Homework Assignment #4

(Due Date: Wednesday, Oct. 08, 01:50 pm, in class)

4.1 Free and In-Medium Nucleon-Nucleon Scattering: T- and G-Matrix (2+3+3+1+2 pts.)

The Lippmann-Schwinger equation (LSE) for the free NN scattering $T$-matrix reads

$$ T(E; \vec{k}', \vec{k}) = V(\vec{k}', \vec{k}) + \int \frac{d^3p}{(2\pi)^3} V(\vec{k}', \vec{p}) \frac{1}{E - p^2/M_N + i\eta} T(E; \vec{p}, \vec{k}) , $$

(1)

where $\pm \vec{k}$ ($\pm \vec{k}'$) denotes the relative momentum of the incoming (outgoing) nucleons in the center of mass, $E$ is the total “on-shell” energy, $E = k^2/M_N = k'^2/M_N$, $M_N = 940$ MeV the nucleon mass and $\eta$ infinitesimal (neglect spin-isospin except for nuclear densities).

(a) Using a partial-wave expansion for potential $V$ and $T$-matrix,

$$ X(\vec{k}', \vec{k}) = 4\pi \sum_{l=0}^{\infty} (2l + 1) P_l(u_{k'k}) X_l(k', k) , $$

(2)

as well as azimuthal symmetry, show that the LSE can be reduced to

$$ T_l(E; k', k) = V_l(k', k) + \frac{2}{\pi} \int p^2 dp \ V_l(k', p) \frac{1}{E - p^2/M_N + i\eta} T_l(E; p, k) , $$

(3)

in each partial wave $l$. In eq. (2), $u_{k'k} = \cos(\alpha_{k'k})$ with $\alpha_{k'k} = \angle(\vec{k}', \vec{k})$; utilize the orthogonality of the Legendre polynomials,

$$ \int du_{pk} \ P_l(u_{k'p}) \ P_l(u_{pk}) = \frac{2\delta_{ll'}}{2l+1} P_{l'}(u_{k'k}) . $$

(4)

(b) Concentrate on the $S$-wave channel ($l=0$) and approximate the $N$-$N$ interaction by an average attractive $\sigma$-meson exchange potential,

$$ V_0^\sigma(k', k) = \frac{g_\sigma^2}{4\pi} \frac{F_\sigma(k) F_\sigma(k')}{m_\sigma^2} , $$

(5)

where a hadronic formfactor, $F_\sigma(k) = \Lambda_\sigma^2/(\Lambda_\sigma^2 + k^2)$, simulates the finite size of the hadronic vertex and ensures convergence of the 1-D LSE. Write down explicitly the first 3 terms of the Born series for the $T$-matrix and exploit the separability of the potential, $V(k', k) \equiv v(k') v(k)$, to resum the geometric series yielding

$$ T_0(E; k', k) = \frac{V_0(k', k)}{1 - \Pi(E)} , \quad \Pi(E) = \frac{2}{\pi} \int p^2 dp \ \frac{V_0(p)}{E - p^2/M_N + i\eta} . $$

(6)

(c) The differential cross section is given in terms of the $T$-matrix as

$$ \frac{d\sigma}{d\Omega}(k, \Theta) = \frac{k^2}{v_{rel}} \frac{|T(k, \Theta)|^2}{4\pi^2} . $$

(7)
Show that for $S$-wave scattering the total cross section reads

$$\sigma^{l=0}_{\text{tot}}(E) = 4\pi M_N^2 |T_0(E)|^2 .$$  \hfill (8)

Evaluate the “loop” function, $\Pi(E)$, utilizing the following decomposition into real and imaginary parts:

$$\int dp \frac{f(p)}{p_0^2 - p^2 + i\eta} = PP \int_0^\infty dp \frac{f(p)}{p_0^2 - p^2} + \int dp f(p) (-i\pi)\delta(p_0^2 - p^2) \quad \hfill (9)$$

(here: $p_0^2 = M_N E$). The principle-value ($PP$) integral for the real part of $\Pi(E)$ requires a numerical integration. To avoid numerical instabilities when integrating over the pole, use the following “regularization” trick

$$PP \int_0^\infty dp \frac{f(p)}{p_0^2 - p^2} = PP \int_0^\infty dp \frac{f(p) - f(p_0)}{p_0^2 - p^2} , \quad \text{since} \quad PP \int_0^\infty \frac{dp}{p_0^2 - p^2} = 0 . \quad \hfill (10)$$

Compute and plot the cross section in $[\text{mb}]=[0.1 \text{ fm}^2]$ from threshold to $E=150 \text{ MeV}$ using $m_\sigma=550 \text{ MeV}$. Adjust $g_\sigma$ and $\Lambda_\sigma$ to obtain a cross section of ca. 160-180 mb at $E\approx 10 \text{ MeV}$ and ca. 20 mb for $E\approx 120 \text{ MeV}$ ($hint:\ look\ for\ g_\sigma \approx 2-3\ and\ \Lambda_\sigma \approx 1000-1500 \text{ MeV}$). Receive an extra 0.5 pts. if you include the experimental $pp$ cross section data in your plot.

(d) Slowly increase the coupling $g_\sigma$ and monitor the real part of the $T$-matrix close to threshold. Comment on and interpret any qualitative change you observe.

(e) Compute the in-medium $N-N G$-matrix (for total pair momentum $P=0$) using $g_\sigma$ from part (e), by implementing a Pauli-Blocking factor, $[1 - f(\epsilon_p; \mu_N, T)]^2$, into the integral of the LSE (3), where

$$f(\epsilon_p; \mu_N, T) = \frac{1}{\exp[(\epsilon_p - \mu_N)/T] + 1} \quad \hfill (11)$$

with $\epsilon_p = p^2/2M_N$ and chemical potential $\mu_N = k_F^2/2M_N$ ($k_F$: Fermi momentum). Plot the $G$-matrix and $S$-wave cross section vs. $E$ for $k_F=265 \text{ MeV}$ and temperatures $T=0.5 \text{ MeV}$ and $T=5 \text{ MeV}$. Interpret your results.