Name: PHY625 (Spring 2011), 02/02/11

Homework Assignment #2

(*Due Date*: Friday, Feb. 11, 9:10am, in class)

2.1 *Liquid Drop Model (LDM) of Nuclei* *(1+3+1+2+1+2 pts.)*

The empirical Weizsäcker formula characterizing the binding energy of nuclei within the LDM is given by

$$E_B = \sum_{i=1}^{5} E_i = -a_1 A + a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} + a_4 \frac{(A-2Z)^2}{A} + a_5 \frac{\lambda}{A^{3/4}} \qquad (1)$$

with A: nuclear mass number, Z: nuclear charge (in units of e) and positive coefficients a_i (in units of MeV).

(a) Briefly discuss the physical motivation for each term.

(b) Calculate the value of a_3 by modeling a nucleus of given A, Z and radius $R_A = r_0\, A^{1/3}$ (with $r_0 = 1.15\,\text{fm}$) by a uniform spherical charge distribution, $\rho_c(r) = \rho_{c,0}\, \Theta(R_A - r)$. First use Poisson's law, $\Delta V(r) = -\rho(r)$, to find the static electric potential for $r < R_A$ (make sure it is continuous with the point-charge potential $V(r) = Ze/4\pi r$ for $r > R_A$). Then calculate the electric potential energy,

$$U(A,Z) = -\frac{1}{2} \int d^3 r\ V_c(r)\ \rho_c(r)\ , \qquad (2)$$

to evaluate a_3.

(c) Derive the value Z^* for the charge which minimizes the binding energy for fixed A. Plot the resulting valley of stability, $Z^*(N)$.

(d) Plot $-E_B(A)$ using $Z^*(A)$ from part (c) by subsequently adding in the terms of the LDM in numerical order as written above up to including $i = 4$. For each curve determine the value of A, if any, for which $-E_B(A)$ is maximal. Here and below use $a_1 = 15.75$ MeV, $a_2 = 17.8$ MeV, $a_3 = 0.71$ MeV, $a_4 = 23.7$ MeV, $a_5 = 34$ MeV and λ=-1,0,1 for e-e,e-o,o-o nuclei, respectively.

(e) Use the LDM to calculate the the binding energies of $^{60}_{34}Fe$ and $^{120}_{68}Te$, and compare to the experimental values of $E_B/A = -8.756$ MeV and -8.477 MeV, respectively.

(f) How much energy is set free when fissioning a $^{120}_{68}Te$ nucleus into two $^{60}_{34}Fe$ nuclei (once the latter are far separated)? Why does a $^{120}_{68}Te$ nucleus then not fission spontaneously into two $^{60}_{34}Fe$ nuclei? (*Hint: compute the Coulomb repulsion between two* $^{60}_{34}Fe$ *nuclei when their surfaces just touch*).