Name: PHY625 (Fall 2013), 11/18/13

Homework Assignment #8

(*Due Date*: Monday, Dec. 02, 01:50 pm, in class)

8.1 *Heavy-Quark Potential and Quarkonium Spectrum* *(2+1+2+2 pts.)*

The static QCD potential between a heavy quark and antiquark is well represented by

$$V_{Q\bar{Q}}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r \tag{1}$$

with a string tension of σ=1 GeV/fm. The ground and first excited state of the Upsilon ($b\bar{b}$) spectrum have masses of $M(\Upsilon(1S))$=9.46 GeV and $M(\Upsilon(2S))$=10.02 GeV.

(a) Assuming α_s=0.5 and neglecting the linear confining term in the potential, use the hydrogen expression for the bound-state energy to estimate the bottomonium binding energies. Can you find a reasonable value for the bottom-quark mass, m_b, to establish approximate agreement with the above masses?

(b) Estimate the correction to the 2 binding energies due to the confining term assuming hydrogen-like radii of the 2 Υ states. Is each correction "large" or "small"?

(c) Generalize the expression for the electromagnetic hyperfine splitting,

$$\Delta E_{hf} = -\frac{2}{3}\,\vec{\mu}_1\cdot\vec{\mu}_2\,|\psi(0)|^2 \tag{2}$$

($\vec{\mu}_i = e_i\vec{\sigma}_i/2m_i$, $|\psi(0)|^2$: onium wave function overlap at the origin; for the $1S$ ground state $\psi(r) = (\pi a_0^3)^{-1/2}\mathrm{e}^{-r/a_0}$), to color charges by replacing $\alpha_{em} \to \frac{4}{3}\alpha_s$. Prove $\vec{\sigma}_1\cdot\vec{\sigma}_2 = 2\vec{S}^2 - 3$; use this to predict the hyperfine splitting between the ground-state bottomonia $\Upsilon(1S)$ (spin S=1) and η_b (spin S=0). Redo this for the ground-state charmonia J/ψ (S=1) and η_c (S=0), using m_c=1.8 GeV and a size of 0.4 fm, and compare ΔE_{hf} to the measured J/ψ-η_c mass splitting, $M_{J/\psi} - M_{\eta_c}$=(3097−2980) MeV≃120 MeV.

(d) Use the semiclassical approximation to obtain the Coulombic binding energy, radius and heavy-quark velocity of topomonium (top quark mass m_t=173 GeV). How many revolutions does the system complete before a top quark decays (Γ_t≃3 GeV)?

8.2 *Constituent-Quark Mass and Quark Condensate* *(3 pts.)*

In the Nambu Jona-Lasinio model, the gap equation for the constituent light-quark mass is

$$m_q^* = m_q + 4N_cN_fG\int\frac{d^3p}{(2\pi)^3}\frac{m_q^*}{(\vec{p}^2+(m_q^*)^2)^{1/2}}\;F(p)^2 \tag{3}$$

where $m_q \simeq 5$ MeV is the bare quark mass, G the 4-point q-$\bar{q}$ coupling constant and $F(p) = \Lambda^2/(\Lambda^2+\vec{p}^2)$ an effective formfactor. Set Λ=0.6 GeV. Compute numerically the value of the coupling constant, G, for which the constituent quark mass acquires a value of 350 MeV. How large (in [fm^{-3}]) is the quark-antiquark condensate in this case?