

Multi-Pion Initial States in Dilepton and Photon Production from Hadron Gas

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1) Dileptons

1.1 A brief history

1.2 Reaction rate formula

1.3 Reactions with resonances and their counterparts

1.4 Four-pion initial states

2) Photons (in collaboration with J. Vysoudil)

2.1 A piece of very early history

2.2 Reactions with resonances and their counterparts

2.3 Three-pion initial states

3) Conclusions

1.1 A brief history of thermal dileptons from hadronic phase

Enhanced dilepton production as a signature of the QGP formation:

E. Shuryak, Phys. Lett. B 78, 150 (1978),
Phys. Rep. 61, 72 (1980)

K. Kajantie and H.I. Miettinen, Z. Phys. C 9,
341 (1981)

G. Domokos and J. Goldman, Phys. Rev. D
23, 203 (1981)

Also L. McLerran, T. Toimela, R. Hwa and K.
Kajantie, J. Kapusta, A. Mekjian, ...

Thermal dileptons from hadron gas as an alternative (or background) to the QGP ones. Only the $\pi^+\pi^-$ annihilation was first used when comparing the dilepton yields from the QGP and hadronic phase at the same temperature. Later on, many processes were added to the list of hadronic sources of dileptons:

C. Gale and J. Kapusta, Phys. Rev. C 38, 2659 (1988) considered the K^+K^- annihilation;

K.K. Gudina, A.I. Titov, and V.D. Toneev, Phys. Lett. B287, 302 (1992) used kinetic model for hA and AA collisions and showed that " $\omega/\rho \rightarrow e^+e^-$ decay contributes as much as $\pi\pi$ annihilation to the same region of invariant masses";

P. Koch, Z. Phys. C 57, 283 (1993) calculated the contribution of many decay modes of mesons to the thermal production of dielectrons;

C. Gale and P. Lichard, Phys. Rev. D 49, 3338 (1994) considered, like P. Koch, the light mesons decays, but added also the reactions among pseudoscalar and vector mesons.

P. L., Phys. Rev. D 49, 5812 (1994): Reaction $\pi\pi \rightarrow \pi e^+e^-$ and the three-pion annihilation $\pi^+\pi^-\pi^0 \rightarrow e^+e^-$.

Many reactions and decays with a resonance (resonances) in the initial state were considered later on and some of them were shown to be important:

C. Song, C. M. Ko, and C. Gale, Phys. Rev. D 50, R1827 (1994)

K. Haglin and C. Gale, Phys. Rev. D 52, 6297 (1995)

J. K. Kim, P. Ko, K. Y. Lee, and S. Rudaz, Phys. Rev. D 53, 4787 (1996)

K. Haglin, Phys. Rev. C 53, R2606 (1996)

C. Song and C. M. Ko, Phys. Rev. C 53, 2371 (1996)

R. Baier, M. Dirks, and K. Redlich, Phys. Rev. D 55, 4344 (1997)

S. Gao and C. Gale, Phys. Rev. C 57, 254 (1998)

G. Q. Li and C. Gale, Phys. Rev. Lett. 81, 1572 (1998)

G. Q. Li and C. Gale, Phys. Rev. C 58, 2914 (1998)

A. Faessler, C. Fuchs, and M. I. Krivoruchenko, Phys. Rev. C 61, 035206 (2000)

List of considered processes (only meson sector)

Annihilation

$$\pi^+\pi^- \rightarrow e^+e^-$$

$$\rho^+\rho^- \rightarrow e^+e^-$$

$$K^+K^- \rightarrow e^+e^-$$

$$K^{*+}K^{*-} \rightarrow e^+e^-$$

$$a_1^+a_1^- \rightarrow e^+e^-$$

$$K^0\bar{K}^0 \rightarrow e^+e^-$$

$$K^{*0}\bar{K}^{*0} \rightarrow e^+e^-$$

Decays

$$\rho^0 \rightarrow e^+e^-$$

$$\pi^0 \rightarrow \gamma e^+e^-$$

$$f_0 \rightarrow \gamma e^+e^-$$

$$\eta \rightarrow \pi^+\pi^-e^+e^-$$

$$\rho \rightarrow \pi e^+e^-$$

$$\omega \rightarrow \eta e^+e^-$$

$$K^* \rightarrow K e^+e^-$$

$$\eta \rightarrow \pi\pi e^+e^-$$

$$\omega \rightarrow \pi\pi e^+e^-$$

$$\omega \rightarrow e^+e^-$$

$$\eta \rightarrow \gamma e^+e^-$$

$$\eta' \rightarrow \rho^0 e^+e^-$$

$$\rho^0 \rightarrow \eta e^+e^-$$

$$\phi \rightarrow \pi^0 e^+e^-$$

$$\phi \rightarrow \eta' e^+e^-$$

$$\eta' \rightarrow \pi\pi e^+e^-$$

$$f_0 \rightarrow \pi\pi e^+e^-$$

$$\phi \rightarrow e^+e^-$$

$$\eta' \rightarrow \gamma e^+e^-$$

$$a_0^0 \rightarrow \gamma e^+e^-$$

$$\eta' \rightarrow \omega e^+e^-$$

$$\omega \rightarrow \pi^0 e^+e^-$$

$$\phi \rightarrow \eta e^+e^-$$

$$a_1 \rightarrow \pi e^+e^-$$

$$\rho \rightarrow \pi\pi e^+e^-$$

$$a_0^0 \rightarrow \pi\pi e^+e^-$$

$$V + P \rightarrow e^+ e^-$$

$$\begin{array}{lll} \omega \pi^0 \rightarrow e^+ e^- & \rho \pi \rightarrow e^+ e^- & \phi \pi^0 \rightarrow e^+ e^- \\ \omega \eta \rightarrow e^+ e^- & \phi \eta \rightarrow e^+ e^- & \rho^0 \eta \rightarrow e^+ e^- \\ \omega \eta' \rightarrow e^+ e^- & \phi \eta' \rightarrow e^+ e^- & \rho^0 \eta' \rightarrow e^+ e^- \\ K^{*\pm} K^\mp \rightarrow e^+ e^- & \bar{K}^{*0} K^0 \rightarrow e^+ e^- & K^{*0} \bar{K}^0 \rightarrow e^+ e^- \end{array}$$

Other reactions

$$\begin{array}{ll} \pi^\pm a_1^\mp \rightarrow e^+ e^- & \pi \pi \rightarrow \pi e^+ e^- \\ \pi^+ \pi^- \pi^0 \rightarrow e^+ e^- & \end{array}$$

Problems and Questions:

- **Double counting** Is it correct to calculate the total dilepton rate as a sum of contributions from all the processes listed above ?

An example:

$$\rho \rightarrow e^+e^- \text{ versus } \pi^+\pi^- \rightarrow e^+e^-$$

Gudina, Titov, and Toneev,
and also

P. L., Phys. Rev. D 49, 5817 (1994):

| T (MeV) | $R(\rho^0 \rightarrow e^+e^-) / R(\pi^+\pi^- \rightarrow e^+e^-)$ |
|-----------|---|
| 100 | 0.83 |
| 150 | 0.98 |
| 200 | 0.98 |

Similar analyzes done also for ω and ϕ peaks.

- **Completeness** Don't we forget about some important contribution? A more systematic approach needed.

- Sharp masses of resonances, ambiguities in smearing
- Shouldn't some contributions be summed coherently?
- Ensemble of stable hadrons and resonances treated as a mixture of ideal gases. Is it correct?

R. F. Dashen and Rajaraman, Phys. Rev. D 10, 694 (1974):

When the dynamics of a system is dominated by narrow- resonance formation it behaves like a non-interacting system with added species of free particles, corresponding to all quantum numbers and the statistics of each of the resonances.

Also: R. Venugopalan and M. Prakash, Nucl. Phys. A 546, 718 (1992) and references therein.

1.2 Reaction rate formula



$$R = K_i K_f \int d\Phi_N \int d\Phi_{ab} (2\pi)^4 \delta(P - p_a - p_b) \\ \times \sum_{\lambda_1, \dots, \lambda_N} \sum_{\lambda_a, \lambda_b} |\mathcal{M}_{1+\dots+N \rightarrow ab}|^2$$

$$d\Phi_N = \prod_{i=1}^N \frac{d^3 p_i}{2E_i} \frac{1}{(2\pi)^3} f_i(E_i)$$

$f_i(E_i)$ are the Fermi-Dirac or Bose-Einstein mean occupation numbers of incoming hadrons.

$$d\Phi_{ab} = \prod_{x=a,b} \frac{d^3 p_x}{2E_x} \frac{1}{(2\pi)^3} F_x(E_x)$$

$F_x(E_x)$ are the Fermi-Dirac suppression or Bose-Einstein enhancement factors for outgoing particles. K_i and K_f account for possible appearance of the identical hadrons in the initial and final states, respectively.

$$R = \frac{K_i K_f}{16\pi^2} \int d\Phi_N \frac{p_a^*}{\sqrt{s}} \int d\Omega_a^* F(E_a) F(E_b) \\ \times \sum_{\lambda_1, \dots, \lambda_N} \sum_{\lambda_a, \lambda_b} \left| \mathcal{M}_{1+\dots+N \rightarrow a+b} \right|^2 .$$

If $a = e^+$, $b = e^-$ then $F_a = F_b = 1$ and a simpler expression for R can be derived, see P.L., Phys. Rev. D 49, 5817 (1994).

If the final state consists of hadron a and a dilepton, we first calculate $R_\gamma(M)$, the rate of the massive photon production

$$1 + 2 + \dots + N \rightarrow a + \gamma^*$$

and then use the formula

$$\frac{dR_{e^+e^-}}{dM} = h(M) R_\gamma(M),$$

where

$$h(M) = \frac{2\alpha M^2 + 2m^2}{3\pi M^3} \left(1 - \frac{4m^2}{M^2} \right)^{1/2} .$$

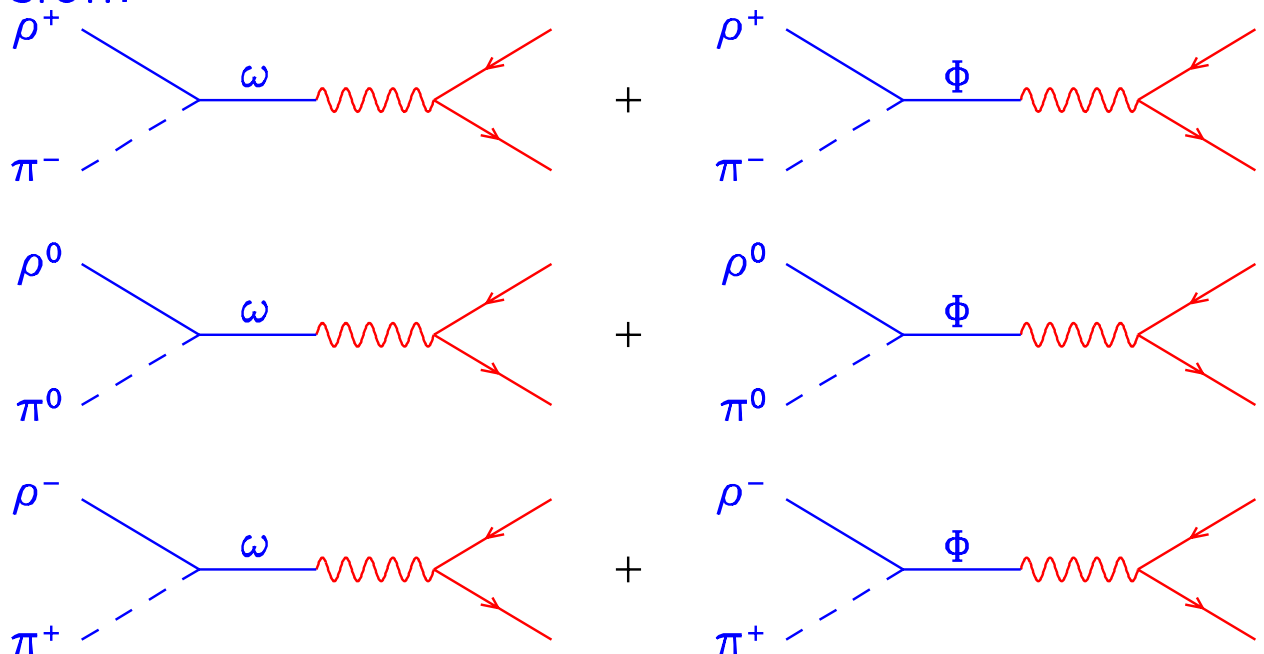
1.3 Reactions with resonances and their counterparts

a) Toward the three-pion initial states

Dielectron producing decays of the ω and ϕ mesons in the VMD approximation:

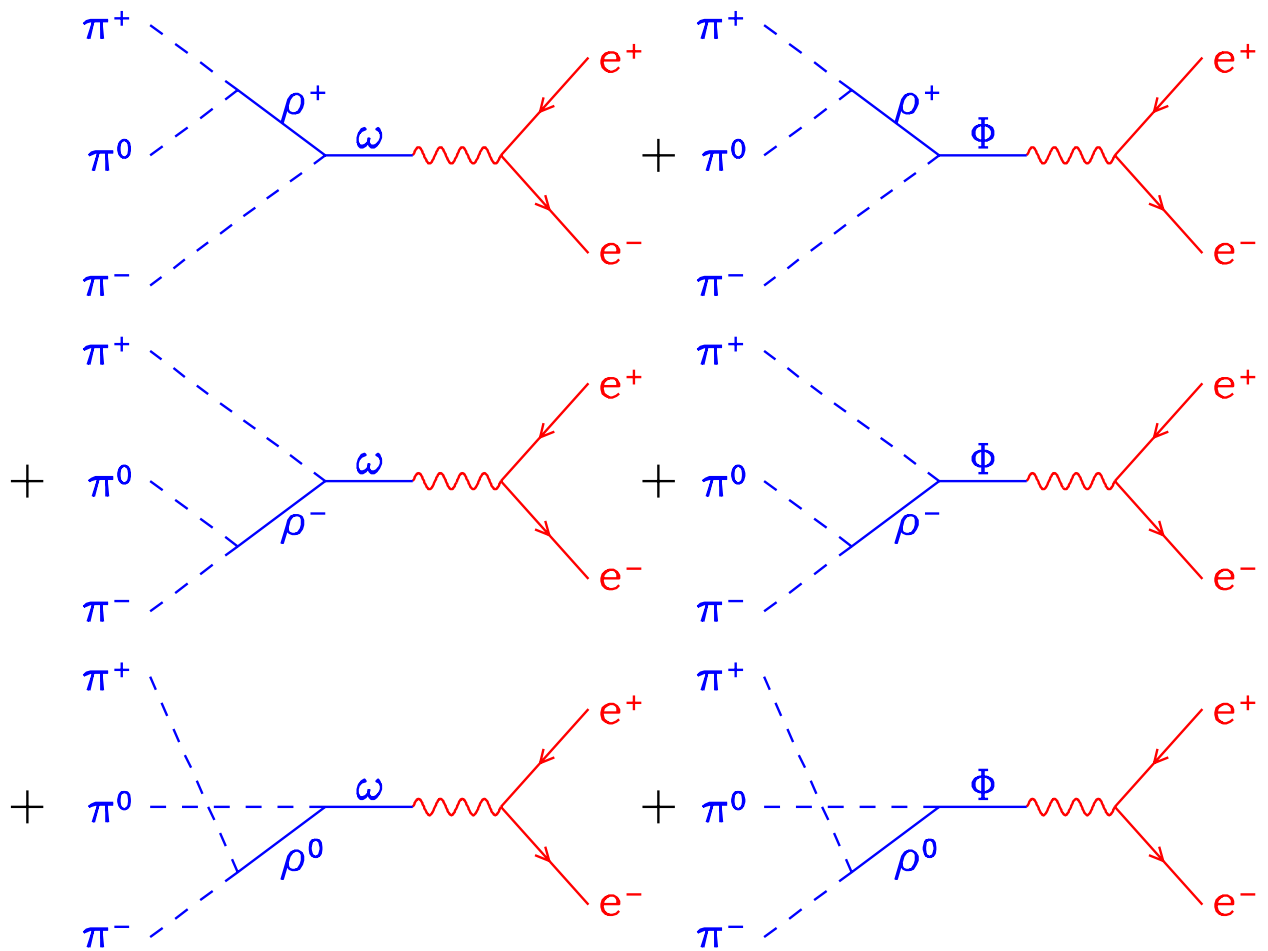


Formation of the ω and ϕ mesons by the ρ - π fusion:



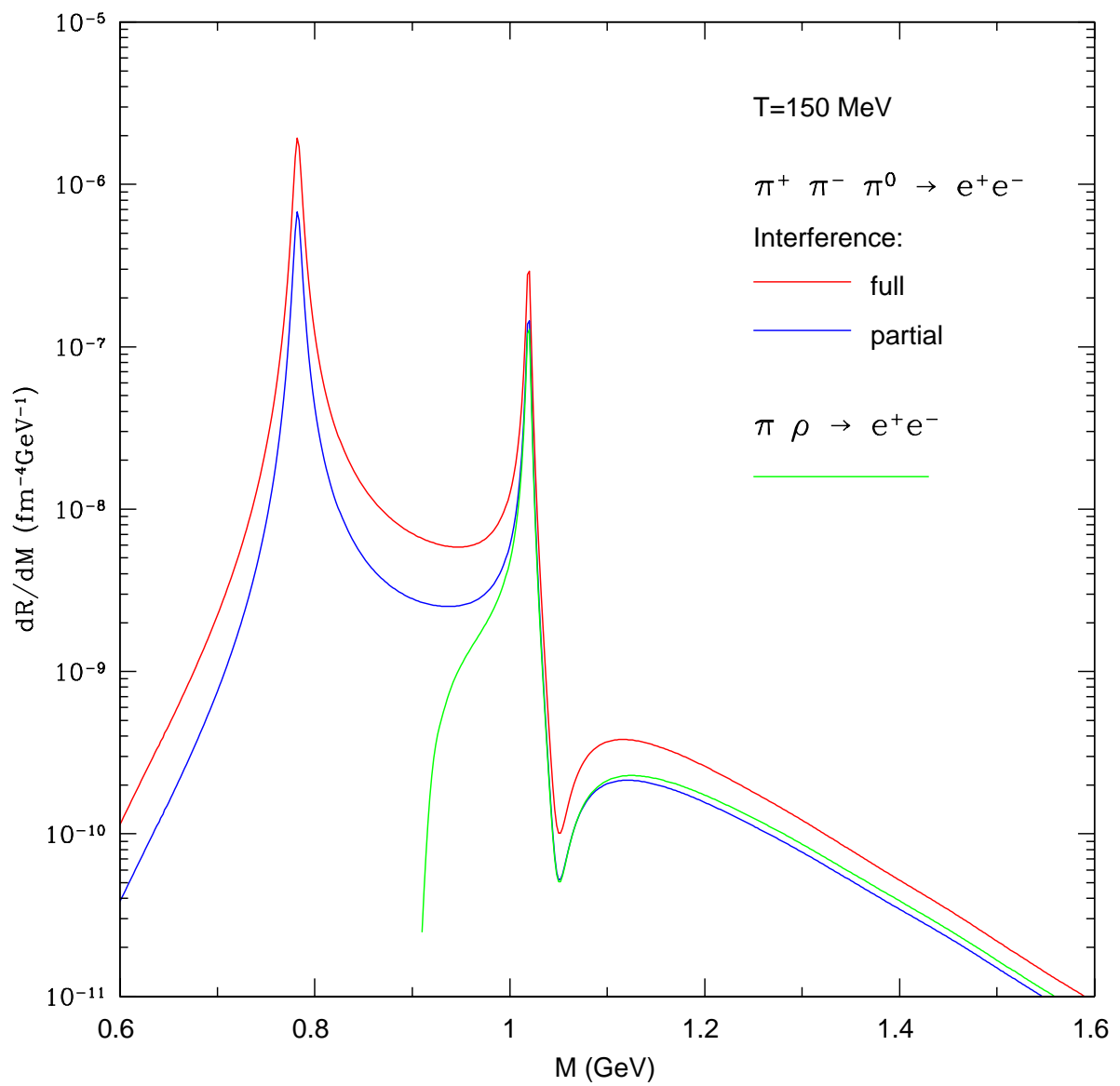
horizontally-coherent
vertically-incoherent

Also the formation of the ρ mesons considered:



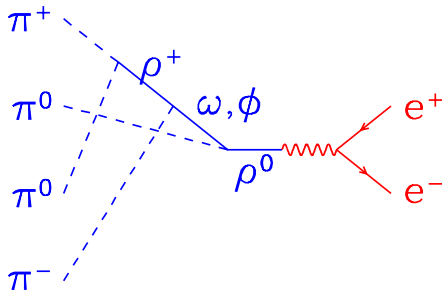
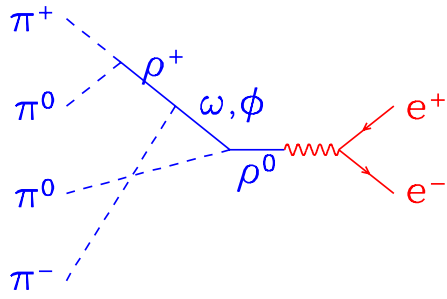
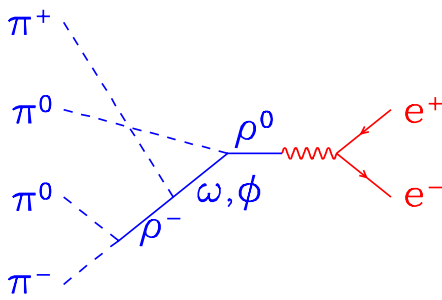
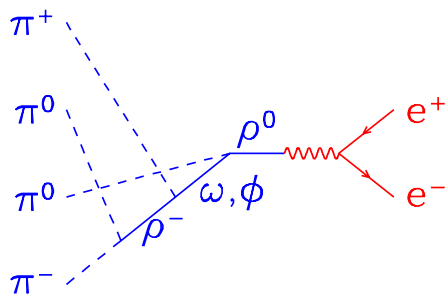
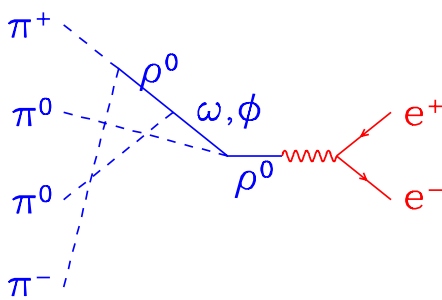
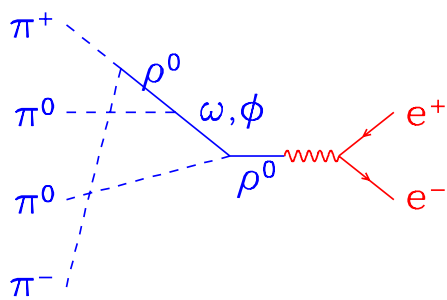
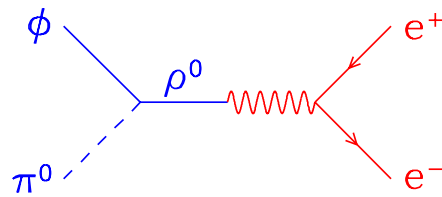
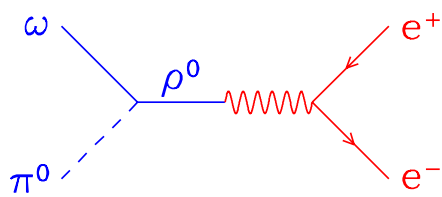
All six diagrams should be summed up coherently.

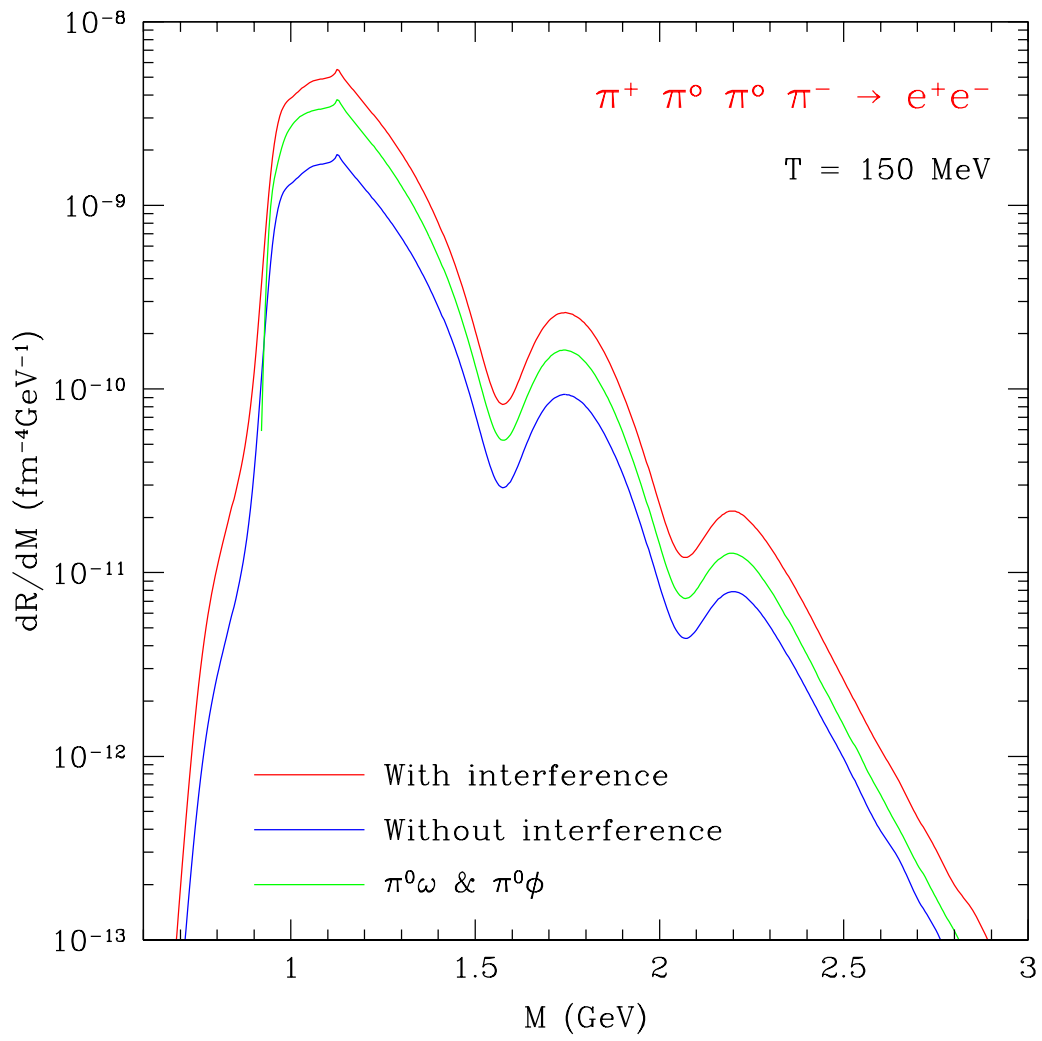
Comparison of the resonance approach with the three-pion one:



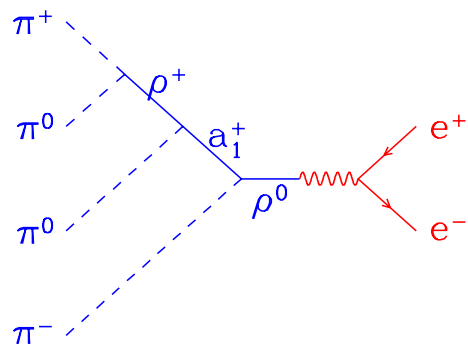
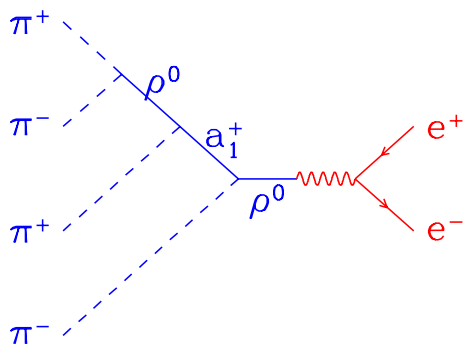
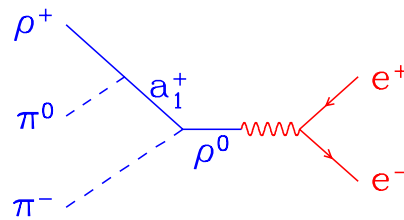
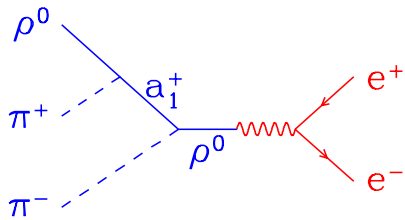
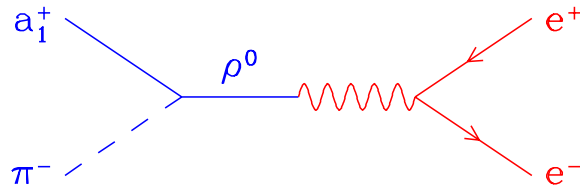
b) Toward the four-pion initial states

b.1) $\pi^0\omega \rightarrow e^+e^-$ and $\pi^0\phi \rightarrow e^+e^-$
 (C. Gale and P.L. 1994)





b.2) $\pi^- a_1^+ \rightarrow e^+ e^-$ and $\pi^+ a_1^- \rightarrow e^+ e^-$
 (C. Song, C.M. Ko, and C. Gale 1994)



(4 diagrams)

(2 diagrams)

Adding the diagrams that develop from $\pi^+ a_1^- \rightarrow e^+ e^-$, we get:
 8 diagrams with the $\pi^+ \pi^- \pi^+ \pi^-$ initial state
 4 diagrams with the $\pi^+ \pi^0 \pi^0 \pi^-$ initial state.

A two-component Lagrangian of the $a_1\pi\rho$ interaction was first introduced by C. Song, C.M. Ko and C. Gale (1994). We write it in the form

$$\begin{aligned}\mathcal{L} &= G(\cos\theta \mathcal{L}_A + \sin\theta \mathcal{L}_B) \\ \mathcal{L}_A &= \mathbf{A}^\mu \cdot (\mathbf{V}_{\mu\nu} \times \partial^\nu \phi) \\ \mathcal{L}_B &= \mathbf{V}_{\mu\nu} \cdot (\partial^\mu \mathbf{A}^\nu \times \phi)\end{aligned}$$

| | $\sin\theta$ |
|-------------------------------|--------------|
| Xiong, Shuryak, Brown (XSB) | 0 |
| Song I | 0.2128 |
| Song II | 0.6306 |
| Janssen, Holinde, Speth (JHS) | 1 |

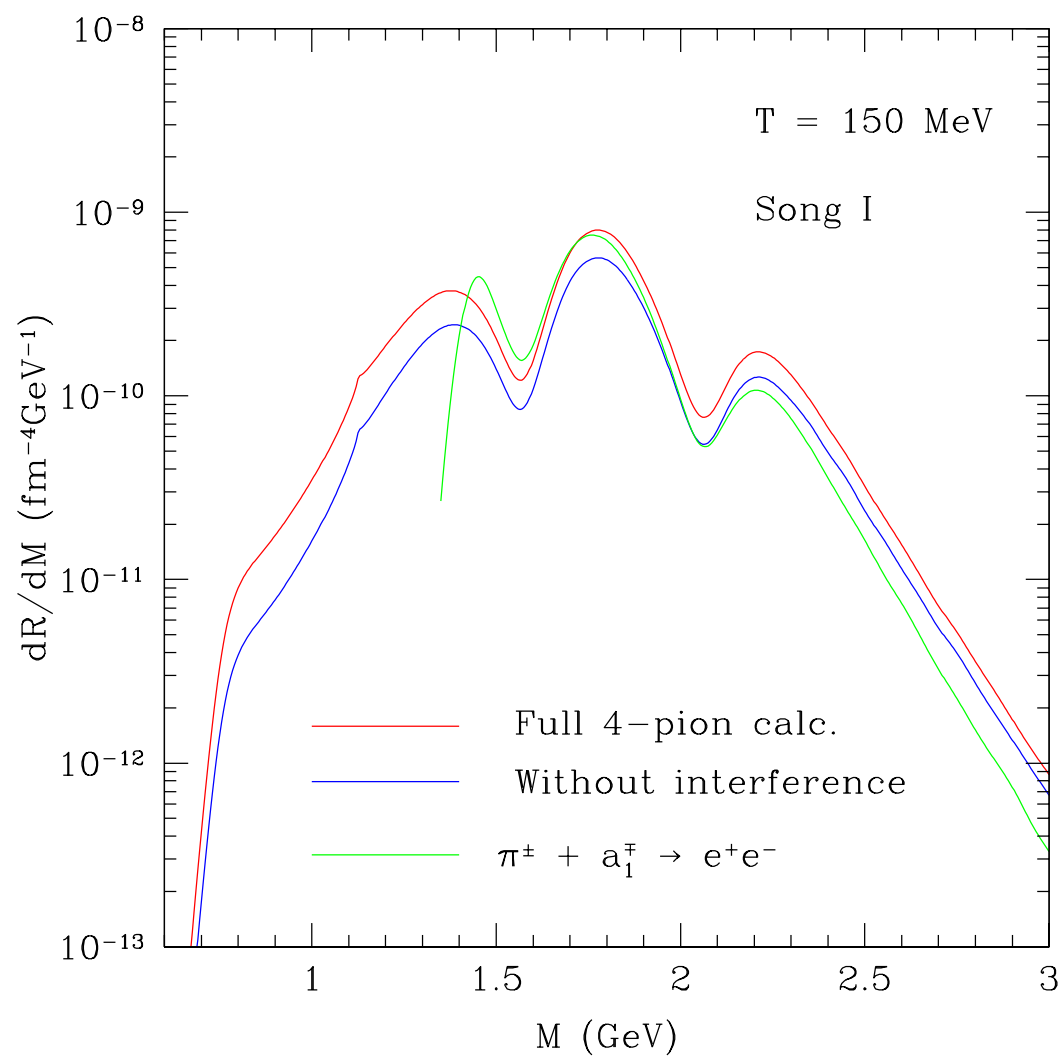
L. Xiong, E. Shuryak, and G. E. Brown, Phys. Rev. D 46, 3798 (1992)

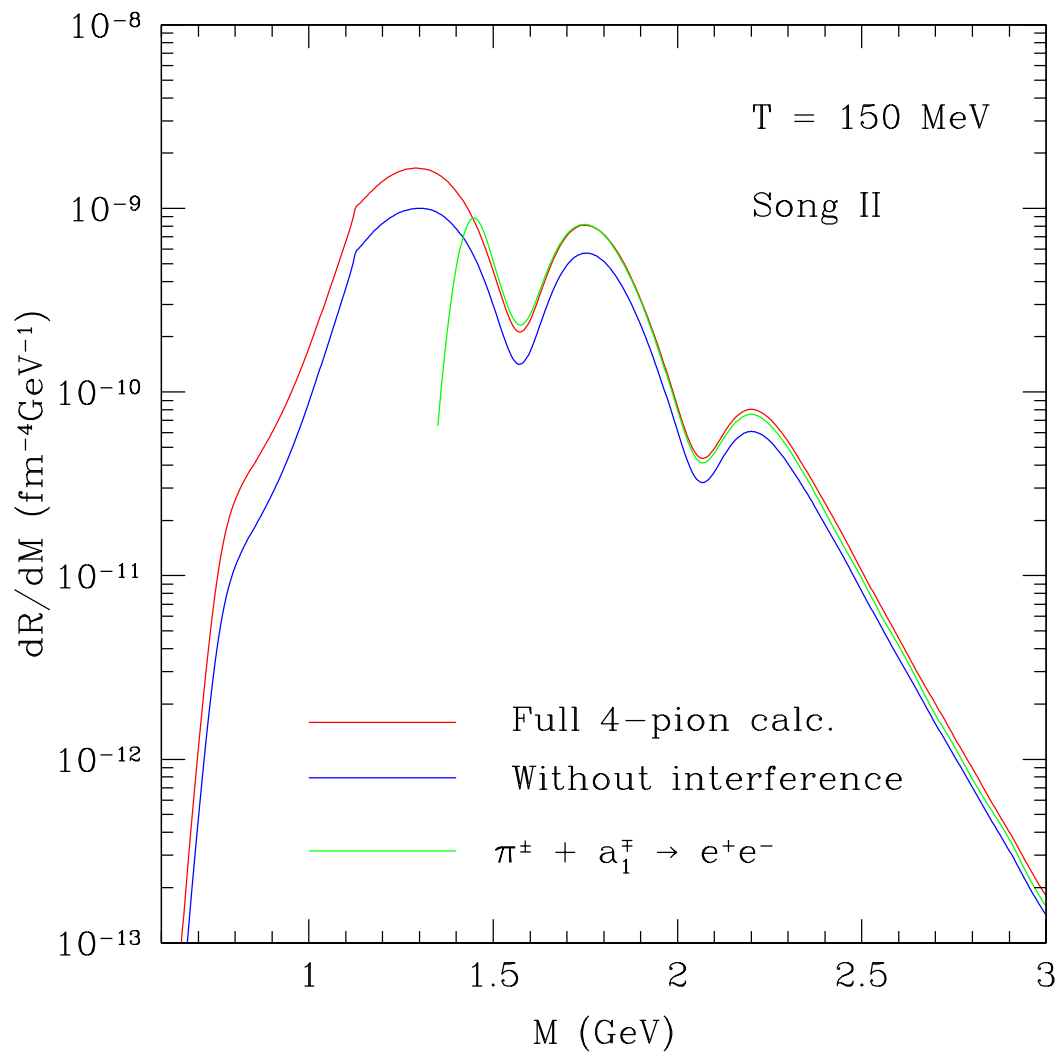
C. Song, Phys. Rev. C 47, 2861 (1993)

G. Janssen, K. Holinde, and J. Speth, Phys. Rev. C 49, 2763 (1994)

G is fixed for a chosen value of $\sin\theta$ by $\Gamma(a_1 \rightarrow \rho\pi) = 400$ MeV.

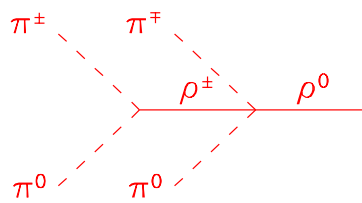
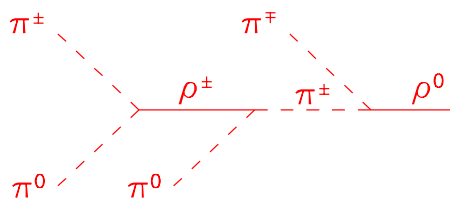
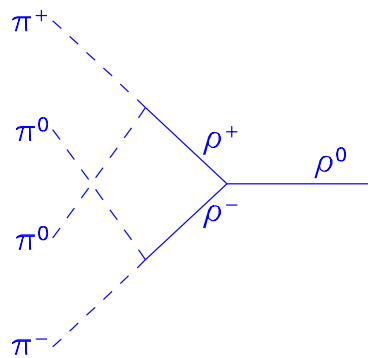
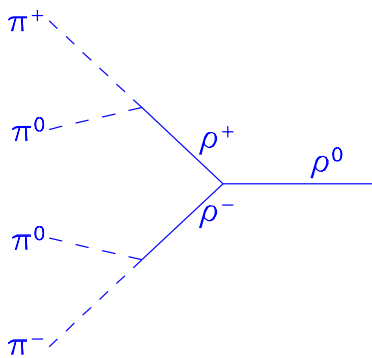
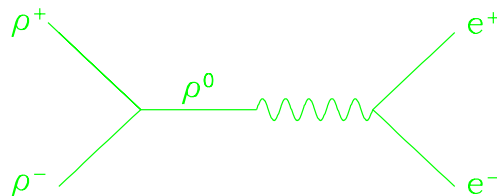
Comparison of the two approaches using the $a_1\rho\pi$ Lagrangians Song I and Song II:





b.3) $\rho^+\rho^- \rightarrow e^+e^-$

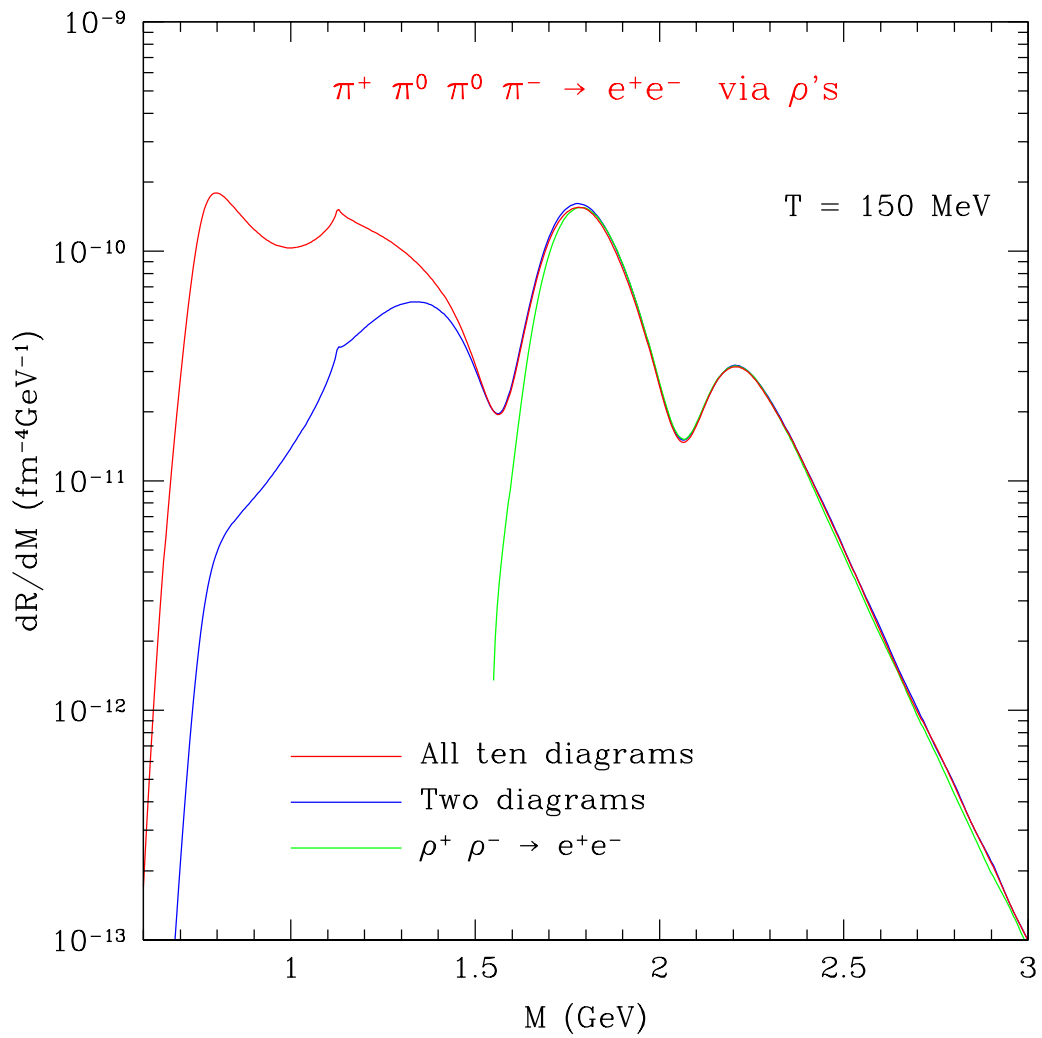
(C. Song, C.M. Ko, and C. Gale 1994)



(four diagrams)

(four diagrams)

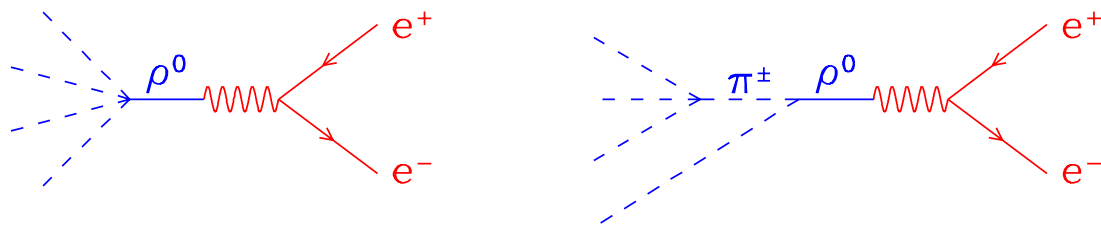
Note: The red diagrams operate also from the $\pi^+\pi^-\pi^+\pi^-$ initial states. The number of diagrams in the first group then becomes eight. In that case the red diagrams form a gauge invariant set.



1.4 Four pion initial states

| origin | $\pi^+ \pi^0 \pi^0 \pi^-$ | $\pi^+ \pi^- \pi^+ \pi^-$ |
|----------------------|---------------------------|---------------------------|
| $\pi\omega, \pi\phi$ | 6 | |
| πa_1 | 4 | 8 |
| $\rho\rho$ | 10 | 12 |
| continuum | 3 | 5 |
| Total | 23 | 25 |

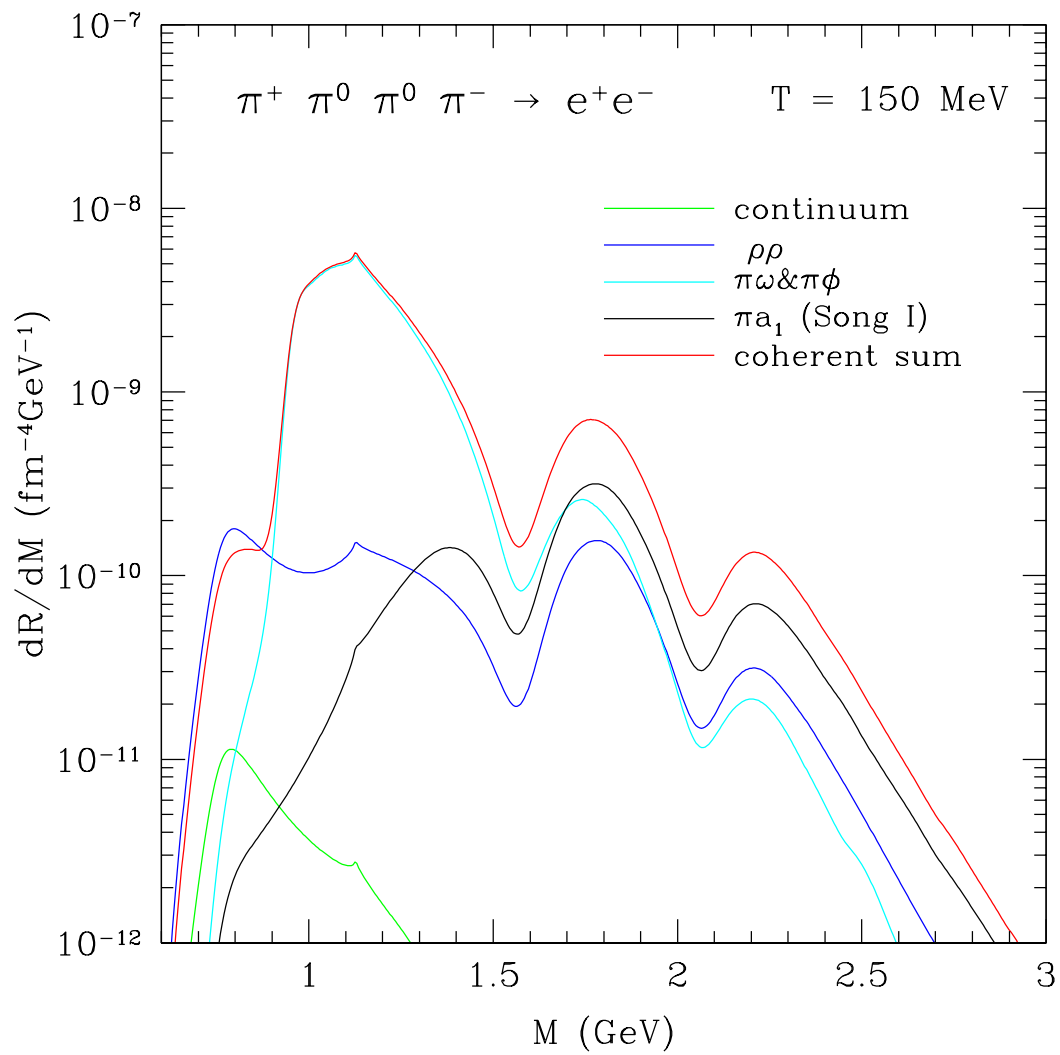
" Continuum" :

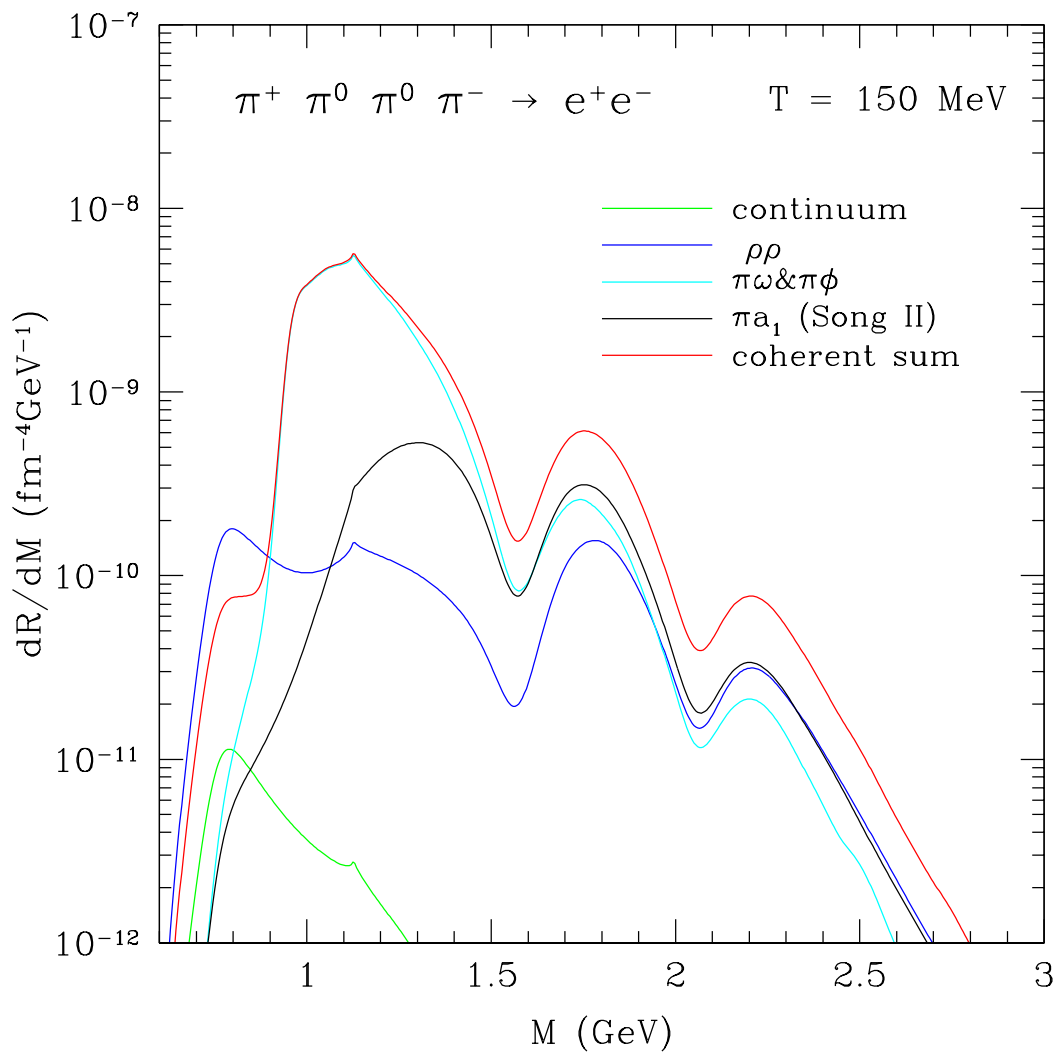


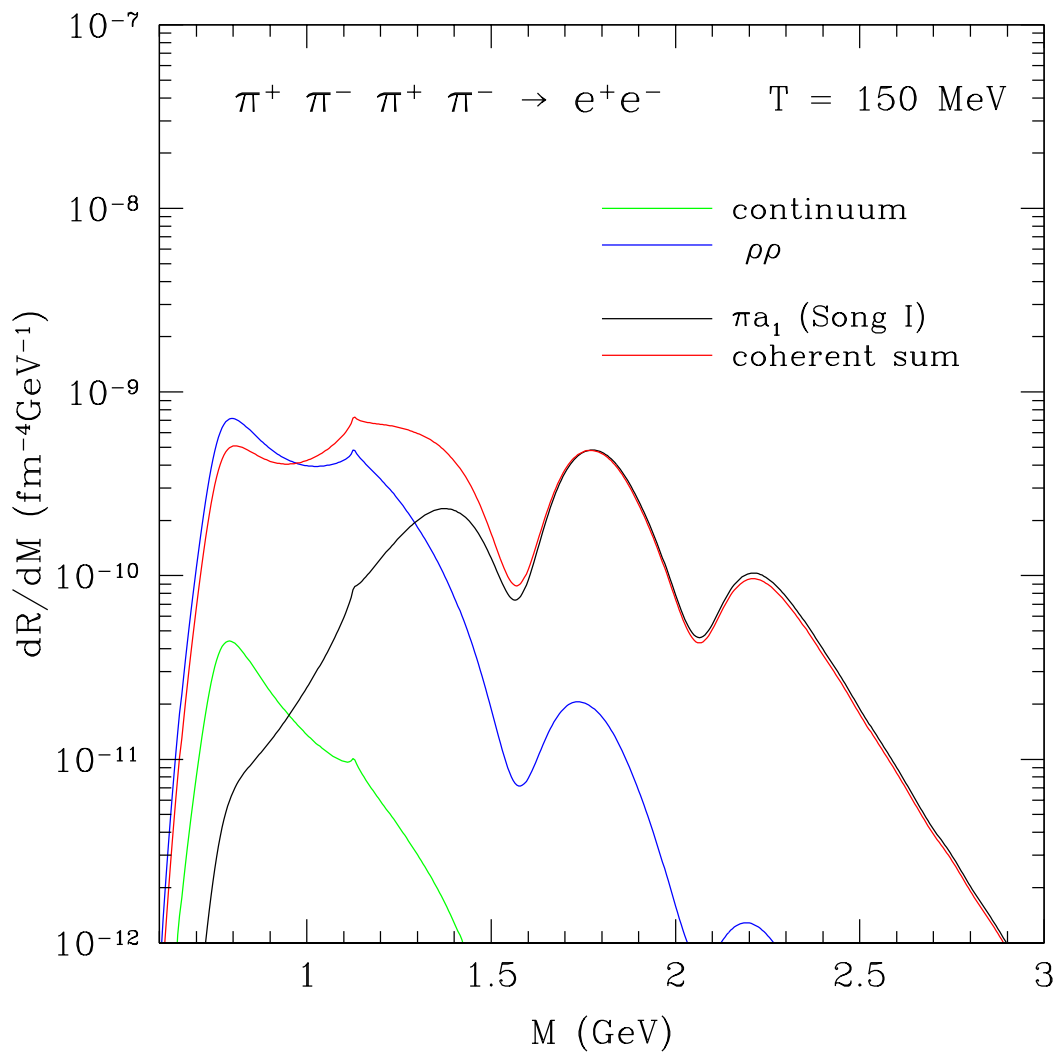
Lagrangian:

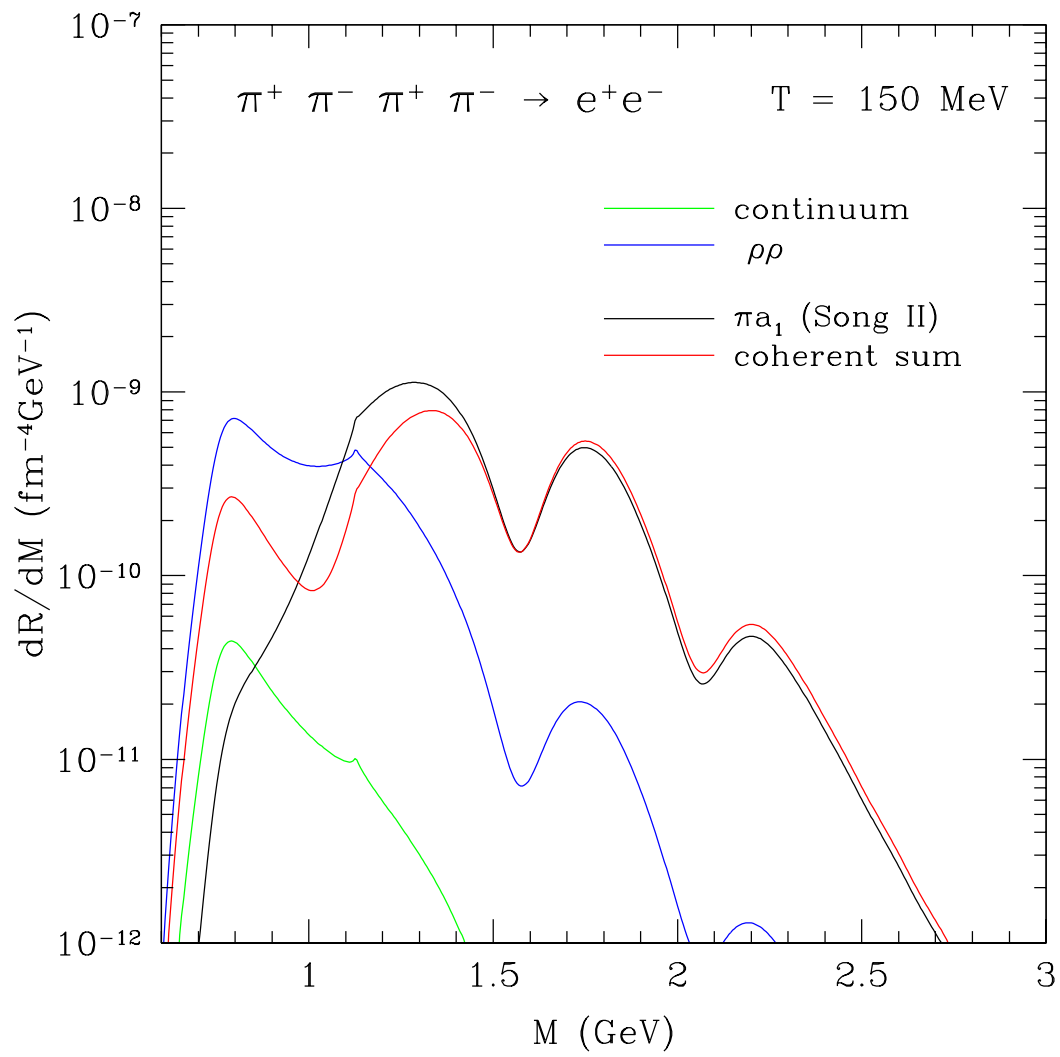
V. Brihaye, N.K. Pak, P. Rossi, Nucl. Phys. B254, 71 (1985); Phys. Lett. B164, 111 (1985).

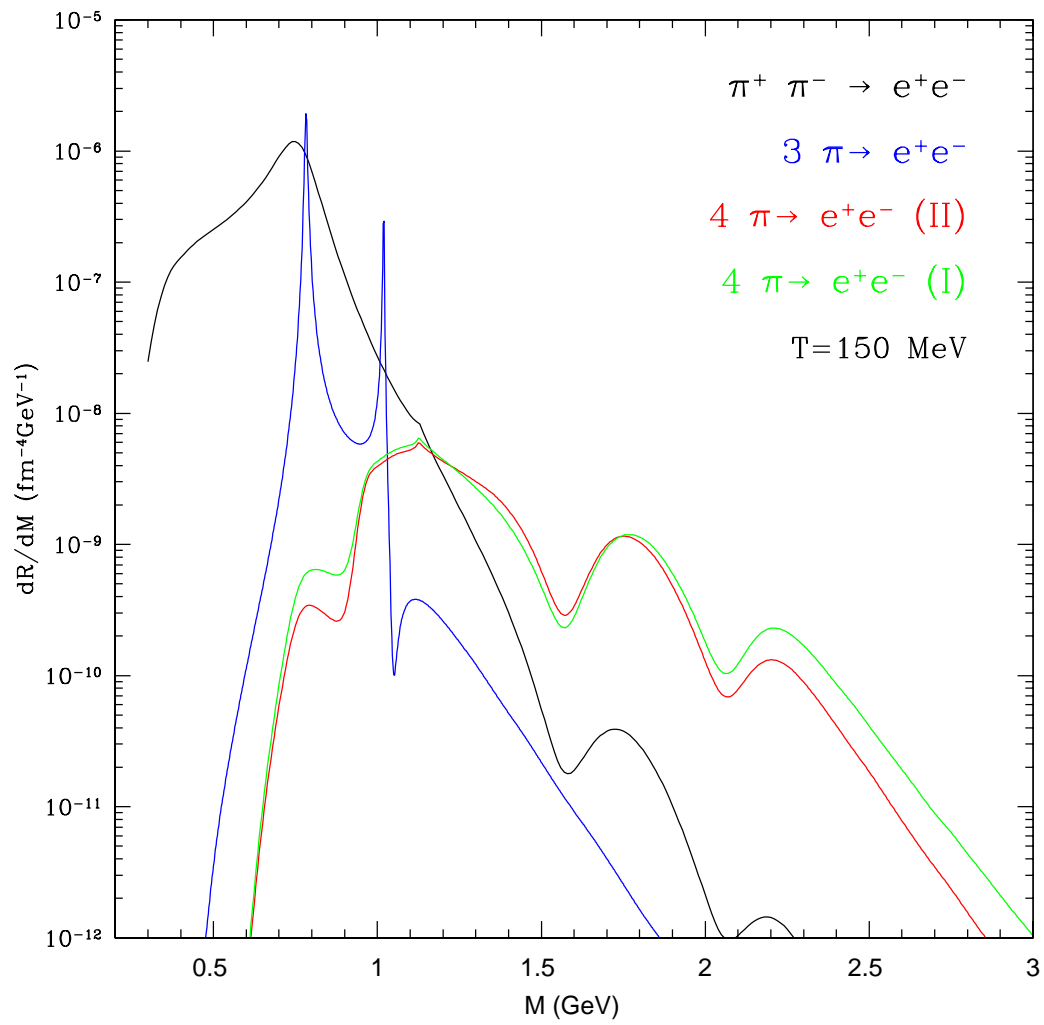
E.A. Kuraev, Z.K. Silagadze, Phys. Lett. B292, 377 (1992).











2.1 A piece of very early history of thermal photons

Enhanced photon production as a signature of the QGP formation:

E. Shuryak, Phys. Lett. B 78, 150 (1978),
Phys. Rep. 61, 72 (1980);

K. Kajantie and H.I. Miettinen, Z. Phys. C 9,
341 (1981);

L. McLerran and T. Toimela, Phys. Rev. D
31, 545 (1985);

R. Hwa and K. Kajantie, Phys. Rev. D 32,
1109 (1985).

Hadronic alternative:

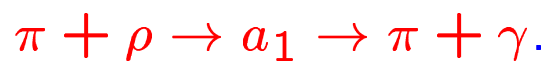
Photon production from various reactions within a thermalized meson gas. The main result:

The hadron gas shines just as brightly as QGP at a given temperature.

J. Kapusta, P. L., and D. Seibert, Phys. Rev. D 44, 2774 (1991);

H. Nadeau, J.I. Kapusta, and P. L., Phys. Rev. C 45, 3034 (1992).

L. Xiong, E. Shuryak, and G. E. Brown, Phys. Rev. D 46, 3798 (1992) showed that an important source of photons is the reaction



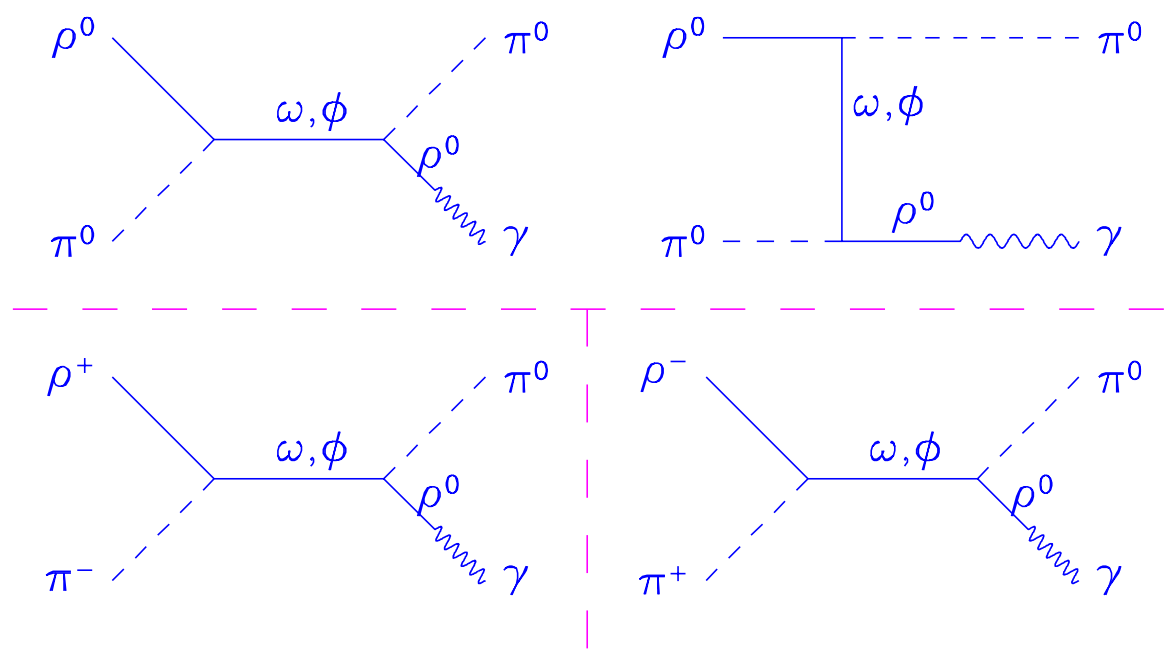
C. Song, Phys. Rev. C 47, 2861 (1993) carefully recalculated $\pi + \rho \rightarrow \pi + \gamma$, $\pi + \pi \rightarrow \rho + \gamma$, and $\rho \rightarrow \pi + \pi + \gamma$ using two different $a_1\rho\pi$ Lagrangians.

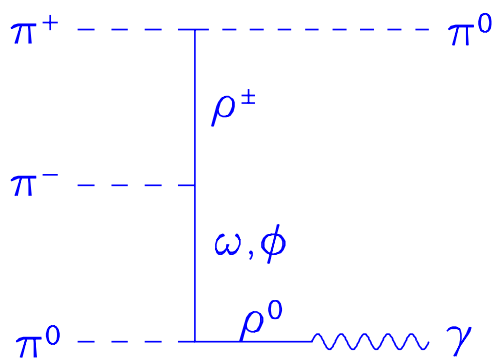
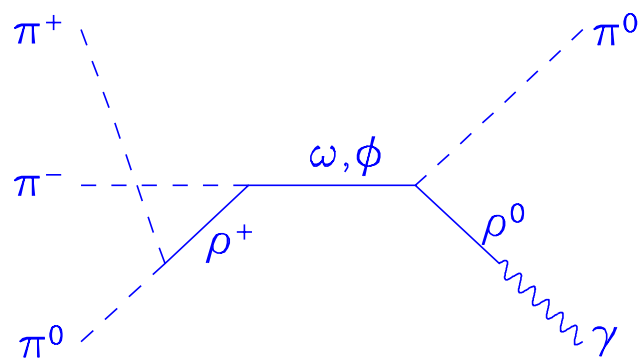
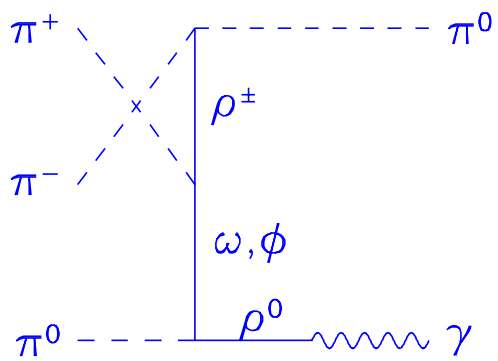
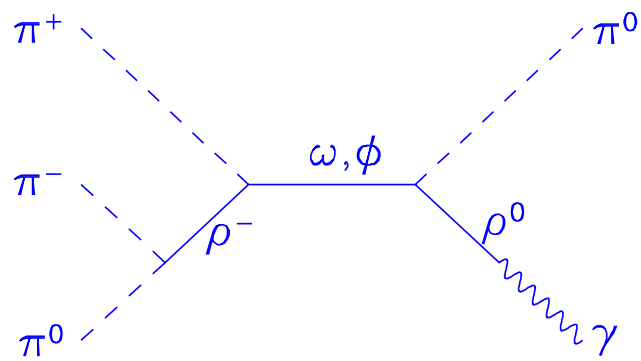
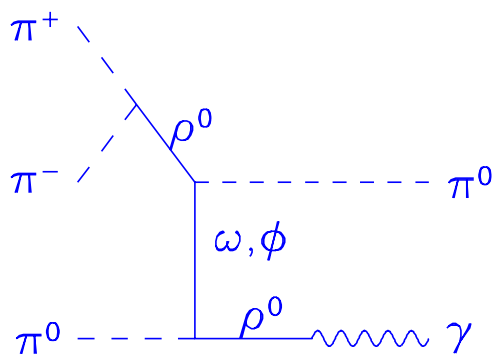
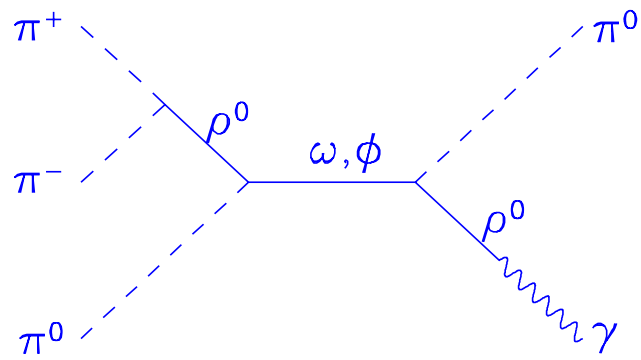


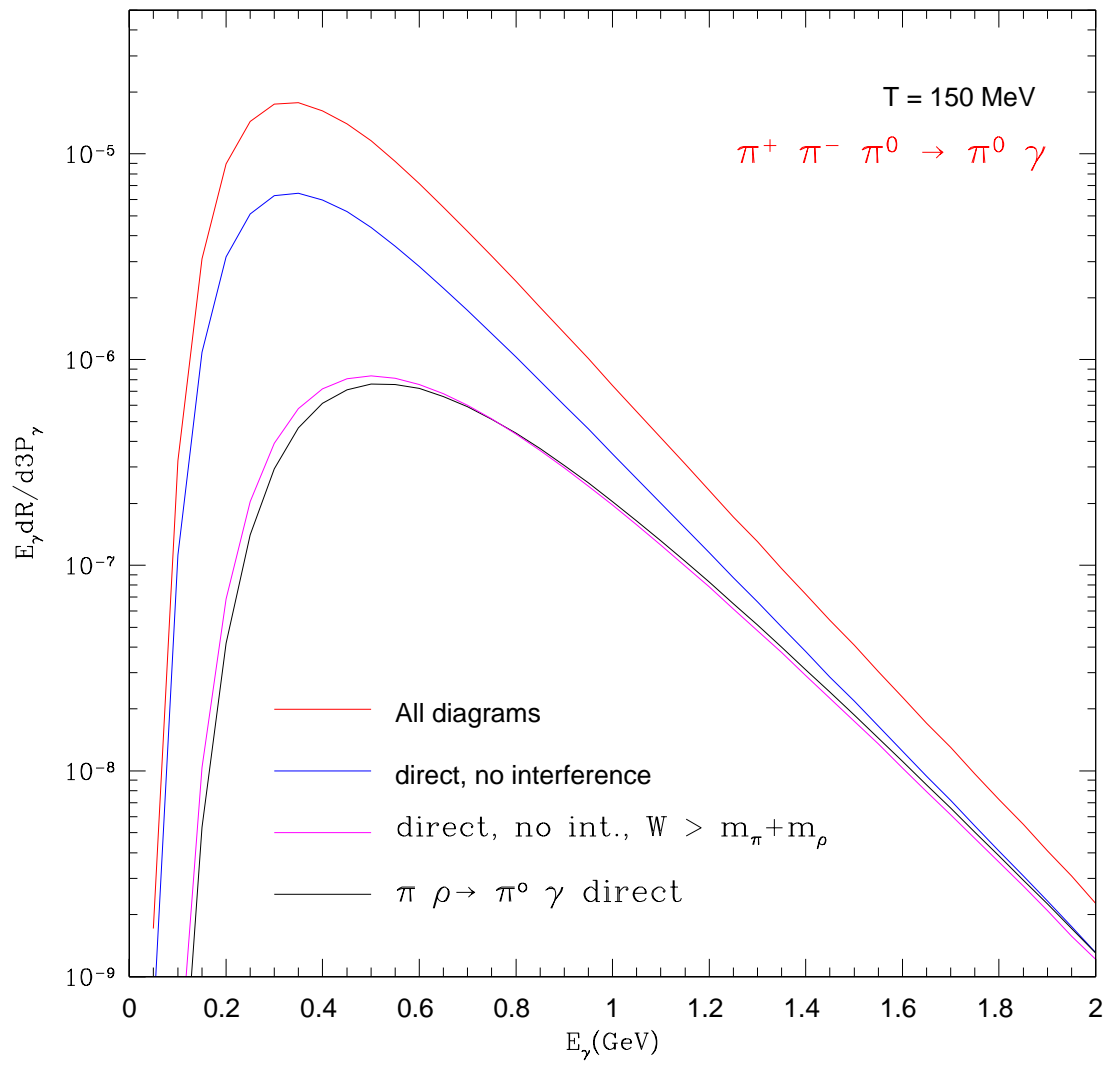
| | Intermediate states |
|---------------------------|---------------------|
| Kapusta, Lichard, Seibert | π |
| Xiong, Shuryak, Brown | a_1 |
| Song | π, ρ, a_1 |
| Nobody yet? | ω, ϕ |

2.2 Reactions with resonances and their counterparts

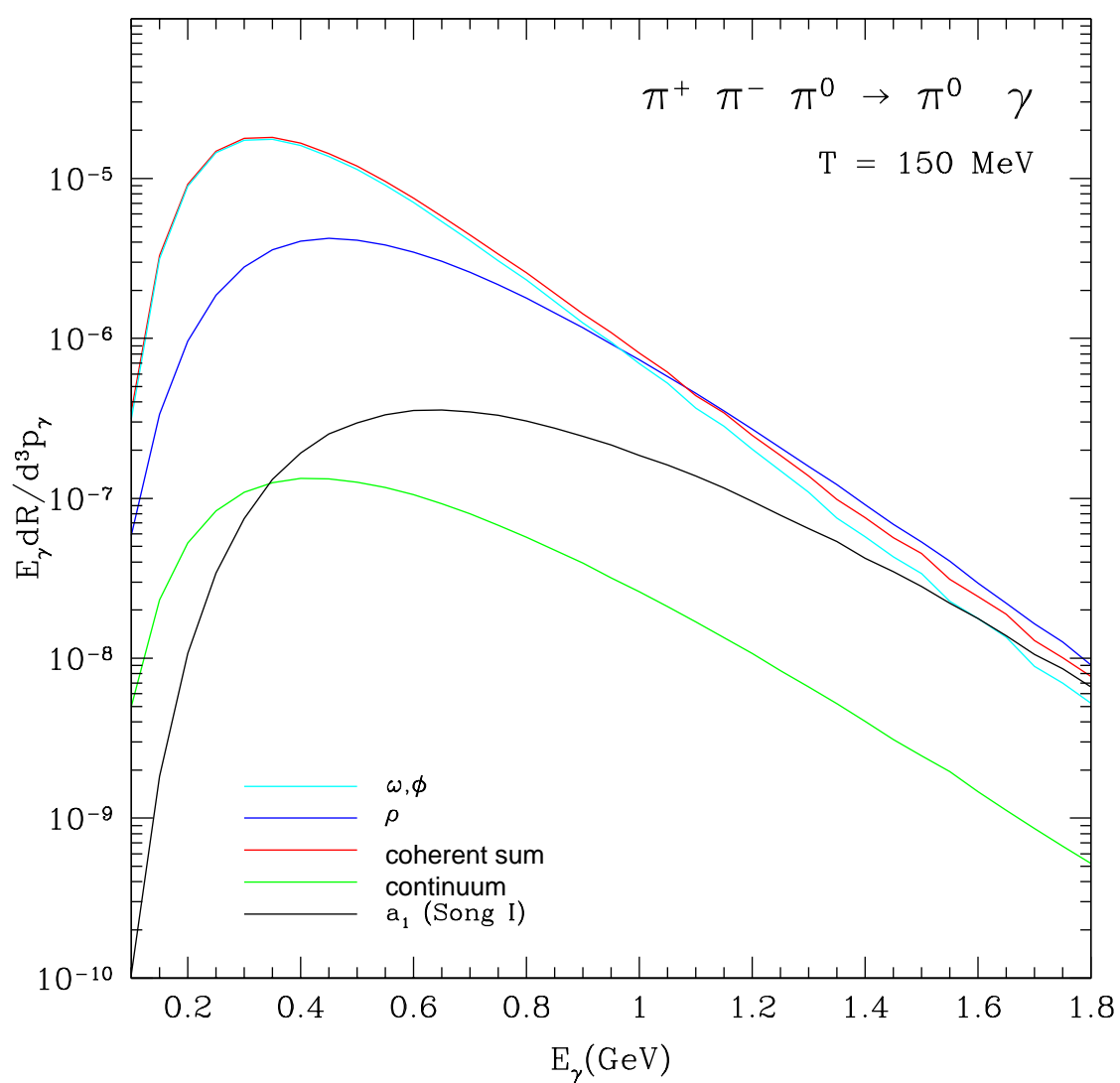
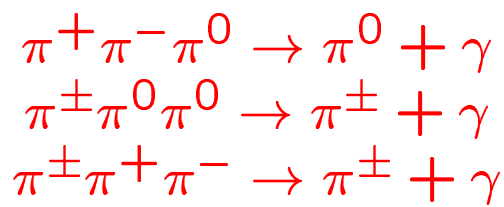
$$\pi + \rho \rightarrow \pi^0 + \gamma$$

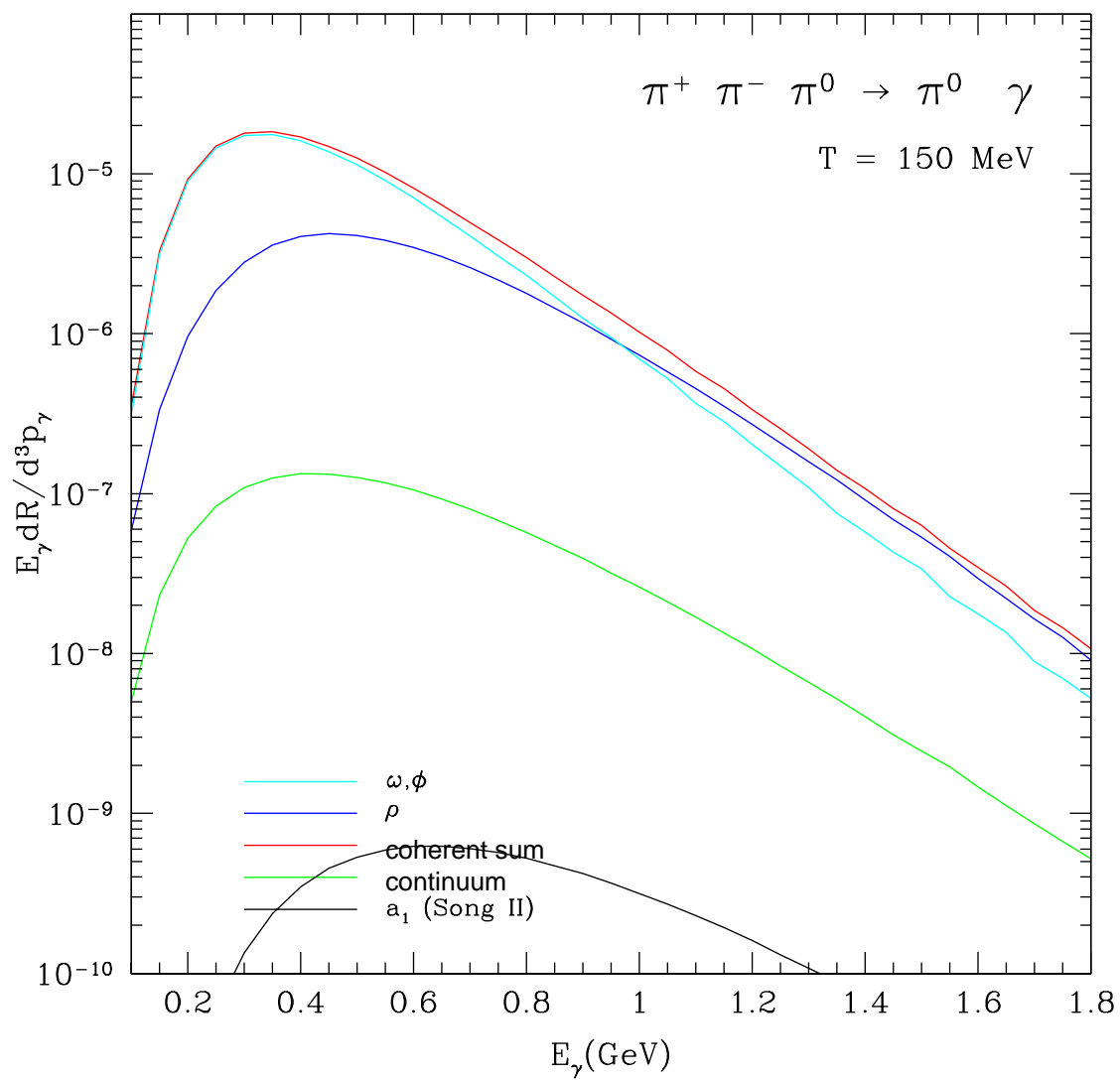


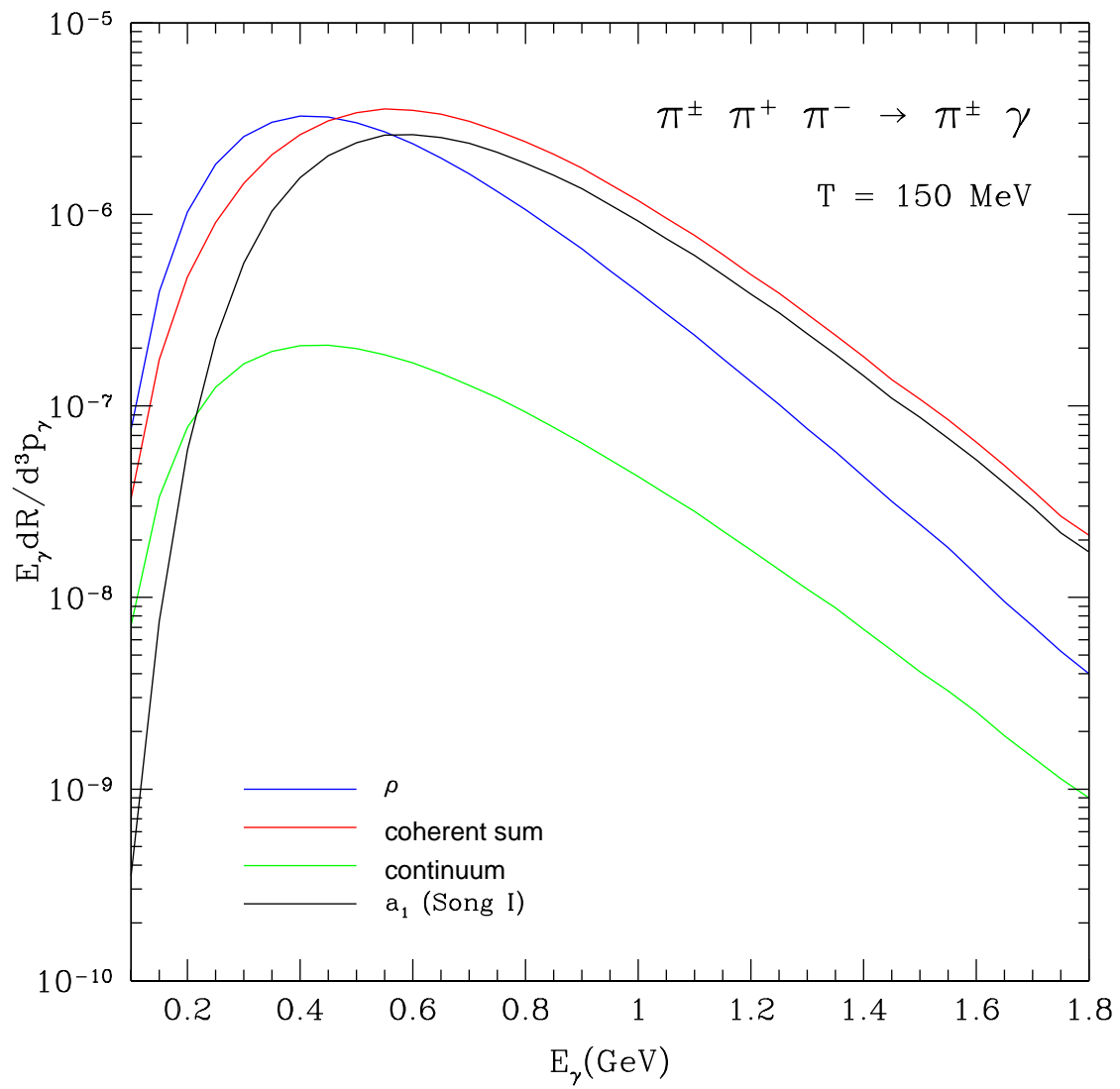


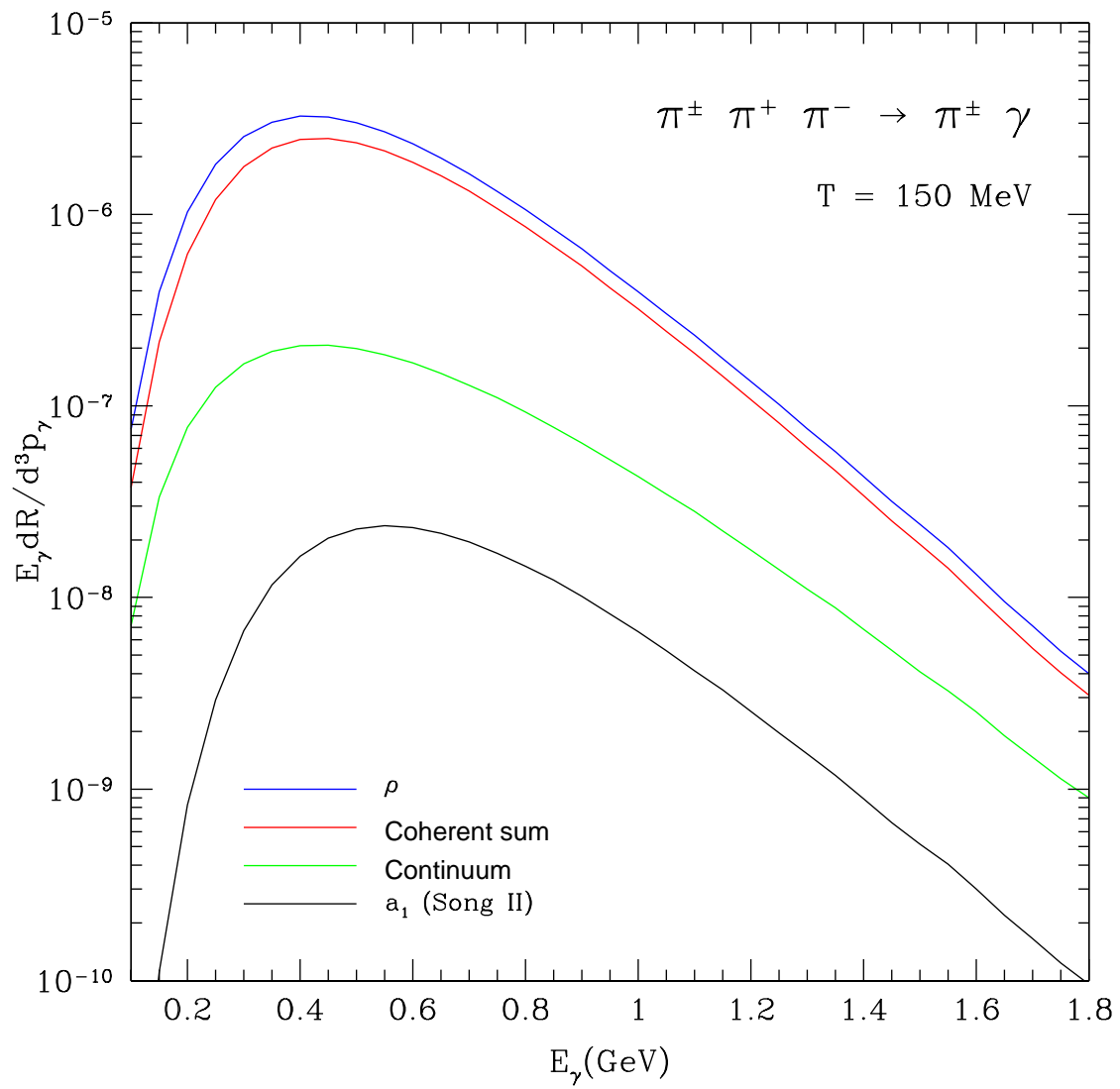


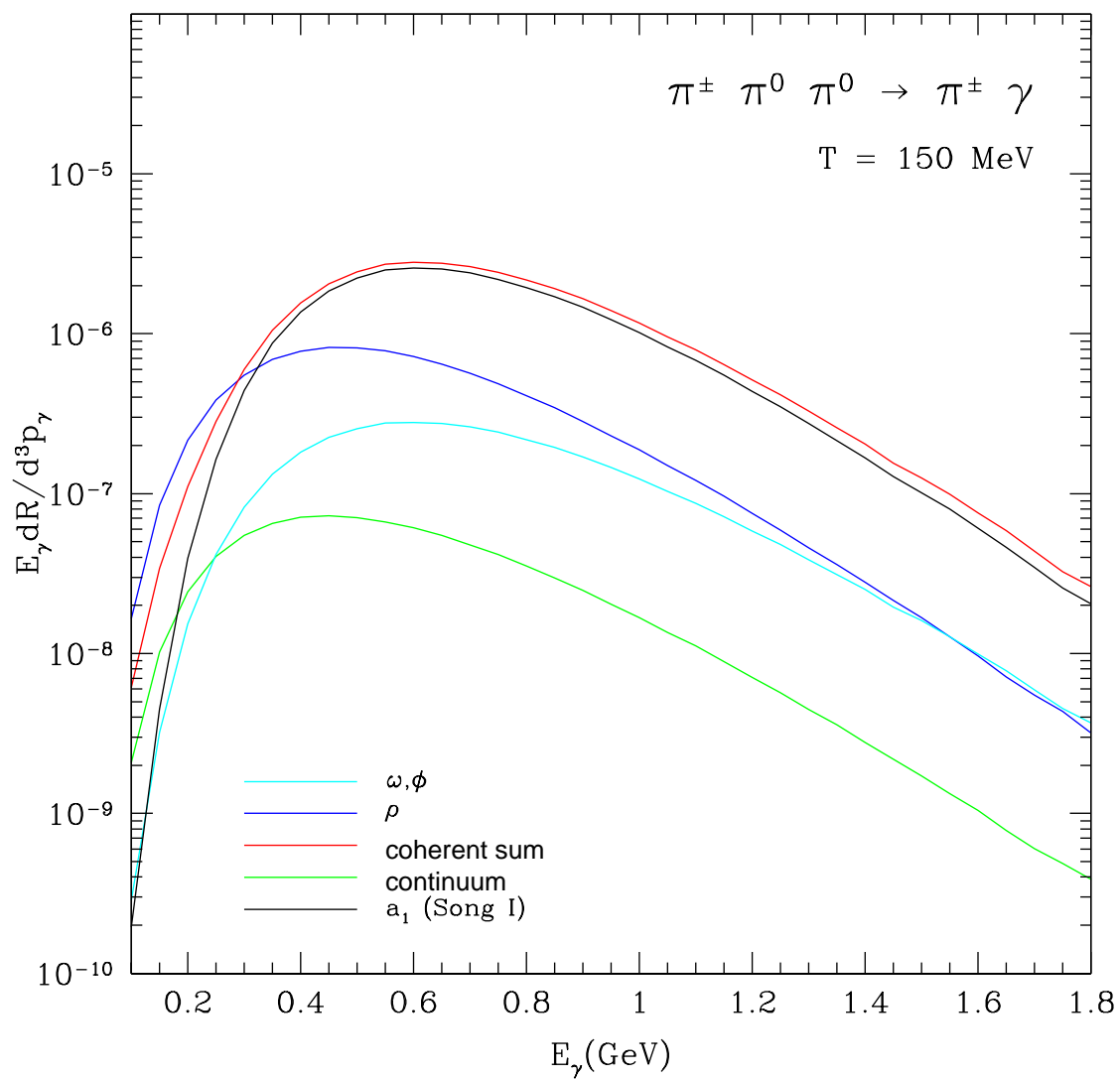
2.3 Three-pion initial states

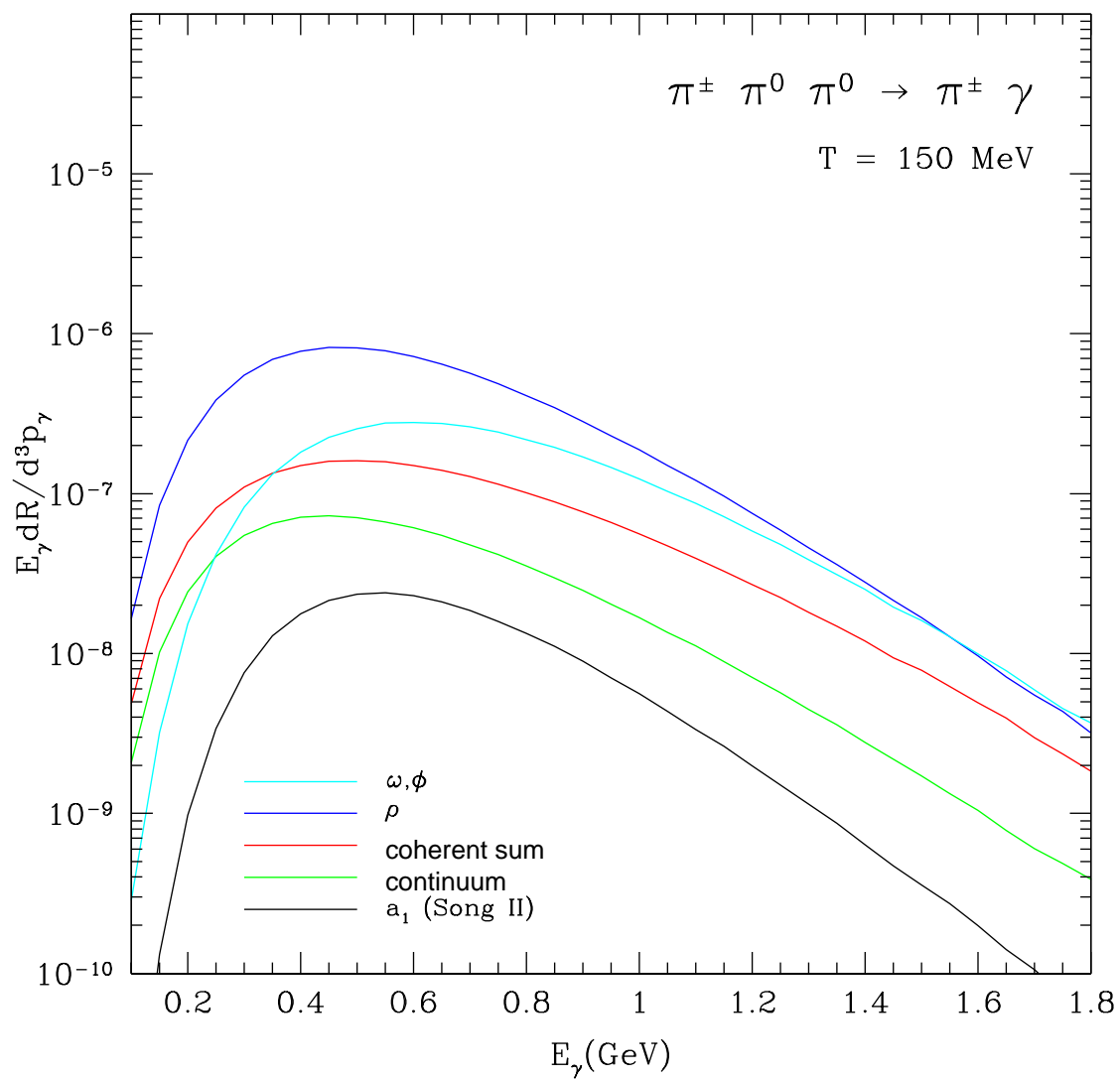


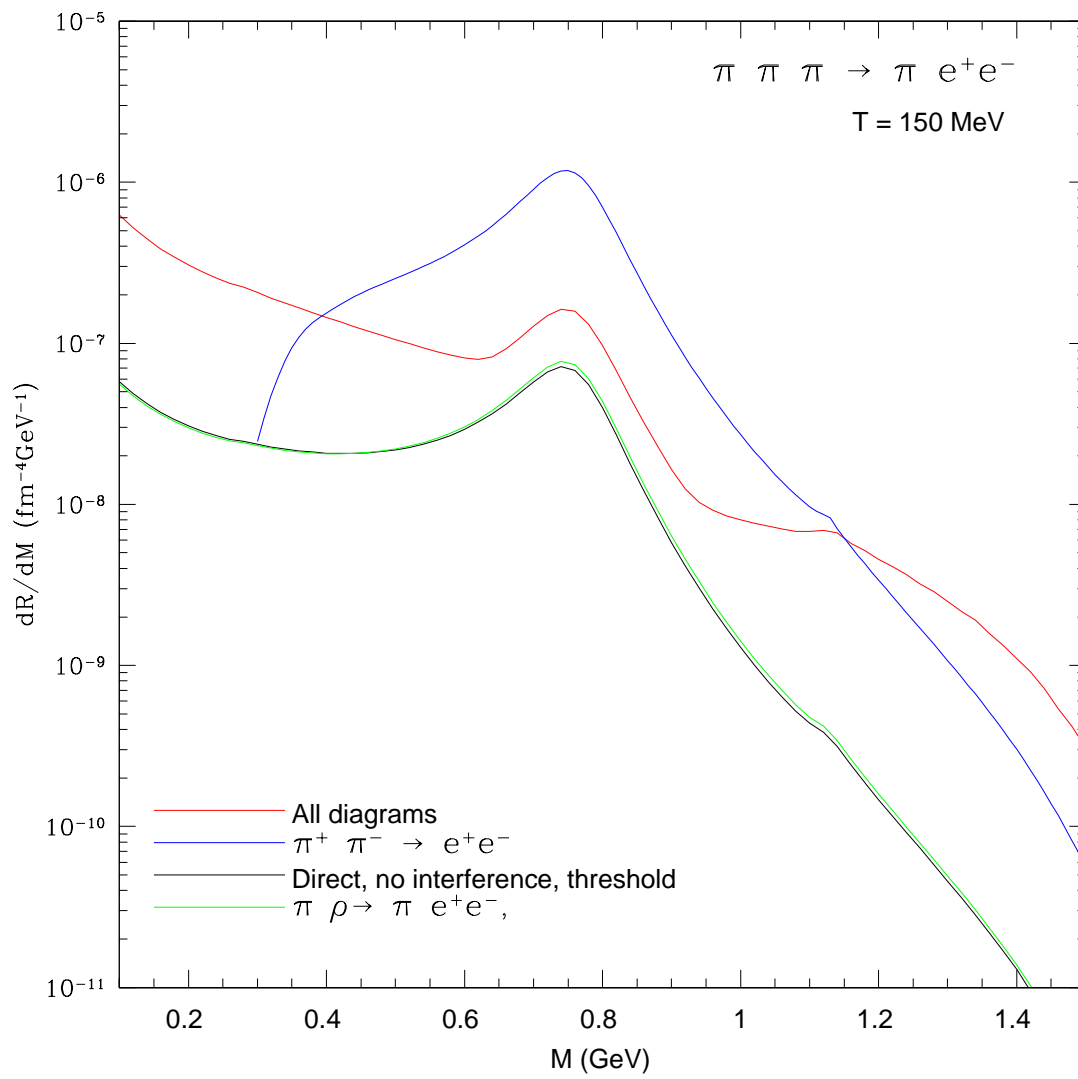












3) Conclusions

Many-body initial state approach is an alternative (and more correct?) description of the reactions with resonance(s) in the initial state.

The calculated yield differs from that of reactions with resonances because of a wider space phase and a possible interference among many diagrams with various resonances in the intermediate states.

Many-body approach deals with stable hadrons in the initial and final states. It thus allows us to classify all reactions by numbers and types of the stable particles in the initial and final state and thus avoid double counting.

| Initial hadrons | Final hadrons | Intermediate states | Replaces (partly) |
|--------------------|---------------|--|--|
| $\pi^+\pi^-$ | none | $\rho, \rho', \rho'', \omega, \dots$ | $\rho^0 \rightarrow e^+e^-$ |
| $\pi\pi$ | π | $\rho, \omega, \phi, \rho', \dots$ | $\rho \rightarrow \pi e^+e^-$ |
| $\pi\pi$ | $\pi\pi$ | $\rho, \rho', \rho'' \dots$ | $\rho \rightarrow \pi\pi e^+e^-$ |
| K^+K^- | none | $\rho, \omega, \phi, \rho', \dots$ | $\rho^0 \rightarrow e^+e^-$, $\omega \rightarrow e^+e^-$, $\phi \rightarrow e^+e^-$ |
| $\pi^+\pi^-\pi^0$ | none | $\rho, \omega, \phi, \rho', \dots$ | $\pi\rho \rightarrow e^+e^-$, $\omega \rightarrow e^+e^-$, $\phi \rightarrow e^+e^-$ |
| $\pi\pi\pi$ | π | $\rho, a_1, h_1,$ ρ', \dots | $\pi\rho \rightarrow \pi e^+e^-$, $a_1 \rightarrow \pi e^+e^-$, $h_1 \rightarrow \pi e^+e^-$ |
| $K^+K^-\pi^0$ | none | $K^{*-}, K^{*+},$ $\rho', \rho'', \phi, \dots$ | $K^{*\pm}K^{\mp} \rightarrow e^+e^-$, $\pi^0\phi \rightarrow e^+e^-$ |
| 4π | none | $a_1, \rho, \omega,$ ϕ, ρ', \dots | $a_1^{\pm}\pi^{\mp} \rightarrow e^+e^-$, $\omega\pi^0 \rightarrow e^+e^-$, $\phi\pi^0 \rightarrow e^+e^-$, $\rho^+\rho^- \rightarrow e^+e^-$ |
| 4π | π | $b_1, \omega, \rho, \rho', \dots$ | $b_1 \rightarrow \pi e^+e^-$, $\pi\omega \rightarrow \pi e^+e^-$, $\pi^0\phi \rightarrow e^+e^-$ |
| $K^+K^-\pi^0\pi^0$ | none | $K^{*-}, K^{*+}, \rho',$ ρ'', ϕ', \dots | $K^{*-}K^{*+} \rightarrow e^+e^-$ |
| $K^+K^-\pi^+\pi^-$ | none | $K^{*0}, K^{*0},$ $\rho', \rho'', \phi', \dots$ | $K^{*0}K^{*0} \rightarrow e^+e^-$ |
| 6π | none | a_1, ρ, ρ', \dots | $a_1^{\pm}a_1^{\mp} \rightarrow e^+e^-$ |