Name: PHY625 (Fall 2013), 11/08/13

Homework Assignment #7

(*Due Date*: Monday, Nov. 18, 01:50 pm, in class)

7.1 *Electron Scattering and Nucleon Structure* *(2+1+2+2+2+1 pts.)*

In Dirac theory the cross section for the elastic scattering of an electron (with initial and final 4-momenta $(E,\vec{k})$ and $(E',\vec{k}')$, respectively) off a spin-$\frac{1}{2}$ point particle of mass M is given by

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4(\theta/2)} \frac{E'}{E} \left(\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2} \sin^2\frac{\theta}{2} \right) \tag{1}$$

where $\theta = \angle(\vec{k}, \vec{k}')$ is the scattering angle and $q = (\nu, \vec{q})$ the 4-momentum transfer (the finite electron mass has been neglected, $m_e \to 0$).

(a) Show that, for fixed incident lab energy E, the scattering angle is the only independent variable, by expressing E' and q^2 in terms of θ.

(b) In elastic e^-p scattering, the proton structure is characterized by electric and magnetic formfactors, $G_{E,M}$. Write down the accordingly modified cross section, known as "Rosenbluth cross section", and sketch the empirical behavior of $G_{E,M}(q^2)$.

(c) Take the limit of a non-relativistic target, i.e., $M_p^2 \gg |q^2|$, to show that $\nu \to 0$ and that the Rosenbluth cross section becomes

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4(\theta/2)} \; \cos^2(\theta/2) \; G_E^2(\vec{q}^{\,2}) \; . \tag{2}$$

What is the difference to the 1911 Rutherford cross section and where does this difference come from?

(d) Recalling that $G_E(\vec{q}) = \int d^3r \; e^{i\vec{q}\cdot\vec{r}} \rho_{\rm ch}(\vec{r})$ where $\rho_{\rm ch}(\vec{r})$ is the target's charge density, show that for small $\vec{q}$ and spherically symmetric $\rho_{\rm ch}$ one has $G_E(\vec{q}) \simeq 1 - \frac{1}{6}\vec{q}^{\,2}\langle R_p^2 \rangle$ and thus

$$\langle R_p^2 \rangle = -6 \frac{dG_E}{d\vec{q}^{\,2}} (\vec{q}^{\,2} \to 0) \tag{3}$$

(e) Empirically, one finds $G_E(q^2) = \Lambda^4/(\Lambda^2 - q^2)^2$ with Λ=0.84 GeV. Use this to calculate the proton's charge radius (in [fm]).

(f) Give a physical interpretation of the empirical form of the electric formfactor.