## Homework Assignment \#3

(Due Date: Fri, Sept. 26, 01:50pm, in class)

### 3.1 Nuclear Binding in the Fermi Gas Model of the Nucleus <br> $(1+1+2+3+3$ pts. $)$

 In the Fermi Gas Model (FGM) the nuclear wave function is approximated by $A$ noninteracting nucleons in a volume $V$ (assume it to be spherical, $V=4 \pi R_{A}^{3} / 3$ ). The corresponding one- and two-particle density matrices in the nuclear ground state, $|0\rangle$, read$$
\begin{align*}
\rho_{00}^{(1)}\left(\vec{r} ; \vec{r}^{\prime}\right) & =\frac{1}{V A} \sum_{j=1}^{A} \exp \left[i \vec{k}_{j} \cdot\left(\vec{r}^{\prime}-\vec{r}\right)\right]  \tag{1}\\
\rho_{00}^{(2)}\left(\vec{r}_{1}, \vec{r}_{2}\right) & \equiv \rho_{00}^{(2)}\left(\vec{r}_{1}, \vec{r}_{2} ; \vec{r}_{1}, \vec{r}_{2}\right)=\frac{1}{16 V^{2}}\left[10 g_{-}(x)+6 g_{+}(x)\right] \tag{2}
\end{align*}
$$

with $x=k_{F} r, \vec{r}=\vec{r}_{2}-\vec{r}_{1}$ and

$$
\begin{equation*}
g_{ \pm}(x)=1 \pm\left[\frac{3}{x^{2}}\left(\frac{\sin x}{x}-\cos x\right)\right]^{2} \tag{3}
\end{equation*}
$$

In the following, the objetive is to evaluate the ground state expectation value (GSEV) of the $A$-body nuclear Hamiltonian,

$$
\begin{equation*}
\langle 0| \hat{H}|0\rangle=\langle 0| \hat{T}|0\rangle+\langle 0| \hat{V}_{12}|0\rangle, \tag{4}
\end{equation*}
$$

and find its minimum.
(a) Use a Taylor expansion of $\sin x$ and $\cos x$ to show that $g_{-}(x \rightarrow 0)=0$ ("Pauli repulsion") and $g_{+}(x \rightarrow 0)=2$ ("Pauli attraction").
(b) Plot $\rho_{00}^{(2)}(r)$ for $r=[0,5] \mathrm{fm}$ at $k_{F}=265 \mathrm{MeV}$.
(c) Calculate the GSEV of the kinetic energy of the nucleus, $\langle 0| \hat{T}|0\rangle=A \operatorname{tr}\left(\hat{T} \hat{\rho}_{00}^{(1)}\right)$, in momentum space; express the result as a function of $R_{A}$ and $A$ (i.e., eliminate the $V$ dependence). Start by calculating the Fourier transform of $\rho_{00}^{(1)}\left(\vec{r}, \vec{r}^{\prime}\right)$.
(d) Numerically compute the GSEV of the potential energy in coordinate space,

$$
\begin{equation*}
\langle 0| \hat{V}_{12}|0\rangle=\frac{A(A-1)}{2} \int_{V} d^{3} r_{1} \int_{V} d^{3} r_{2} \rho_{00}^{(2)}(r) V_{12}(r), \tag{5}
\end{equation*}
$$

with a 2-body potential $V_{12}(r)=-V_{0} \Theta(b-r)$. For $R_{A}>\frac{b}{2}$ compare your results to

$$
\begin{equation*}
\langle 0| \hat{V}_{12}|0\rangle\left(R_{A}\right) \simeq-\frac{A^{2} V_{0}}{2}\left(\frac{b}{R_{A}}\right)^{3}\left[1-\frac{9}{16} \frac{b}{R_{A}}+\frac{1}{32}\left(\frac{b}{R_{A}}\right)^{3}\right] \tag{6}
\end{equation*}
$$

(you can use $A=100, V_{0}=25 \mathrm{MeV}, b=2 \mathrm{fm}$ ).
(e) Plot the results for kinetic and potential energy, as well as their sum, as a function of $R_{A}$ for $A=50$ and $A=200$, using $b=2 \mathrm{fm}$ and $V_{0}=25 \mathrm{MeV}$. Determine the minimum of the GSEV in each case and extract the corresponding binding energy per nucleon and nuclear density. Do your results agree with experiment? What important ingredient is missing? What problem arises when using a potential with short-range repulsion?

