Name: PHY625 (Fall 2014), 09/10/14

# Homework Assignment #2

(*Due Date*: Wednesday, September 17, 01:50 pm, in class)

2.1 *Electron Scattering off Nuclei* *(6 pts.)*

In Born approximation, the differential cross section for an ultrarelativistic electron of energy $E \gg m_e$ ($m_e$=0.511 MeV) elastically scattering off an electrostatic potential $A_0(r)$ is given by (neglecting any intrinsic spin dependencies)

$$\frac{d\sigma}{d\Omega} = \frac{E^2}{4\pi^2}\left[e\int d^3r A_0(r)\ \mathrm{e}^{i\vec{q}\cdot\vec{r}}\right]^2 , \tag{1}$$

where $\vec{q} \equiv \vec{p}_f - \vec{p}_i$: momentum transfer from the electron, and $e$: electron charge.
(use units of MeV or fm, as appropriate, with $\hbar c$=197.33 MeVfm)

(a) Show that the relation between the magnitude of the momentum transfer, $q = |\vec{q}|$, and the scattering angle, $\theta$, is given by $q = 2E\sin(\theta/2)$.

(b) Using Possion's equation, $\vec{\nabla}^2 A_0 = -Ze\hat{\rho}_{\mathrm{ch}}(r)$, as well as $\vec{\nabla}^2\mathrm{e}^{i\vec{q}\cdot\vec{r}} = -\vec{q}^{\,2}\mathrm{e}^{i\vec{q}\cdot\vec{r}}$ and partial integration, to show that the cross section takes the form

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma_R}{d\Omega}\right)|F(q^2)|^2 , \tag{2}$$

where $(d\sigma_R/d\Omega) = Z^2\alpha^2/(4E^2\sin^4(\theta/2))$ is the Rutherford cross section for a point charge $Z$, $\alpha = e^2/4\pi$, and the formfactor of the charge distribution defined as

$$F(q) = \int d^3r\hat{\rho}_{\mathrm{ch}}(r)\mathrm{e}^{i\vec{q}\cdot\vec{r}} . \tag{3}$$

(c) Sketch the angular dependence of the cross section for a uniform charge distribution, $\hat{\rho}_{\mathrm{ch}}(r) = C\ \theta(R_0 - r)$, with radius $R_0$=5 fm ($C$: constant) and electron energies $E$=$E_i$=$E_f$=150 MeV. Compare it to the result for a point charge, $\rho_{ch}(r) = Ze\,\delta^{(3)}(\vec{r})$. Start by determining the constant $C$ to normalize $\hat{\rho}_{\mathrm{ch}}(r)$ to one.

2.2 *Liquid Drop Model (LDM) of Nuclei* *(4 pts.)*

The empirical Weizsäcker formula for the binding energy of nuclei is given by

$$E_B = \sum_{i=1}^{5} E_i = a_1 A + a_2 A^{2/3} + a_3\frac{Z^2}{A^{1/3}} + a_4\frac{(A-2Z)^2}{A} + a_5\frac{\lambda}{A^{3/4}} \tag{4}$$

with $A$: nuclear mass number, $Z$: nuclear charge in units of $e$, $a_1$=−15.75 MeV, $a_2$=17.8 MeV, $a_3$=0.71 MeV, $a_4$=23.7 MeV and $a_5$=34 MeV with $\lambda$=-1,0,1 for e-e,e-o,o-o nuclei.

(a) Briefly discuss the physical motivation ($A$ and $Z$ dependence) for each term.

(b) Derive the value $Z^*$ for the charge which minimizes the binding energy for fixed $A$. Plot the resulting "valley of stability", $Z^*(N)$.

(c) Plot $|E_B(A)|/A$ using $Z^*(A)$ from part (b) by subsequently adding the terms of the LDM in numerical order up to (including) $i$=4. In each step estimate the value for $A$ (if any) for which $|E_B(A)|/A$ is maximal.

(d) How much fission energy is released in ${}^{235}_{92}U \to {}^{144}_{56}Ba + {}^{89}_{36}Kr + 2n$? How many kg of ${}^{235}U$ are spent in 1 year at an output of 1000 MW (neglect any losses; 1 eV=$1.6\cdot 10^{-19}$J)?