

Effects of Finite Size of Nuclei on Thermodynamic Properties

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Introduction

An important ingredient of the theory of nuclear structure and nuclear reactions is the single-particle (SP) level density, $g(\epsilon)$, associated with the nucleus mean field. In particular, $g(\epsilon)$ is a basic ingredient in the evaluation of the nucleus level density $\rho(E)$, needed for the description of nuclear reactions, the partition function, and the ground state and deformation energy by the Strutinsky shell correction method. We emphasize that for an accurate description of these physical quantities, $g(\epsilon)$ should be evaluated for a wide range of ϵ , including the continuum region.

Calculation of $g_s(\epsilon)$

We consider a particle, such as the nucleon, subject to the spherically symmetric Hamiltonian

$$\hat{H} = \frac{P^2}{2m} + V(r). \quad (1)$$

The energies ϵ_i of the bound states are obtained from $H\psi_i = \epsilon_i\psi_i$ and the level density is determined from

$$g_B(\epsilon) = \sum_i \delta(\epsilon - \epsilon_i). \quad (2)$$

We will adopt the Thomas-Fermi approximation for the smooth SP level density. Taking into account the spin-isospin degeneracy (neglecting spin orbit interaction), we have

$$g_{TF}(\epsilon) = \frac{1}{\pi^2} \left(\frac{2m}{\hbar^2} \right)^{3/2} \int d^3\vec{r} (\epsilon - V)^{1/2} \Theta(\epsilon - V). \quad (3)$$

In the case of finite potential wells, the results of eq. (3) should be corrected when considering the level density in the continuum by subtracting the contribution of the free gas $g_f(\epsilon)$ when $\epsilon > 0$. In the Thomas-Fermi Approximation we have

$$g_f = \frac{1}{2\pi^2} \Omega \left(\frac{2m}{\hbar^2} \right)^{3/2} \sqrt{\epsilon}, \quad \epsilon > 0 \quad (4)$$

for a Fermi gas. This expression agrees with the quantum mechanical expression.

Thermodynamic Properties

The entropy $S(T)$ and the internal energy $E(T)$ can be calculated using,

$$S(T) = - \int g(\epsilon) [f \ln f + (1-f) \ln(1-f)] d\epsilon$$

$$E(T) = \int \epsilon g(\epsilon) f(\epsilon, \mu, T) d\epsilon \quad (5)$$

where $f(\epsilon, \mu, T) = 1 / \{ 1 + \exp[(\epsilon - \mu)/T] \}$ is the occupation probability. The chemical potential μ is determined by the particle number conservation condition

$$A = \int g(\epsilon) f(\epsilon, \mu, T) d\epsilon. \quad (6)$$

In the zero temperature limit we have for the entropy

$$S = 2aT, \quad (7)$$

and the excitation energy E^*

$$E^* \equiv E(T) - E(0) = aT^2, \quad (8)$$

where a is the level density parameter

$$a = \frac{1}{6} \pi^2 g_s(\epsilon_F). \quad (9)$$

Here ϵ_F is the Fermi energy and g_s is the smooth level density.

Model Potentials

Harmonic Oscillator (HO) Potential

The HO single-particle potential is an infinite potential well with a smooth surface and applicable only to small nuclei. It is given by

$$V^{HO}(r) = \frac{1}{2} m \omega^2 r^2, \quad (10)$$

where $\sqrt{(m\omega/\hbar)}$ is the size parameter of the potential. The HO potential well is very popular in various branches of physics since its eigenvalues and eigenstates are known analytically. Using eq. (10) in eq (3), we obtain the simple expression,

$$g_{TF}^{HO}(\epsilon) = \epsilon^2 / (\hbar\omega)^3. \quad (11)$$

Trapezoid (TR) Potential

The TR Potential well is of the form

$$V^{TR}(r) = \begin{cases} V_0 & r < R-D \\ \frac{1}{2} V_0 [1 - (r-R)/D] & r > R-D, \end{cases} \quad (12)$$

where V_0 , R , and D are the depth, size and surface thickness parameters of the potential well, respectively. Taking $V=0$ for $r > R+D$ in (12) we obtain a finite trapezoidal well with R being its half-radius. Using (12) in (3) we have

$$g_{TF}^{TR}(\epsilon) = \frac{1}{2\pi^2} \left(\frac{2m}{\hbar^2} \right)^{3/2} \frac{4\pi(R-D)^3}{3} \sqrt{\epsilon - V_0} \left[1 + 2x + \frac{8}{5}x^2 + \frac{16}{35}x^3 \right] \quad (13)$$

where

$$x = - \frac{2D(\epsilon - V_0)}{(R-D)V_0}.$$

Note that for the finite trapezoidal potential well the contribution due to the free-gas level density, eq. (4), should be subtracted from (13) for the case of $\epsilon > 0$. Equation (13) should be corrected by subtracting

$$\frac{1}{2\pi^2} \left(\frac{2m}{\hbar^2} \right)^{3/2} \frac{4\pi(R-D)^3}{3} \sqrt{\epsilon} \left[1 + 2u + \frac{8}{5}u^2 + \frac{16}{35}u^3 \right],$$

$$\text{with } u = - \frac{2D\epsilon}{(R+D)V_0}, \text{ for } \epsilon > 0. \quad (14)$$

The parameters of the single-particle potential well are taken to be

$$V_0 = -50 \text{ MeV}$$

$$D = 0.70 \pi \text{ fm}, \quad (15)$$

and R is determined by iteration,

$$R = R_v / [1 + (\pi d + R)^2]^{1/3},$$

$$R_v = 1.12 A^{1/3} + 0.80 \text{ fm} \quad (16)$$

Results

Adopting the trapezoid potential as our model potential (see figure 1) we calculated the level densities for various nuclei of $N = Z$. In order to check the validity of the Fermi gas approximation for large hot nuclei we used nuclei with much larger A than are stable. Figure 2 depicts level densities of nuclei with $A = 40, 216$ and 1000 , normalized with respect to A and with the free gas states subtracted, so the level density decreases above zero.

We then fit our level densities for $\epsilon < 0$ to the power law $g = C\epsilon^\alpha$ using the method of least-squares. We wanted to compare them with the level density obtained using the Fermi gas approximation, which is a power law with $\alpha = 3/2$. Three of these fits are displayed in figure 3. The values of α for our fits as a function of $A^{1/3}$ are plotted in figure 4. Values of α for large, stable nuclei are not larger than 1, and for light nuclei are closer to 2.

Finally, we calculated the entropy and the excitation energies of our nuclei as a function of temperature, and compared these results with those obtained using the Fermi gas

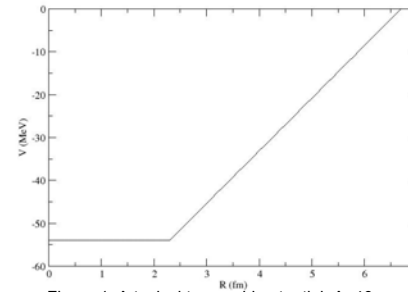


Figure 1: A typical trapezoid potential A=40

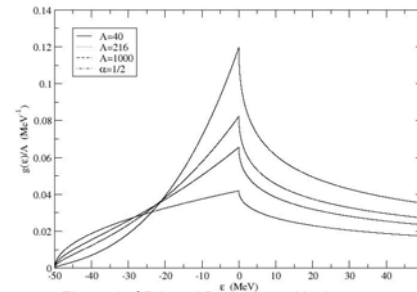


Figure 2: SP Level Density per Nucleon

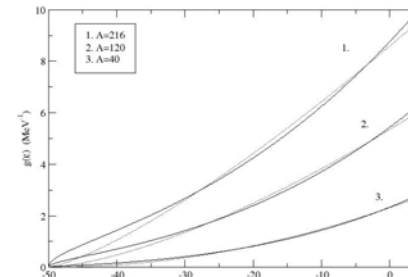


Figure 3: SP Level Densities in a Trapezoid potential fit to $g = C\epsilon^\alpha$

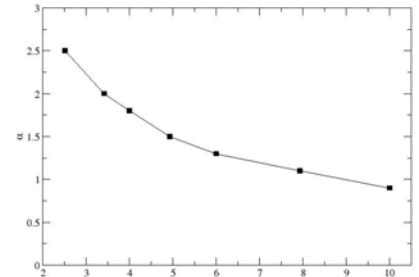


Figure 4: α vs. $A^{1/3}$

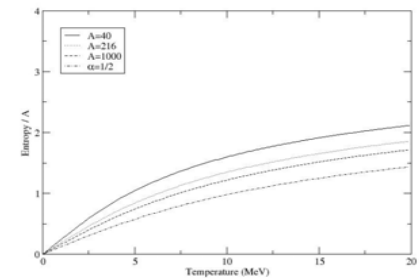


Figure 5: Entropy per Nucleon

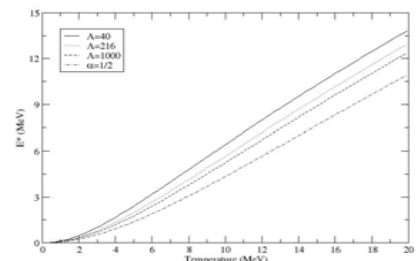


Figure 6: Excitation Energy per Nucleon

Conclusion

We have calculated the smooth single-particle level density, entropy and excitation energy for several nuclei, using a realistic potential well and properly accounting for the continuum. We demonstrated that the commonly adopted Fermi gas model, $g(\epsilon) = C\sqrt{\epsilon}$, is not appropriate for finite nuclei.

