Name: PHY625 (Fall 2014), 11/19/14

Homework Assignment #8

(*Due Date*: Monday, December 1, 01:50 pm, in class)

8.1 *Heavy Quarkonium Spectroscopy* *(6 pts.)*
The static QCD potential between a heavy quark and antiquark can be represented by

$$V_{Q\bar{Q}}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r \tag{1}$$

with a string tension of $\sigma \simeq 1$ GeV/fm and $\alpha_s \simeq 1/3$. The ground-state charmonium and bottomonium have masses of $M(J/\Psi)$=3.10 GeV and $M(\Upsilon(1S))$=9.46 GeV, respectively.

(a) Neglect the confining term in the potential and use the semiclassical approximation to the $Q\bar{Q}$ Hamiltonian for the relative motion, $H = T+V$, to find the Coulombic $1S$ bound-state radii and total energies. Adjust the charm- and bottom-quark masses, $m_{c,b}$, to recover the experimental ground-state masses. Use the obtained radii to estimate the "correction" to the binding energy due to the confining term.

(b) Repeat part (a) by numerically solving the semicalssical expression for r and determining the total masses of the two $1S$ states.

(c) Generalize the expression for the electromagnetic hyperfine splitting,

$$\Delta E_{hf} = -\frac{2}{3}\,\vec{\mu}_1 \cdot \vec{\mu}_2\,|\psi(0)|^2 \tag{2}$$

($\vec{\mu}_i = e_i\vec{\sigma}_i/2m_i$, $|\psi(0)|^2$: onium wave function overlap at the origin [approximate this with the hydrogen expression]), to color charges by replacing $\alpha_{em} \to \frac{4}{3}\alpha_s$. Prove $\vec{\sigma}_1 \cdot \vec{\sigma}_2 = 2\vec{S}^2 - 3$ and use this relation to calculate the hyperfine splitting between the ground-state bottomonia $\Upsilon(1S)$ (spin S=1) and η_b (spin S=0) and the ground-state charmonia $J/\psi(S$=1) and $\eta_c(S$=0) using your results from part (b). Compare ΔE_{hf} to the measured J/ψ-η_c mass splitting, $M_{J/\psi} - M_{\eta_c} \simeq 0.12$ GeV.

8.2 *QCD Vacuum: Constituent-Quark Mass and Quark Condensate* *(4 pts.)*
In the Nambu Jona-Lasinio model, the gap equation for the constituent light-quark mass is given by

$$m_q^* = m_q + 4N_cN_fG \int \frac{d^3p}{(2\pi)^3}\frac{m_q^*}{(\vec{p}^2 + (m_q^*)^2)^{1/2}}\; F(p)^2 \tag{3}$$

where $m_q \simeq 5$ MeV is the bare quark mass, G the chiral 4-point q-$\bar{q}$ coupling constant and $F(p) = \Lambda^2/(\Lambda^2 + \vec{p}^2)$ an effective formfactor. Set Λ=0.6 GeV.

(a) Determine numerically the value of the coupling constant, G, for which the constituent quark mass becomes m_q^*=350 MeV. How large is the pertinent quark-antiquark condensate (in [fm^{-3}])?

(b) For cold quark matter and m_q=0, compute and plot $\chi_0(\mu_q) = m_q^*(\mu_q)/2G$, to numerically determine the critical chemical potential and quark density for which m_q^* and $\langle\bar{q}q\rangle$ vanish. What is the pertinent critical baryon density?