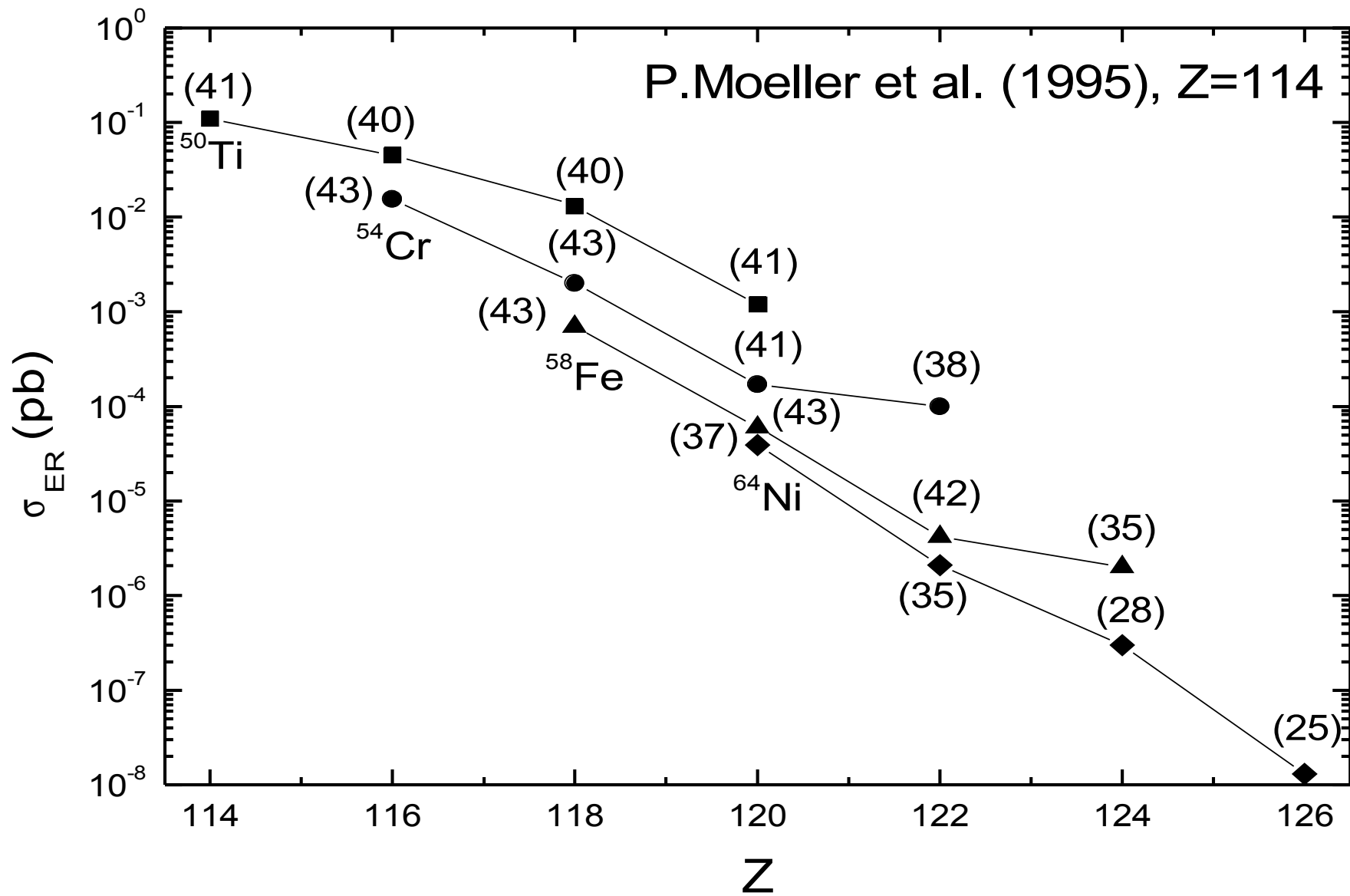
Within the dinuclear system model we analysed the production of SHE in various actinide-based complete fusion reactions with projectiles heavier than  $^{48}\text{Ca}$ .

Different predictions of the properties of heaviest nuclei were used:

- 1) mass table from TCSM (2011),  $Z=120-126$ ;
- 2) mass table by P. Moeller et al.(1995),  $Z=114$ ;







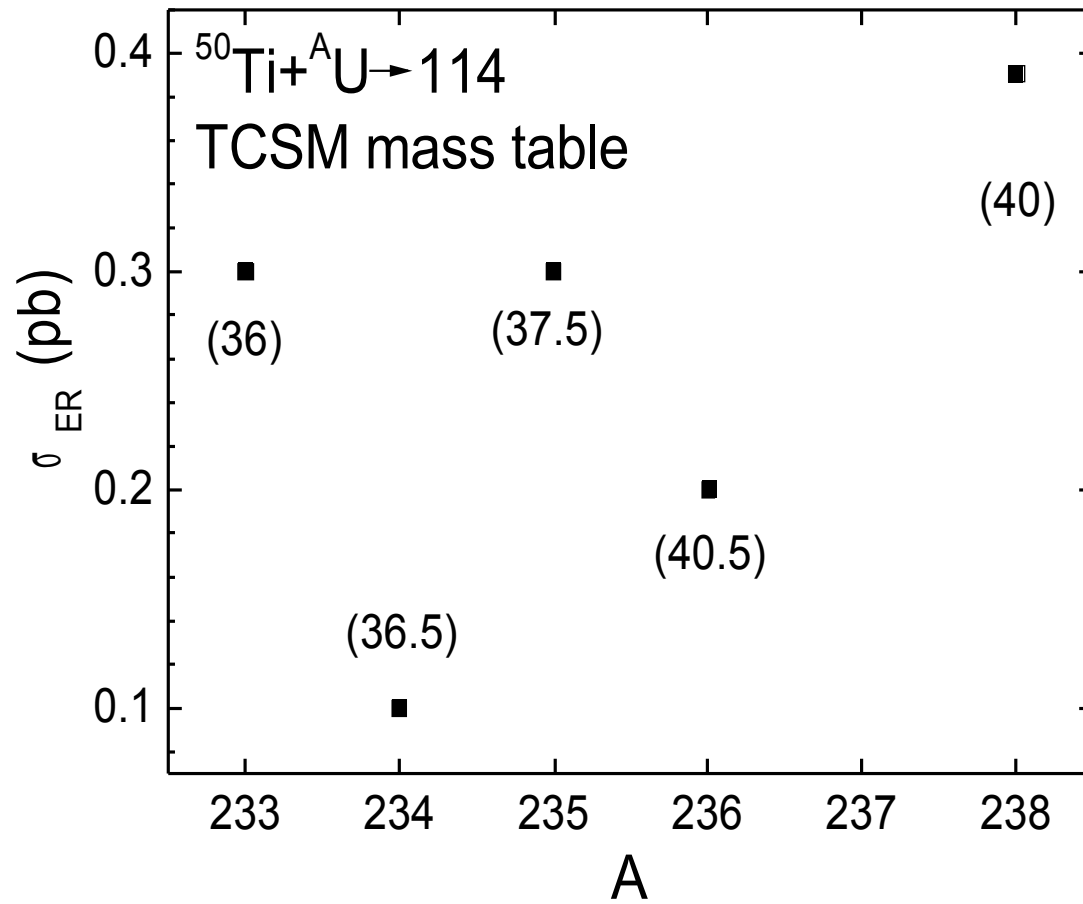
# Summary

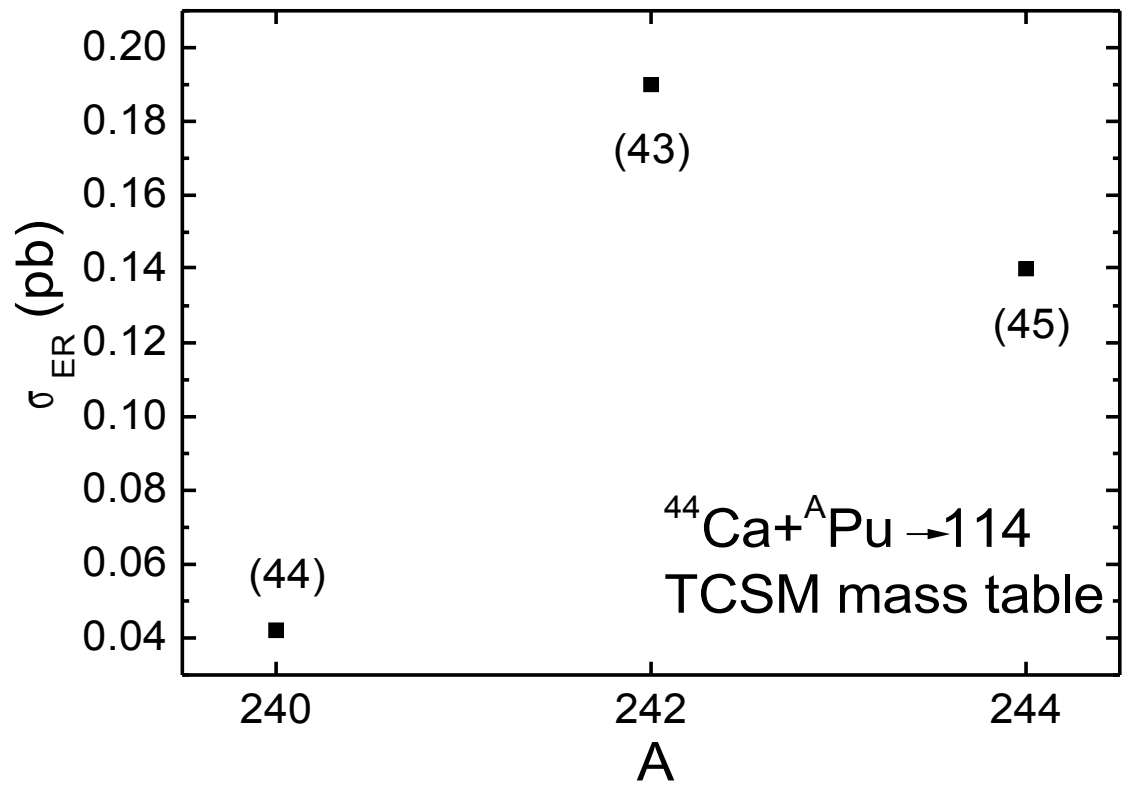
The calculations performed with the modified **TCSM** reveal quite strong shell effects at **Z=120-126** & **N=184** as in the self-consistent mean-field treatments.

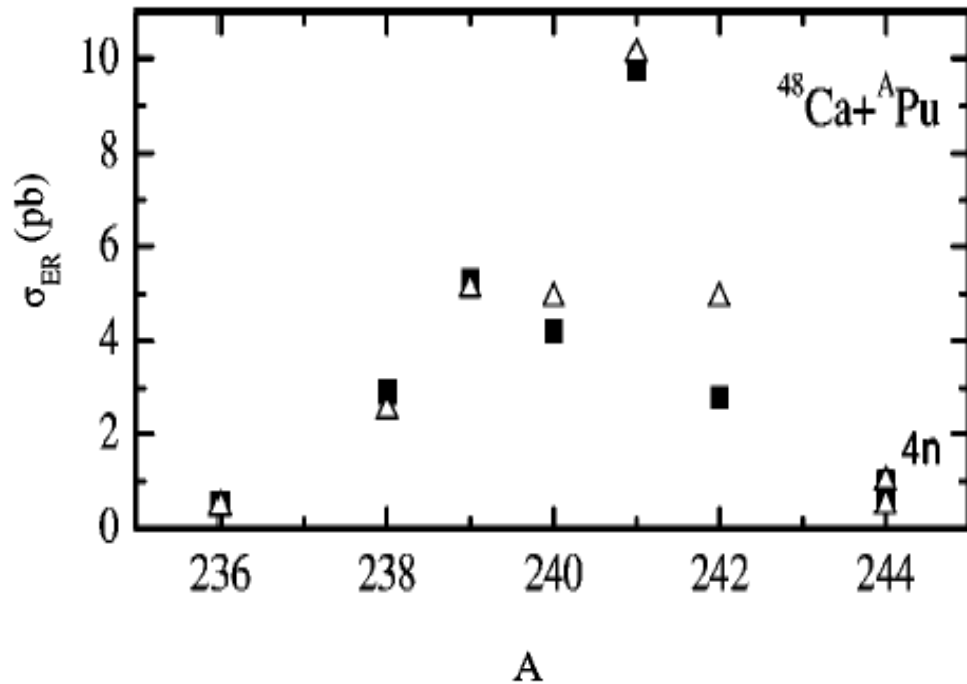
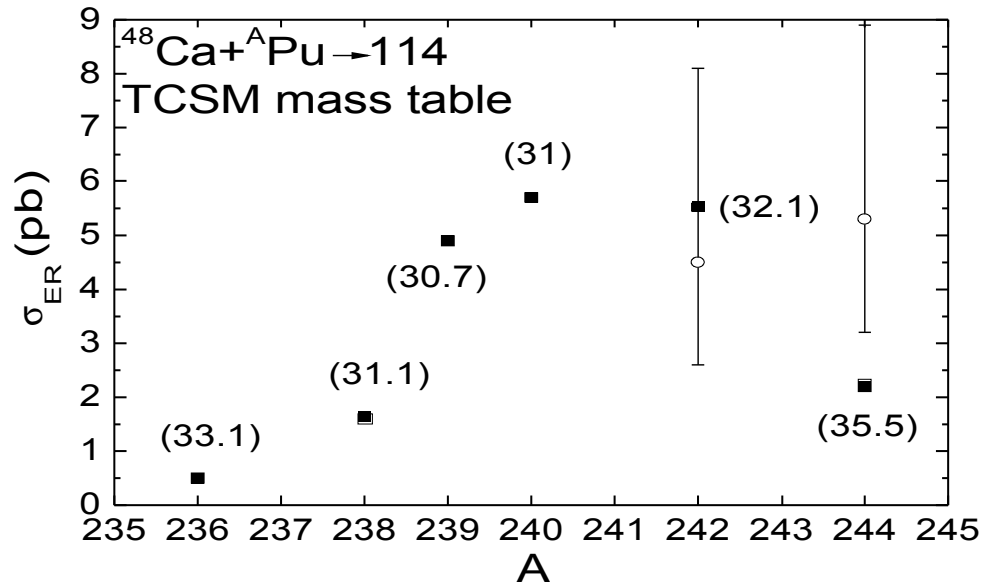
If our prediction of the structure of heaviest nuclei is correct, than one can expect the production of **Z=120** in the reactions  **$^{50}\text{Ti}+^{249}\text{Cf}$**  and  **$^{54}\text{Cr}+^{248}\text{Cm}$**  with the cross sections **23** and **10 fb**, respectively.

**Z=120** nuclei with **N=175-179** are expected to have  $Q_{\alpha}$  about **12.1 - 11.2 MeV** and lifetimes **1.7 ms - 0.16 s** in accordance with our predictions. These  $Q_{\alpha}$  are in fair agreement with **S.Liran et al.(2000)** and about **2 MeV** smaller than those from **P.Möller et al.(1995)** & **A.Sobiczewski et al.(2003)**.

Experimental measurement of  $Q_{\alpha}$  for at least one isotope of **Z=120** nucleus would help us to set proper shell model for the **SHE** with **Z>118**.

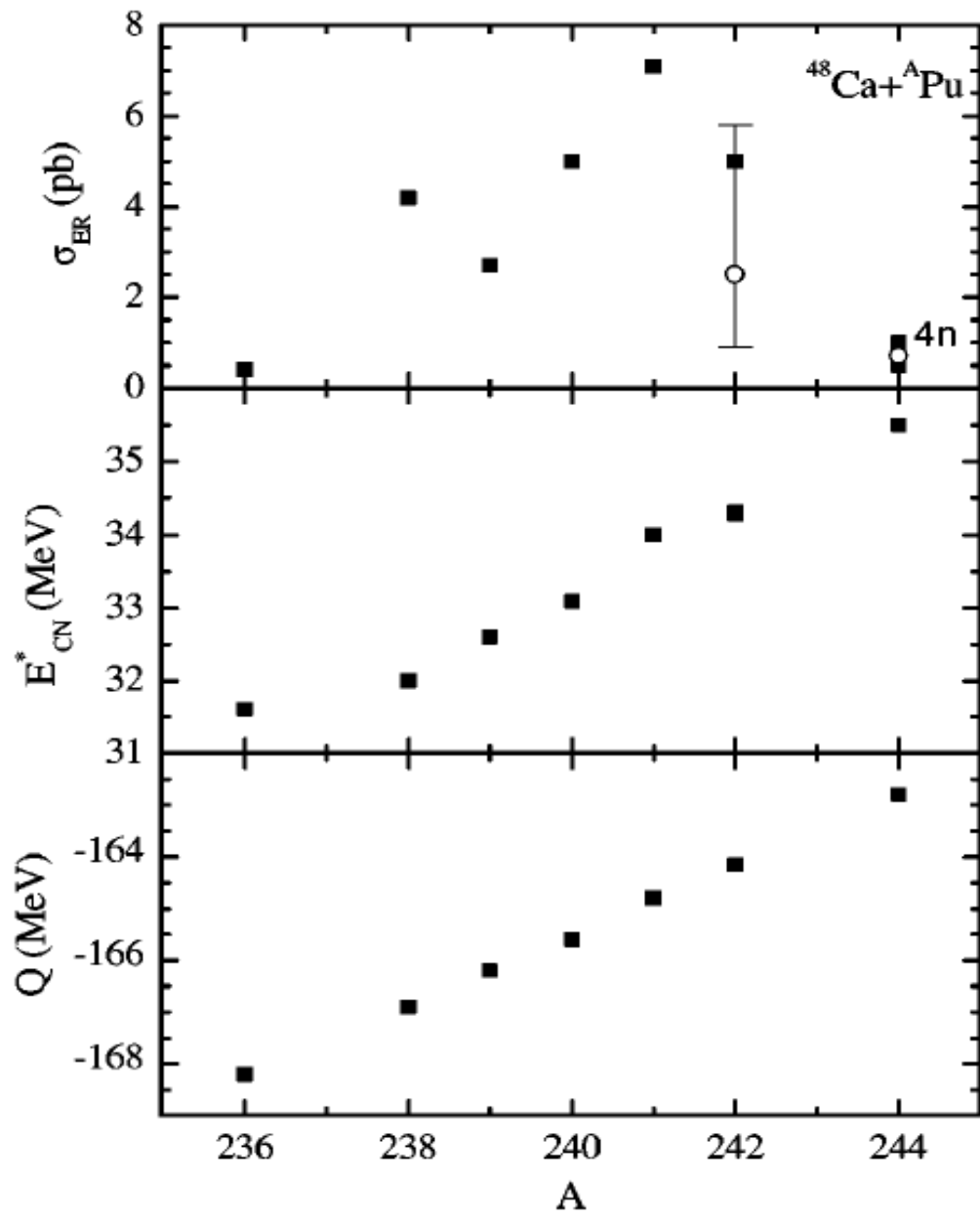




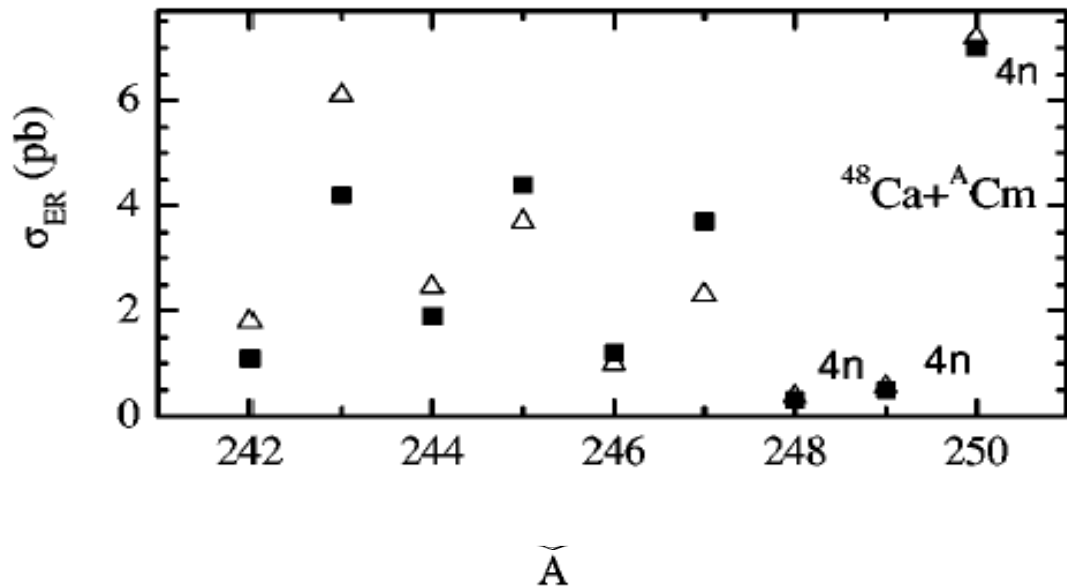
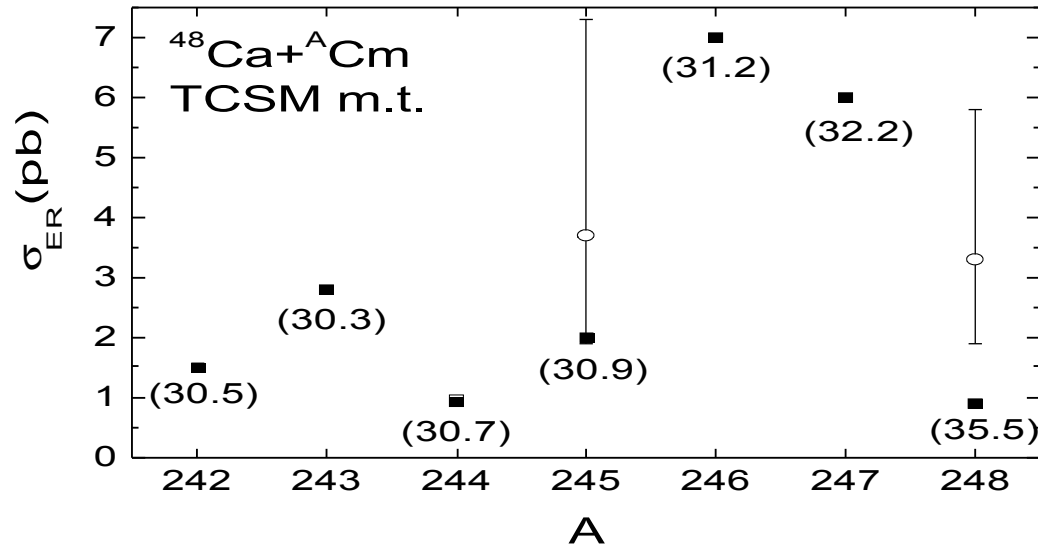


P.Möller et al.

W.D. Myers,  
W.J. Swiatecki



P.Möller et al.

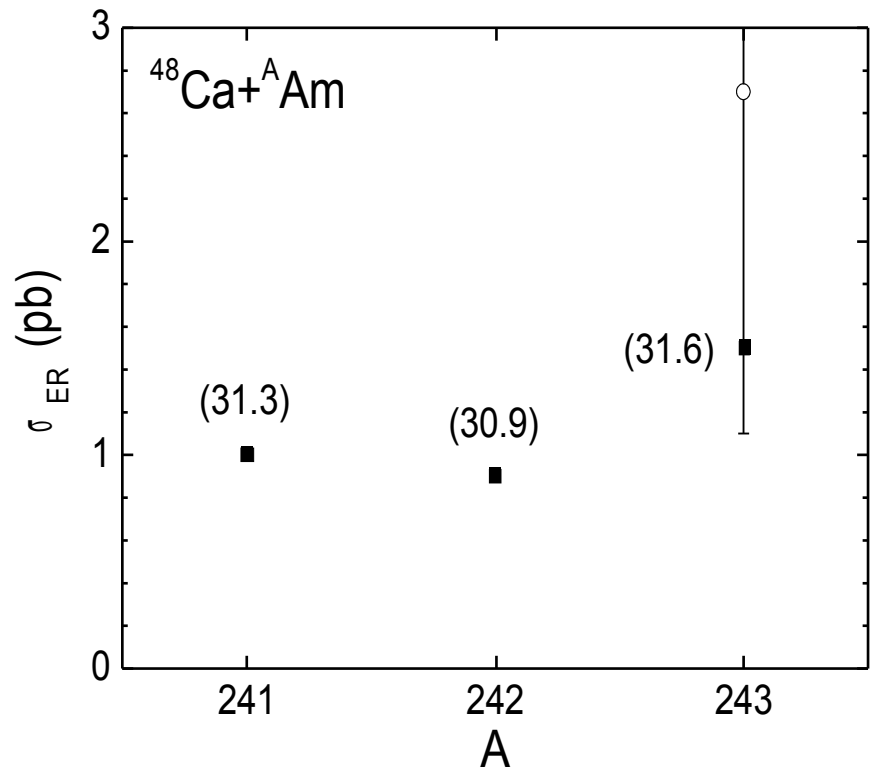


P.Möller et al.

W.D. Myers,

W.J. Swiatecki.





**Cross sections calculated with different mass tables are comparable because:**

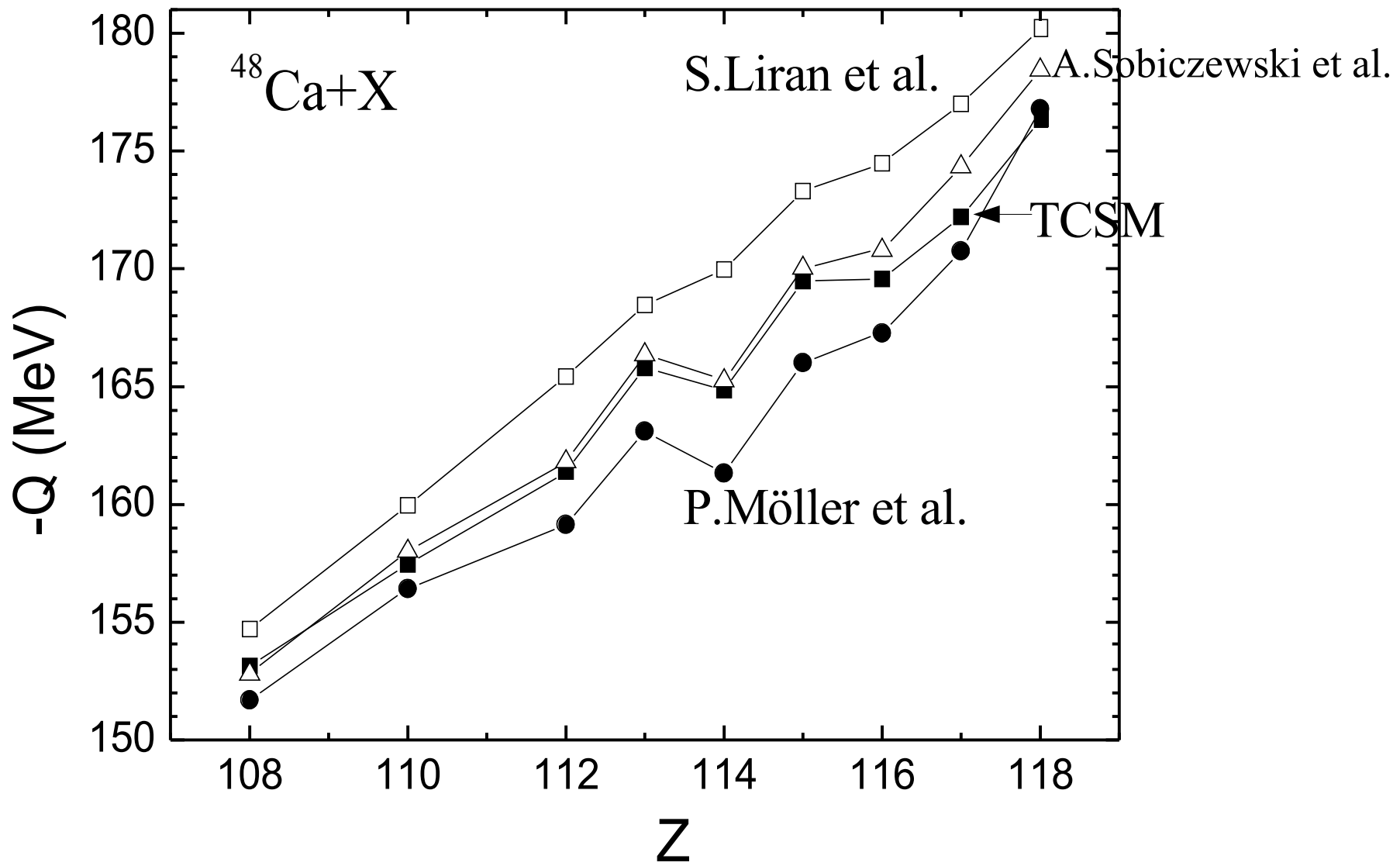
- 1. Fusion probability depends on the predicted mass of nuclei.**
- 2. Smaller barrier requires smaller  $a_f/a_n$**

## Summary

**In  $^{48}\text{Ca}$ -induced reactions the actinide targets with smaller neutron excess (within certain intervals of mass) are even more favorable than those with larger neutron excess.**

**This result is almost independent of mass table!**

**Thank you.**



## Parameter of level density

$$\rho = \frac{\exp[S(\beta, \lambda_Z, \lambda_N)]}{(2\pi)^{3/2} \sqrt{D}},$$

where  $S$  is the entropy,  $\beta = T^{-1}$  is the inverse temperature,  $\lambda_Z$  and  $\lambda_N$  are the chemical potentials for protons and neutrons, respectively, and  $D$  is the determinant of the matrix comprised of the second derivatives of the entropy

$$D = \begin{vmatrix} \frac{\partial^2 S}{\partial \beta^2} & \frac{\partial^2 S}{\partial \beta \partial \mu_Z} & \frac{\partial^2 S}{\partial \beta \partial \mu_N} \\ \frac{\partial^2 S}{\partial \beta \partial \mu_Z} & \frac{\partial^2 S}{\partial \mu_Z^2} & 0 \\ \frac{\partial^2 S}{\partial \beta \partial \mu_N} & 0 & \frac{\partial^2 S}{\partial \mu_N^2} \end{vmatrix},$$

where  $\mu_k = \beta \lambda_k$ , ( $k = N, Z$ ).

$$S = 2 \sum_{k=Z,N} \sum_{\nu} \left\{ \ln[1 + \exp(-\beta E_{\nu k})] + \frac{\beta E_{\nu k}}{1 + \exp(\beta E_{\nu k})} \right\},$$

$$E_{\nu k} = \sqrt{(\varepsilon_{\nu k} - \lambda_k)^2 + \Delta_k^2}$$

$$Z = \sum_{\nu} \left( 1 - \frac{\varepsilon_{\nu Z} - \lambda_Z}{E_{\nu Z}} \tanh\left[\frac{1}{2}\beta E_{\nu Z}\right] \right),$$

$$N = \sum_{\nu} \left( 1 - \frac{\varepsilon_{\nu N} - \lambda_N}{E_{\nu N}} \tanh\left[\frac{1}{2}\beta E_{\nu N}\right] \right),$$

$$\frac{2}{G_Z} = \sum_{\nu} \frac{\tanh[\beta E_{\nu Z}/2]}{E_{\nu Z}},$$

$$\frac{2}{G_N} = \sum_{\nu} \frac{\tanh[\beta E_{\nu N}/2]}{E_{\nu N}},$$

where  $G_Z$  and  $G_N$  are the constants of pairing interaction.

The total  $E$  and excitation  $U$  energies of the nucleus at temperature  $T$  are calculated as

$$E(T) = \sum_{k=Z,N} \left\{ \sum_{\nu} \varepsilon_{\nu k} \left( 1 - \frac{\varepsilon_{\nu k} - \lambda_k}{E_{\nu k}} \tanh \frac{1}{2} \beta E_{\nu k} \right) - \frac{\Delta_k^2}{G_k} \right\},$$

$$U = E(T) - E(0).$$

**Level-density parameter**

$$\rho_{FG}(U) = \frac{\sqrt{\pi}}{12a^{1/4} E^{*5/4}} \exp 2\sqrt{aE^*},$$

where  $E^* = U - \Delta_Z - \Delta_N$  is the excitation energy back-shifted

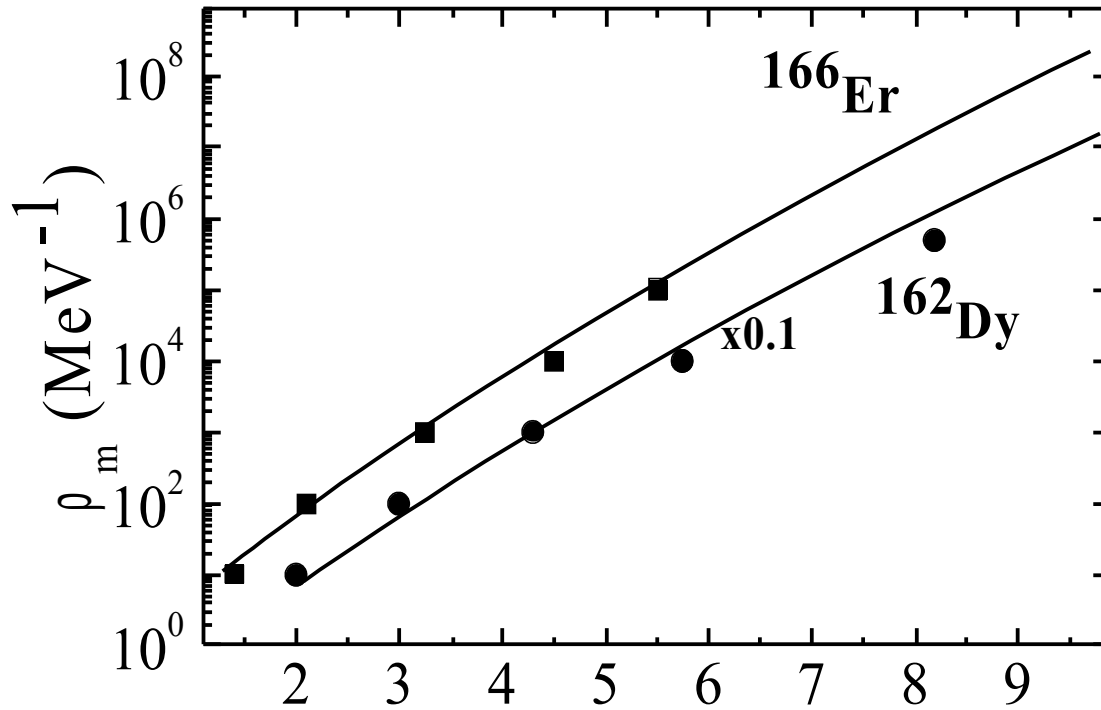
$$a = U/T^2$$

or

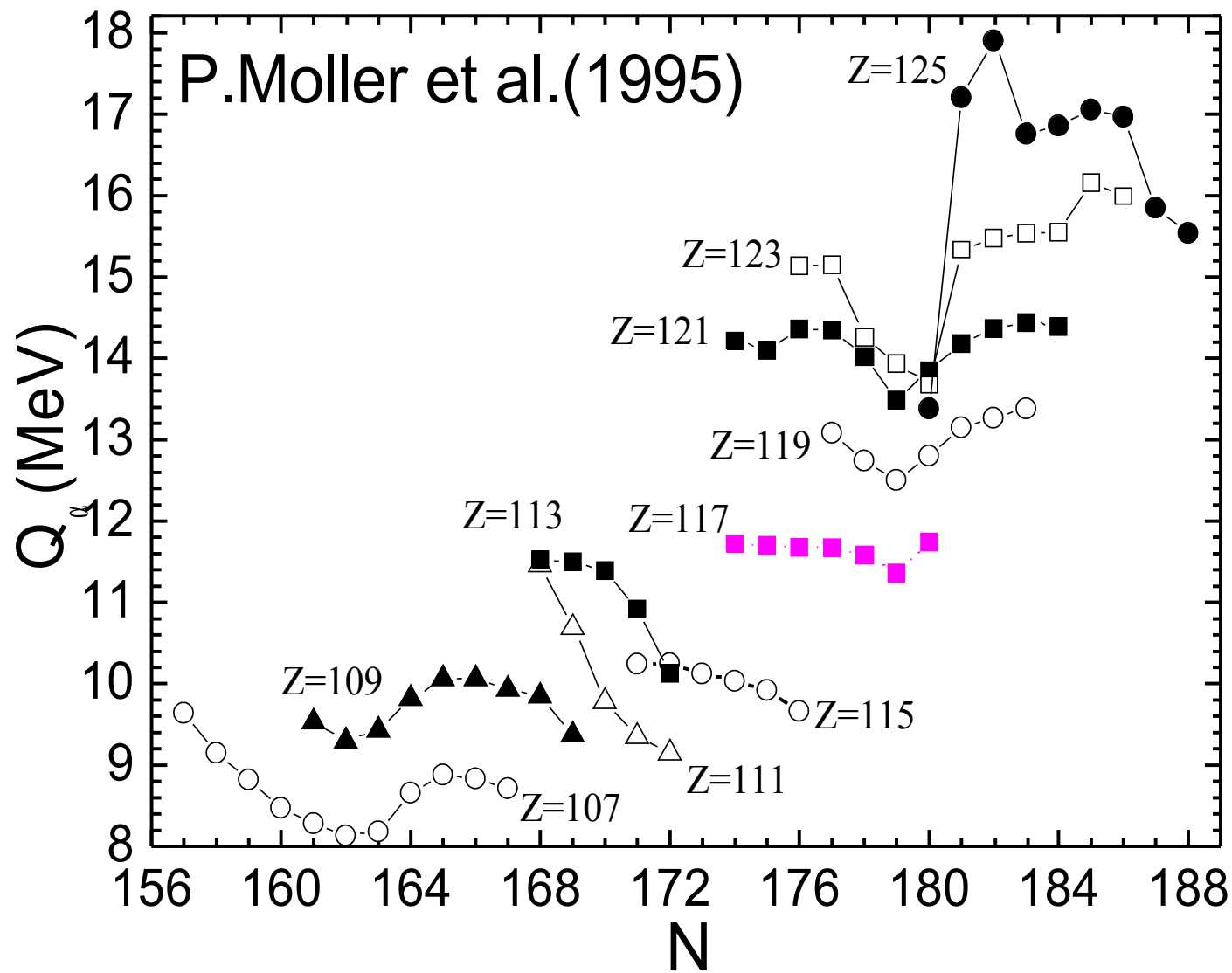
$$a = \frac{\pi^2}{6T} \sum_{k=Z,N} \sum_{\nu} \bar{n}_{k\nu} (1 - \bar{n}_{k\nu})$$

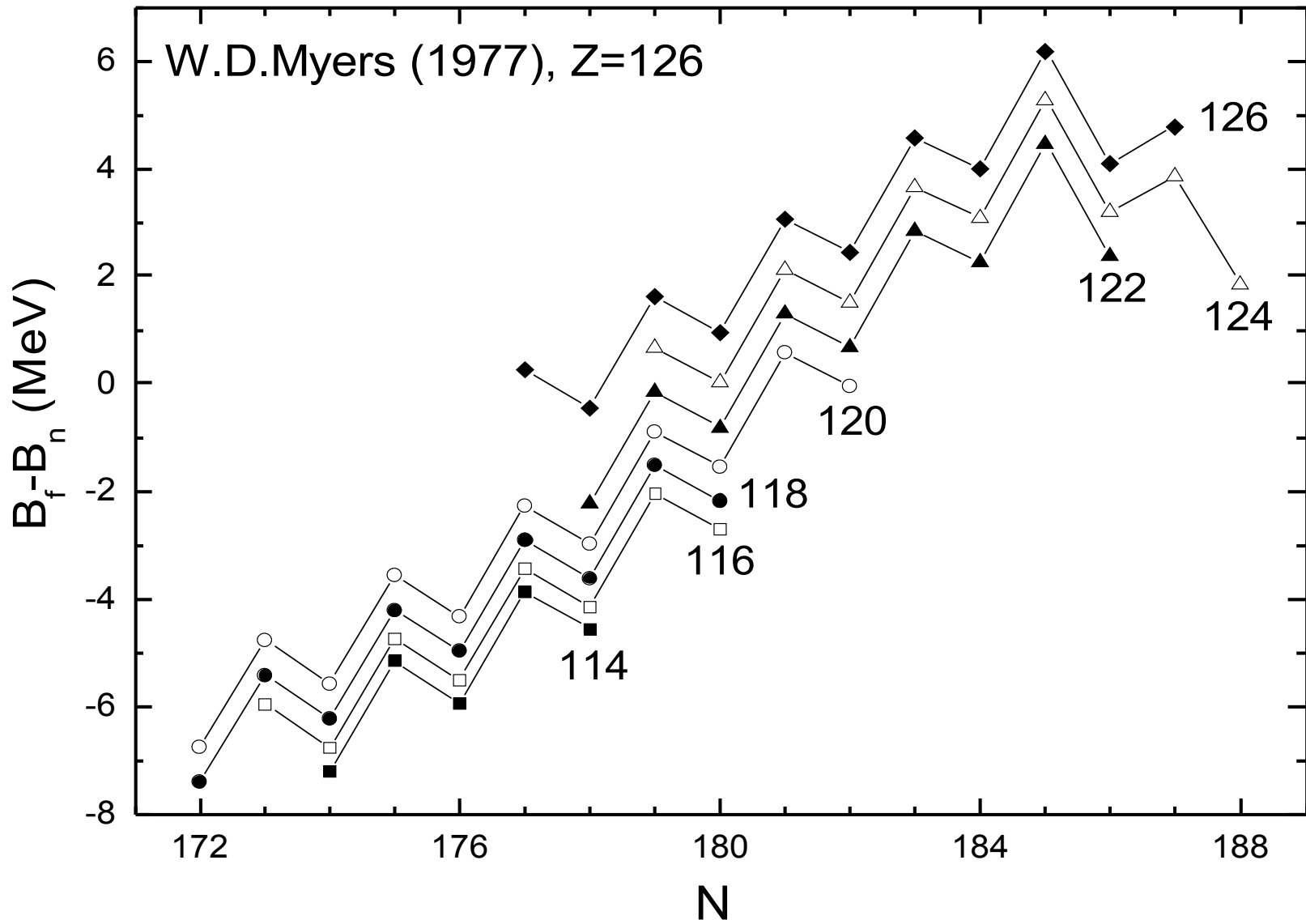
$$\bar{n}_{k\nu} = [1 + \exp(\beta E_{k\nu})]^{-1}$$

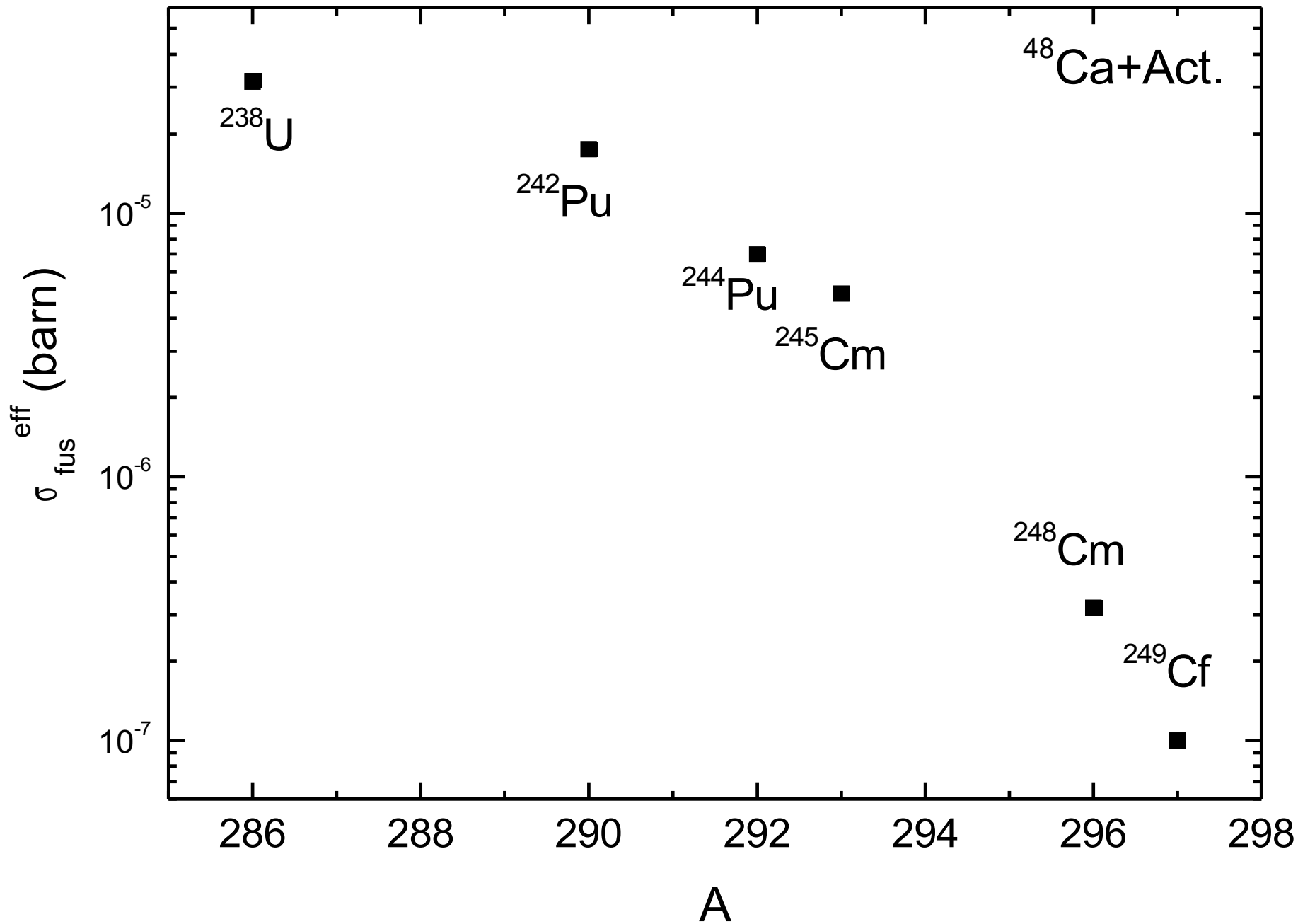


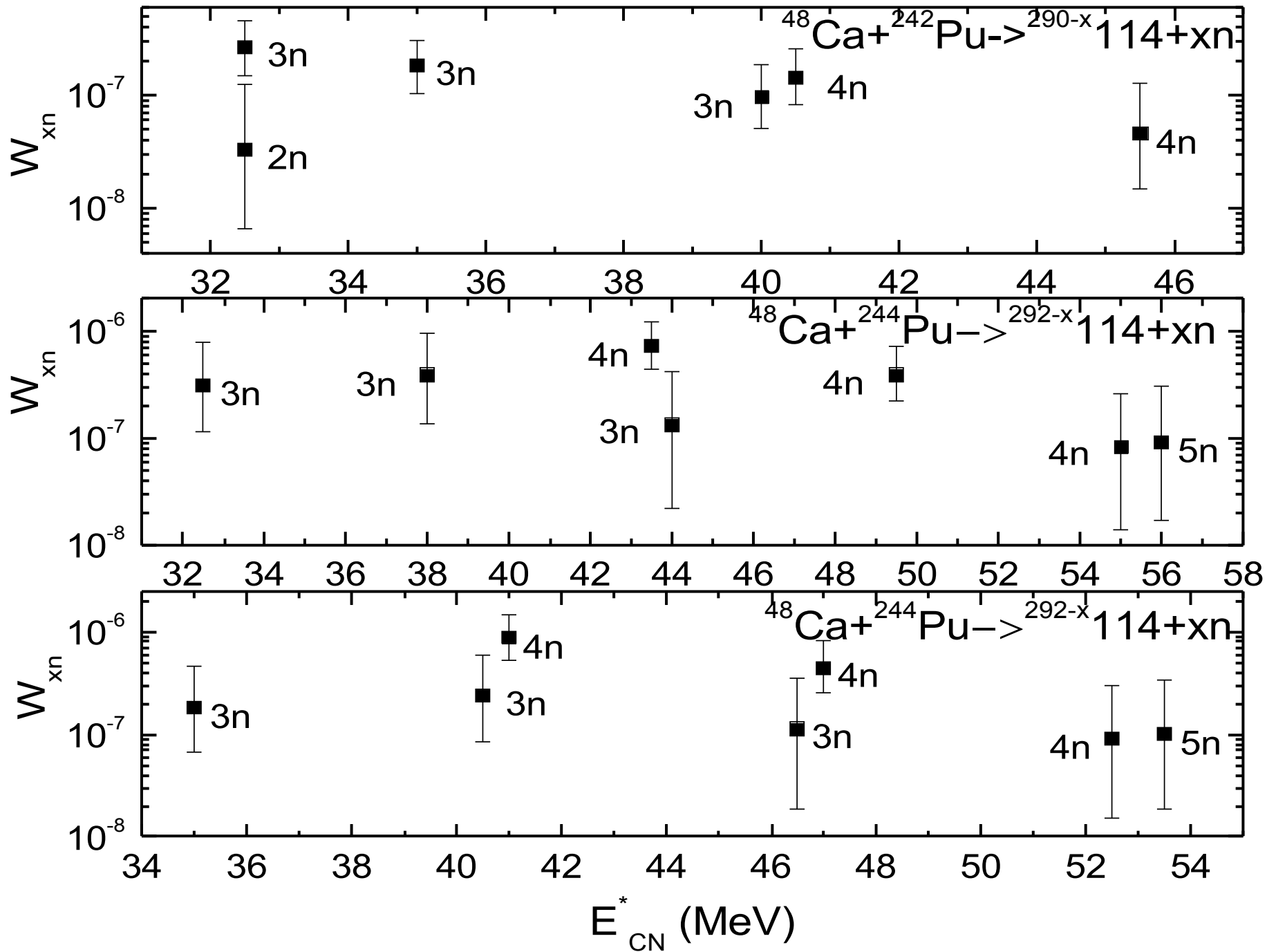


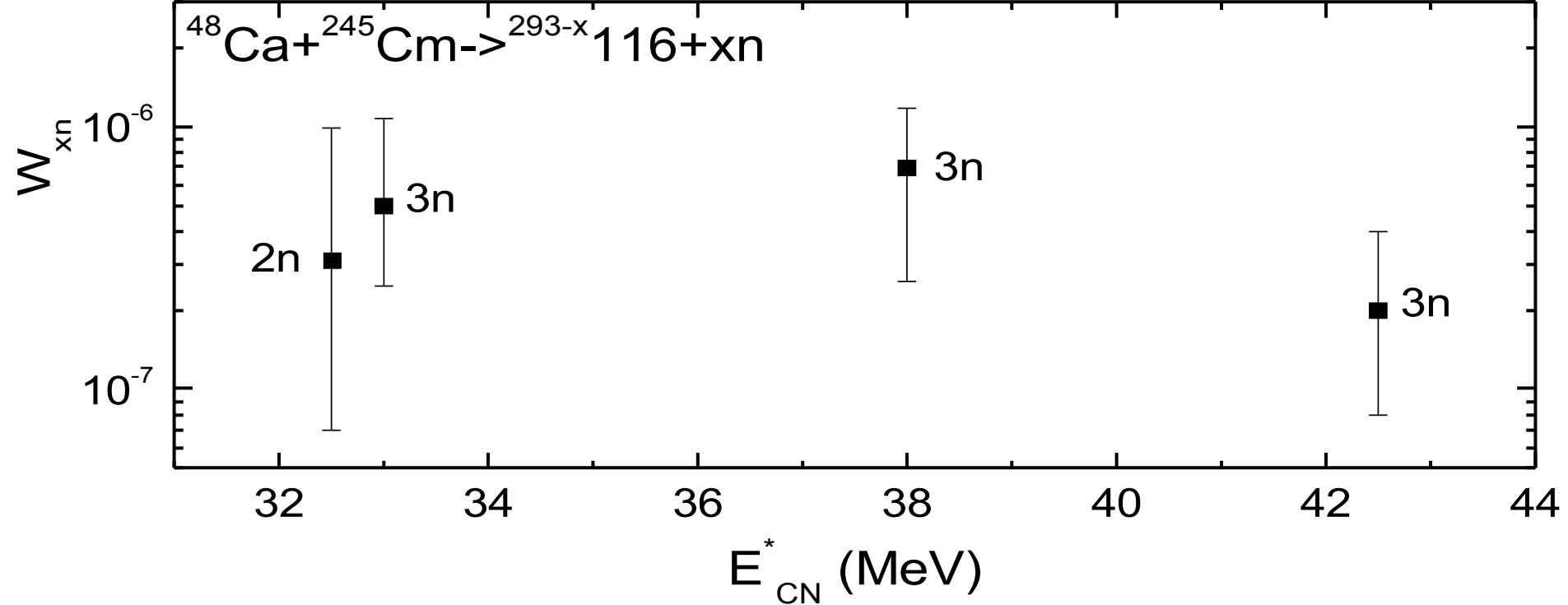
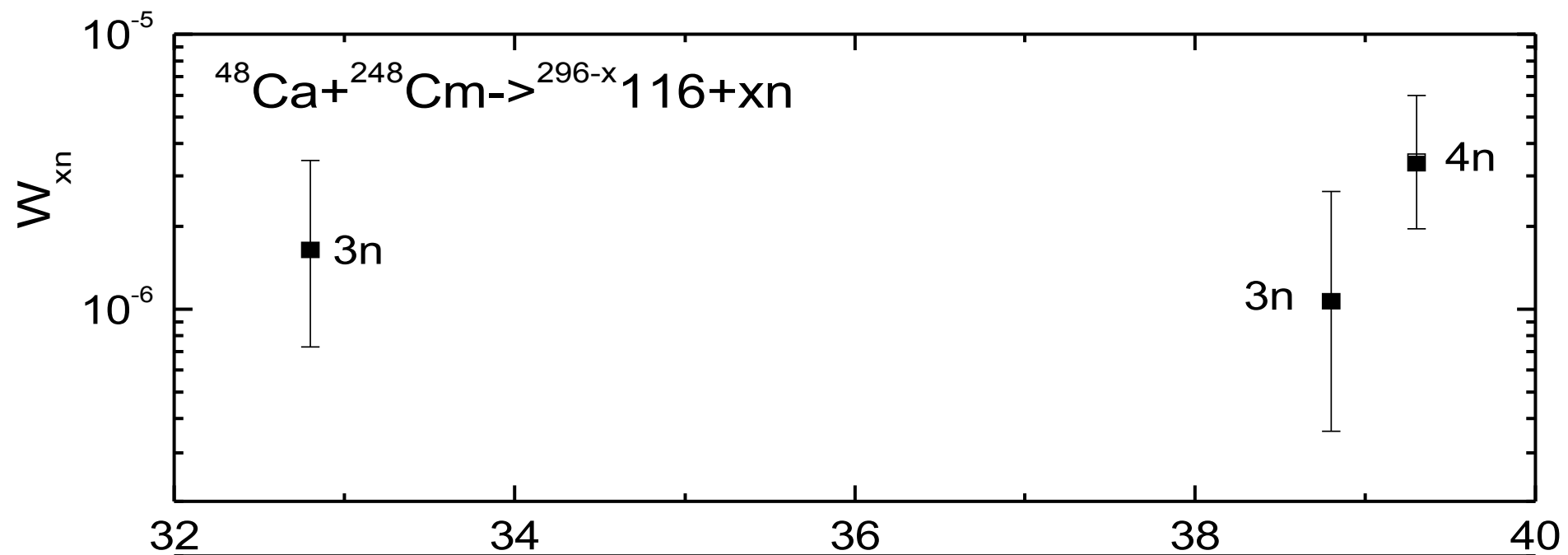
E. Melby, M. Guttormsen, J. Rekstad, A. Schiller, and S. Siem, Phys. Rev. C **63**, 044309 (2001).

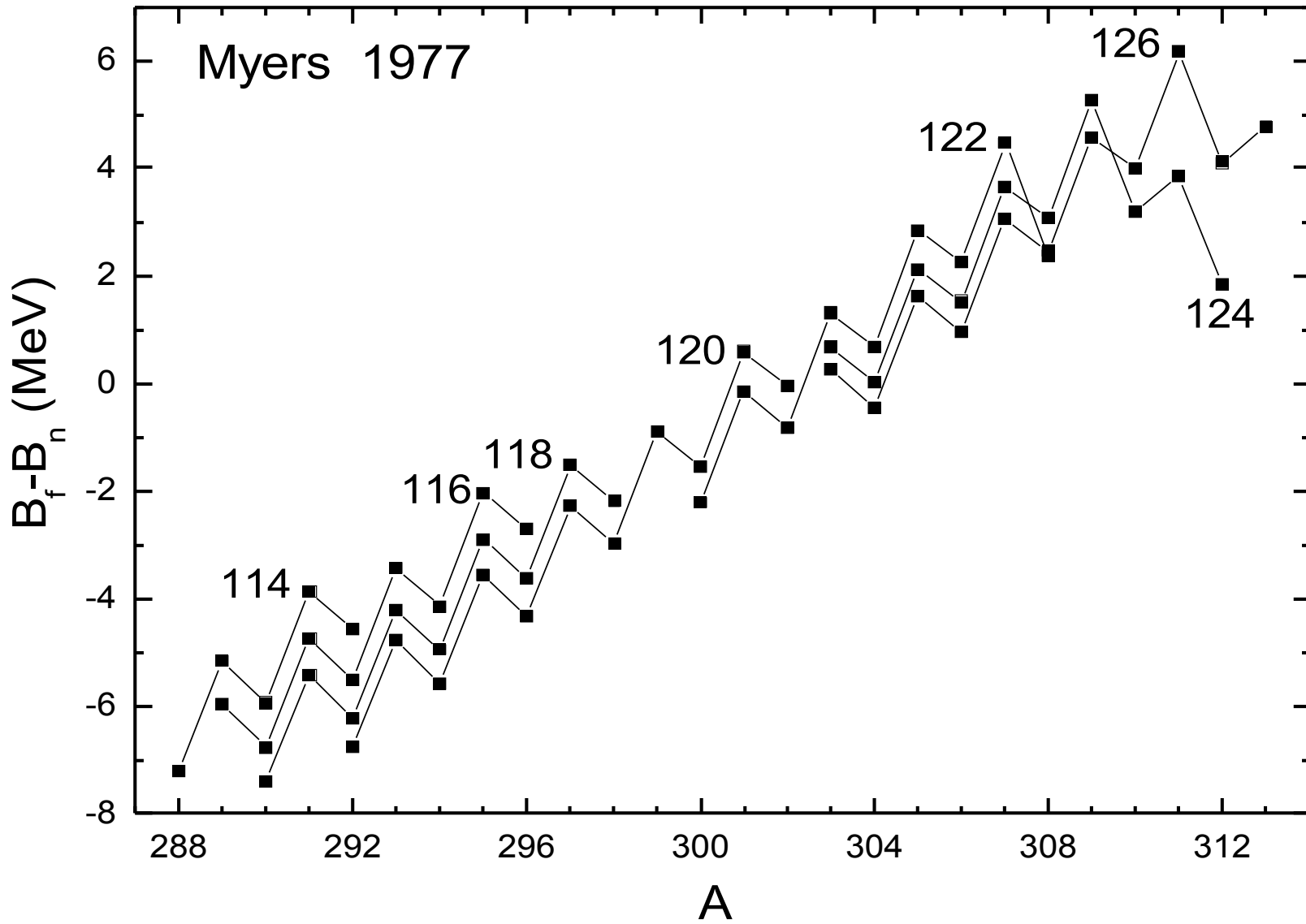






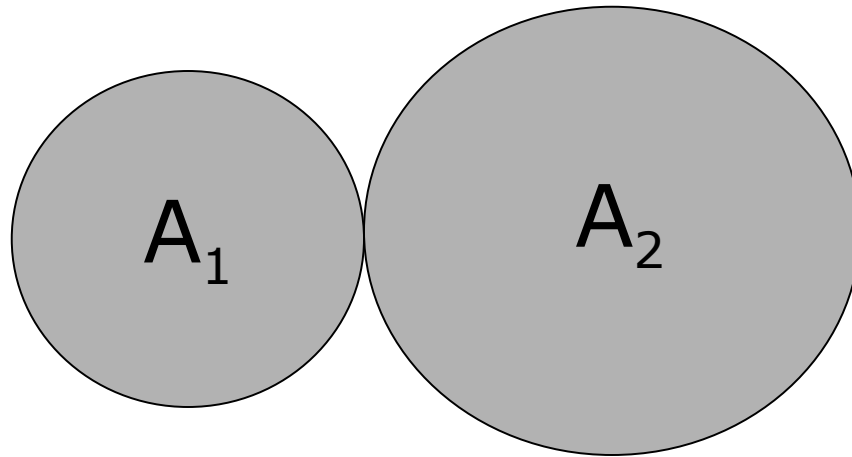






# Mass asymmetry coordinate

$$\eta = \frac{A_1 - A_2}{A_1 + A_2}$$



$\eta = 0$  for  $A_1 = A_2$ ,  $\eta = \pm 1$  for  $A_1$  or  $A_2 = 0$



# Potential energy

$$U(Z, A, \lambda, \beta) = U_{LDM}(Z, A, \lambda, \beta) + \delta U_{mic}(Z, A, \lambda, \beta)$$

# Binding energy

$$B(Z, A) = U(Z, A, \lambda_{gs}, \beta_{gs}) - a_v \left(1 - a_s \left(\frac{N-Z}{A}\right)^2\right) A$$

$$+ W \left| \frac{N-Z}{A} \right| + \delta + c[(N-Z) - 58]$$

$$a_s = 1.778, W = 30 \text{ MeV}, c = 0.25 \text{ MeV},$$

$$\delta = 4.8 / N^{1/3} \delta_{N, odd} + 4.8 / Z^{1/3} \delta_{Z, odd},$$

$$a_v = 15.703 \text{ MeV at } N - Z < 52; 15.715 \text{ MeV at } 54 \leq N - Z \leq 61,$$

## Potential energy

$$U(Z, A, \lambda, \beta) = U_{LDM}(Z, A, \lambda, \beta) + \delta U_{mic}(Z, A, \lambda, \beta)$$

## Binding energy

$$B(Z, A) = U(Z, A, \lambda_{gs}, \beta_{gs}) - a_v \left( 1 - 1.78 \left( \frac{N-Z}{A} \right)^2 \right) A + \dots$$
$$a_v = 15.83 \text{ MeV}$$

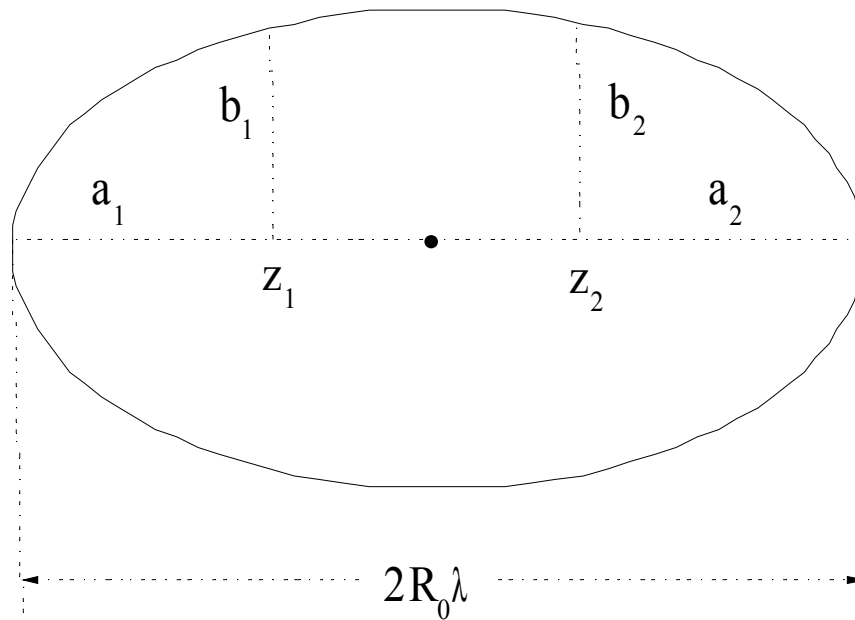
## $Q_\alpha$ energy

$$Q_\alpha(Z, A) = B(Z, A) + 28.296 - B(Z-2, A-4)$$

## Alpha decay half-lives $T_\alpha$ (A. Sobiczewski et al.)

$$\log_{10} T_\alpha(Z, A) = 1.5372 Z Q_\alpha^{-1/2} - 0.1607 Z - 36.573$$

$$\beta_i = a_i/b_i$$



$\beta_1 = \beta_2$ , even multipolarities;       $\beta_1 \neq \beta_2$ , odd and even multipolarities

$R_0$  is the radius of spherical nucleus

Parametrisation of nuclear shape with TCSM

J.Maruhn and W.Greiner, Z. Physik, **251**, 431 (1972)

# Strength parameters of pairing interaction

$$G_{\begin{matrix} n \\ p \end{matrix}} = \left( 19.2 \mp 7.4 \frac{N-Z}{A} \right) A^{-1} \text{ MeV}$$

$$A \approx 250 \rightarrow G_n \approx 0.075 \text{ MeV}, G_p \approx 0.085 \text{ MeV}$$

## One-quasiparticle excitations

$$E_{\mu} = \sqrt{(e_{\mu} - e_F)^2 + \Delta^2} - \sqrt{(e_{\mu}' - e_F)^2 + \Delta^2}$$

## Two-quasiparticle excitations

$$E_{\mu} = \sqrt{(e_{\mu} - e_F)^2 + \Delta^2} + \sqrt{(e_{\mu}' - e_F)^2 + \Delta^2}$$

# Parameters

$$35 \leq N - Z \leq 56$$

*for neutrons*

$$\kappa_n = -0.076 + 0.0058(N - Z) - 6.53 \times 10^{-5}(N - Z)^2 + 0.002 A^{1/3},$$

$$\mu_n = 1.598 - 0.0295(N - Z) + 3.036 \times 10^{-4}(N - Z)^2 - 0.095 A^{1/3},$$

*for protons*

$$\kappa_p = 0.0383 + 0.00137(N - Z) - 1.22 \times 10^{-5}(N - Z)^2 - 0.003 A^{1/3},$$

$$\mu_p = 0.335 + 0.01(N - Z) - 9.367 \times 10^{-5}(N - Z)^2 + 0.003 A^{1/3},$$

The parts in front of the terms with  $A^{1/3}$  vary:

(0.05-0.053) for  $\kappa_n$  , (0.075-0.0768) for  $\kappa_p$  ,

(0.88-0.92) for  $\mu_n$  , (0.58-0.61) for  $\mu_p$

# Dynamics of fusion in the dinuclear system model

Evaporation residue cross section for the production of superheavy nuclei:

$$\sigma_{ER}^s(E_{c.m.}) = \sum_{J=0} \sigma_c(E_{c.m.}, J) P_{CN}(E_{c.m.}, J) W_{sur}^s(E_{c.m.}, J)$$

$$\sigma_c(E_{c.m.}, J) = \pi \lambda^2 (2J+1) T(E_{c.m.}, J)$$

$$\sigma_{ER}^s(E_{c.m.}) \approx P_{CN}(E_{c.m.}, J=0) \sum_J \sigma_c(E_{c.m.}, J) W_{sur}^s(E_{c.m.}, J)$$

$$\approx \sigma_c(E_{c.m.}) P_{CN}(E_{c.m.}) W_{sur}^s(E_{c.m.})$$

**The stronger shell effects revealed for nuclei with  $Z > 118$  result larger survival probabilities and larger values of the cross section.**