

Homework Assignment #7

(Due Date: Wednesday, November 12, 01:50 pm, in class)

7.1 Kinematics in Hadron Scattering (2 pts.)

- (a) Fully ionized ^{12}C nuclei are directed toward a ^{208}Pb target. What is the minimal kinetic energy (in [MeV]) for a carbon nucleus to “touch” a lead nucleus in a central collision (use the nuclear radius formula $R_A = 1.2 A^{1/3}$ fm in this estimate)?
- (b) High-energy protons are directed toward a hydrogen target. What is the minimal momentum (in [GeV]) to being able to create antiprotons? (*hint: watch out for baryon number conservation*)

7.2 Isospin Invariance of π - N Interactions (4 pts.)

A simple π - N - N interaction Lagrangian may be written as

$$\mathcal{L}_{\pi NN} = g_{\pi NN} \bar{\psi}_N i\gamma_5 \vec{\pi} \cdot \vec{\tau} \psi_N , \quad (1)$$

where the arrows indicate vectors in isospin space (ψ_N : isospin doublet, $\vec{\tau}$: isospin Pauli matrices).

- (a) Show that the above Lagrangian is invariant under rotations in isospin space by applying an infinitesimal rotation to all field operators about an angle $\epsilon \ll 1$, $\psi_N \rightarrow (1 - i\vec{\epsilon} \cdot \vec{\tau}/2)\psi_N$, $\vec{\pi} \rightarrow (1 + \vec{\epsilon} \times) \vec{\pi}$, and verifying the invariance to leading order in ϵ .
- (b) Show that the above Lagrangian predicts relations between the physical couplings as

$$g_{pp\pi^0} = -g_{nn\pi^0} = \frac{1}{\sqrt{2}}g_{pn\pi^+} = \frac{1}{\sqrt{2}}g_{pn\pi^-} \quad (2)$$

where the physical (charged and neutral) pion fields are related to the cartesian ones, $\vec{\pi} = (\pi_1, \pi_2, \pi_3)$, as $\pi^\pm = (\pi_1 \pm i\pi_2)/\sqrt{2}$ and $\pi^0 = \pi_3$.

7.3 Hadron Quantum Numbers (2+1+1 pts.)

Consider the constituent-quark model for hadron structure.

- (a) In the meson sector ($\bar{q}q$ states), $SU(3)$ -flavor multiplets are characterized by different parity (P) and particle-antiparticle conjugation (C) eigenvalues. Explain how these emerge (for neutral mesons) from the total quark spin, S , and relative angular momentum, L .
- (b) Identify the members of the pseudoscalar- and vector-meson nonets, along with their masses as quoted by the Particle Data Group. Comment on the main origin of the non-degeneracies within the multiplets, i.e., on the $SU(3)$ -flavor symmetry breaking.
- (b) Give two empirical evidences that suggest an additional intrinsic quark quantum number (beyond spin and flavor).