Name: PHY625 (Fall 2014), 10/24/14

Homework Assignment #6

(*Due Date*: Monday, Nov. 03, 01:50 pm, in class)

6.1 *Relativistic Mean-Field Theory of Nuclear Matter* *(1+3+2+1+3+1 pts.)*

In mean-field approximation, the Hamiltonian, $\mathcal{H} = \pi\dot{q} - \mathcal{L}$, of the σ-ω model reads

$$\mathcal{H}_{\rm MFA} = \frac{1}{2}m_\sigma^2\phi_0^2 - \frac{1}{2}m_\omega^2 V_0^2 + g_\omega V_0 \hat{\rho}_N + \frac{1}{V}\sum_\alpha E_k^*(b_\alpha^\dagger b_\alpha + d_\alpha^\dagger d_\alpha) + \delta\mathcal{H} \ . \tag{1}$$

In numerical calculations use M_N=939 MeV, m_σ=550 MeV, m_ω=782.6 MeV, ρ_0=0.16 fm^{-3}.

(a) Show that the energy density takes the form

$$\epsilon(\rho_N;\phi_0) = \frac{1}{2}m_\sigma^2\phi_0^2 + \frac{1}{2}\frac{g_\omega^2}{m_\omega^2}\rho_N^2 + d_{\rm SI}\int_0^{k_F}\frac{d^3k}{(2\pi)^3}E_k^* \tag{2}$$

with $E_k^* = [(M_N^*)^2 + \vec{k}^2]^{1/2}$, $M_N^* = M_N - g_\sigma\phi_0$, $\rho_N = 2k_F^3/3\pi^2$, $d_{\rm SI}$=4.

(b) Using $dE = -PdV$, derive the expression for the pressure, $P(\rho_N;\phi_0)$.

(c) Derive the selfconsistency equation for the scalar mean field by minimizing the energy at fixed A and V, and rewrite it in terms of the effective nucleon mass as

$$M_N^* = M_N - \frac{g_\sigma^2}{m_\sigma^2}\rho_S \quad , \quad \rho_S = d_{\rm SI}\int_0^{k_F}\frac{d^3k}{(2\pi)^3}\frac{M_N^*}{E_k^*} \ . \tag{3}$$

(d) Show that energy density and pressure can be written as

$$\varepsilon(\rho_N) = \frac{g_\sigma^2}{2m_\sigma^2}\rho_S^2 + \frac{g_\omega^2}{2m_\omega^2}\rho_N^2 + \langle E_k^*\rangle\rho_N \ , \ \ P(\rho_N) = -\frac{g_\sigma^2}{2m_\sigma^2}\rho_S^2 + \frac{g_\omega^2}{2m_\omega^2}\rho_N^2 + \langle\frac{\vec{k}^2}{3E_k^*}\rangle\rho_N \ . \tag{4}$$

(e) Compute numerically and plot the binding energy $E_B/A = \varepsilon/\rho_N - M_N$ (in [MeV]) and pressure for ρ_N=0-2ρ_0 in steps of 0.1. For each ρ_N, start by finding the selfconsistent solution for M_N^* (and ρ_S) by numerical iteration of eqs. (3). Adjust g_σ and g_ω to obtain the empirical saturation point (minimum) at $E_B/A(\rho_0) \simeq -16$ MeV. Interpret any results for negative pressure.

(f) Replot your results of part (e) upon replacing ρ_S by ρ_N. Intepret your findings with regards to the mechanism for nuclear saturation.