Homework Assignment #4

(*Due Date*: Wednesday, Oct. 08, 01:50 pm, in class)

4.1 *Free and In-Medium Nucleon-Nucleon Scattering: T- and G-Matrix (2+3+3+1+2 pts.)*
The Lippmann-Schwinger equation (LSE) for the free NN scattering T-matrix reads

$$T(E;\vec{k}',\vec{k}) = V(\vec{k}',\vec{k}) + \int \frac{d^3p}{(2\pi)^3} V(\vec{k}',\vec{p})\ \frac{1}{E - p^2/M_N + i\eta}\ T(E;\vec{p},\vec{k}) \ , \tag{1}$$

where $\pm\vec{k}$ ($\pm\vec{k}'$) denotes the relative momentum of the incoming (outgoing) nucleons in the center of mass, E is the total "on-shell" energy, $E = k^2/M_N = k'^2/M_N$, M_N=940 MeV the nucleon mass and η infinitesimal (neglect spin-isospin except for nuclear densities).

(a) Using a partial-wave expansion for potential V and T-matrix,

$$X(\vec{k}',\vec{k}) = 4\pi \sum_{l=0}^{\infty} (2l+1)\ P_l(u_{k'k})\ X_l(k',k) \ , \tag{2}$$

as well as azimuthal symmetry, show that the LSE can be reduced to

$$T_l(E;k',k) = V_l(k',k) + \frac{2}{\pi} \int p^2 dp\ V_l(k',p)\ \frac{1}{E - p^2/M_N + i\eta}\ T_l(E;p,k) \ , \tag{3}$$

in each partial wave l. In eq. (2), $u_{k'k} = \cos(\alpha_{k'k})$ with $\alpha_{k'k} = \angle(\vec{k}',\vec{k})$; utilize the orthogonality of the Legendre polynomials,

$$\int du_{pk}\ P_{l'}(u_{k'p})\ P_l(u_{pk}) = \frac{2\delta_{ll'}}{2l+1}\ P_l(u_{k'k}) \ . \tag{4}$$

(b) Concentrate on the S-wave channel (l=0) and approximate the N-N interaction by an average attractive σ-meson exchange potential,

$$V_0^\sigma(k',k) = -\frac{g_\sigma^2}{4\pi} \frac{F_\sigma(k) F_\sigma(k')}{m_\sigma^2} \quad , \tag{5}$$

where a hadronic formfactor, $F_\sigma(k) = \Lambda_\sigma^2/(\Lambda_\sigma^2 + k^2)$, simulates the finite size of the hadronic vertex and ensures convergence of the 1-D LSE. Write down explicitly the first 3 terms of the Born series for the T-matrix and exploit the separability of the potential, $V(k',k) \equiv v(k')\, v(k)$, to resum the geometric series yielding

$$T_0(E;k',k) = \frac{V_0(k',k)}{1 - \Pi(E)} \quad , \quad \Pi(E) = \frac{2}{\pi} \int p^2 dp\ \frac{V_0(p)}{E - p^2/M_N + i\eta} \ . \tag{6}$$

(c) The differential cross section is given in terms of the T-matrix as

$$\frac{d\sigma}{d\Omega}(k,\Theta) = \frac{k^2}{v_{\rm rel}^2} \frac{|T(k,\Theta)|^2}{4\pi^2} \ . \tag{7}$$

(Θ: scattering angle). Show that for S-wave scattering the total cross section reads

$$\sigma_{\rm tot}^{l=0}(E) = 4\pi M_N^2 |T_0(E)|^2 \ . \tag{8}$$

Evaluate the "loop" function, $\Pi(E)$, utilizing the following decomposition into real and imaginary parts:

$$\int dp \frac{f(p)}{p_0^2 - p^2 + i\eta} = PP \int_0^\infty dp \frac{f(p)}{p_0^2 - p^2} + \int dp \ f(p) \ (-i\pi)\delta(p_0^2 - p^2) \tag{9}$$

(here: $p_0^2 = M_N E$). The principle-value (PP) integral for the real part of $\Pi(E)$ requires a numerical integration. To avoid numerical instabilities when integrating over the pole, use the following "regularization" trick

$$PP \int_0^\infty dp \frac{f(p)}{p_0^2 - p^2} = PP \int_0^\infty dp \frac{f(p) - f(p_0)}{p_0^2 - p^2} \ , \quad \text{since} \quad PP \int_0^\infty \frac{dp}{p_0^2 - p^2} = 0 \ . \tag{10}$$

Compute and plot the cross section in [mb]=[0.1 fm^2] from threshold to E=150 MeV using m_σ=550 MeV. Adjust g_σ and Λ_σ to obtain a cross section of ca. 160-180 mb at $E \simeq$ 10 MeV and ca. 20 mb for $E \simeq$120 MeV (*hint: look for $g_\sigma \simeq$ 2-3 and $\Lambda_\sigma \simeq$1000-1500 MeV*). Receive an extra *0.5 pts.* if you include the experimental *pp* cross section data in your plot.

(d) Slowly increase the coupling g_σ and monitor the real part of the T-matrix close to threshold. Comment on and interpret any qualitative change you observe.

(e) Compute the in-medium N-N G-matrix (for total pair momentum P=0) using g_σ from part (c), by implementing a Pauli-Blocking factor, $[1 - f(\epsilon_p; \mu_N, T)]^2$, into the integral of the LSE (3), where

$$f(\epsilon_p; \mu_N, T) = \frac{1}{\exp[(\epsilon_p - \mu_N)/T] + 1} \tag{11}$$

with $\epsilon_p = p^2/2M_N$ and chemical potential $\mu_N = k_F^2/2M_N$ (k_F: Fermi momentum). Plot the G-matrix and S-wave cross section vs. E for k_F=265 MeV and temperatures T=0.5 MeV and T=5 MeV. Interpret your results.