Name: PHY625 (Spring 2011), 02/14/11

# Homework Assignment #3

(*Due Date*: Wed, Feb. 23, 9:10am, in class)

3.1 *Saturation in the Fermi Gas Model of the Nucleus* *(2+1+3+2+2 pts.)*
In the Fermi Gas Model (FGM) the nucleus is approximated by $A$ non-interacting nucleons in a volume $V$ (you can assume it to be spherical, $V = 4\pi R_A^3/3$). The one- and two-particle density matrices in the nuclear ground state, $|0\rangle$, are given by

$$\rho_{00}^{(1)}(\vec{r};\vec{r}\,') = \frac{1}{VA}\sum_{j=1}^{A} \exp[i\,\vec{k}_j \cdot (\vec{r}\,' - \vec{r})] \quad (1)$$

$$\rho_{00}^{(2)}(\vec{r}_1,\vec{r}_2) \equiv \rho_{00}^{(2)}(\vec{r}_1,\vec{r}_2;\vec{r}_1,\vec{r}_2) = \frac{1}{16V^2}[10g_-(x) + 6g_+(x)] \quad (2)$$

with $x = k_F r$, $\vec{r} = \vec{r}_2 - \vec{r}_1$ and

$$g_\pm(x) = 1 \pm \left[\frac{3}{x^2}\left(\frac{\sin x}{x} - \cos x\right)\right]^2 \ . \quad (3)$$

The goal is to evaluate the ground-state expectation value (GSEV) of the $A$-body nuclear Hamiltonian,

$$\langle 0|\hat{H}|0\rangle = \langle 0|\hat{T}|0\rangle + \langle 0|\hat{V}_{12}|0\rangle \quad (4)$$

and find its minimum.

(a) Use the Taylor expansion of $\sin x$ and $\cos x$ to show that $g_-(x \to 0) = 0$ ("Pauli repulsion") and $g_+(x \to 0) = 2$ ("Pauli attraction").

(b) Use plotting software to graph $\rho_{00}^{(2)}(r)$ for $r = [0, 5]$ fm using $k_F = 265$ MeV.

(c) Calculate the GSEV of the kinetic energy of the nucleus by evaluating $\langle 0|\hat{T}|0\rangle = A\ \mathrm{tr}\left(\hat{T}\hat{\rho}_{00}^{(1)}\right)$ in momentum space; express the result as a function of $R_A$ and $A$ (eliminate the $V$ dependence). Start by calculating the Fourier transform of $\rho_{00}^{(1)}(\vec{r},\vec{r}\,')$.

(d) Calculate the GSEV of the potential energy in coordinate space,

$$\langle 0|\hat{V}_{12}|0\rangle = \frac{A(A-1)}{2}\int_V d^3r_1 \int_V d^3r_2\ \rho_{00}^{(2)}(r)\ V_{12}(r)\ , \quad (5)$$

assuming a pairwise 2-body potential $V_{12}(r) = -V_0\ \Theta(b - r)$. You may ignore 25% variations in $\rho_{00}^{(2)}(r)$ and use

$$\frac{1}{V^2}\int_V d^3r_1 \int_V d^3r_2\ \Theta(b - |\vec{r}_2 - \vec{r}_1|) = \left(\frac{b}{R_A}\right)^3 \left(1 - \frac{9}{16}\frac{b}{R_A} + \frac{1}{32}\left(\frac{b}{R_A}\right)^3\right) \quad (6)$$

for $R_A > b/2$.

(e) Plot the results for kinetic and potential energy, as well as their sum, as a function of $R_A$ for $A = 50$ and $A = 200$. From both graphs estimate the minimum and corresponding saturation density.