Homework Assignment #6

(*Due Date*: Wednesday, Nov. 06, 01:50 pm, in class)

6.1 *Translation Group* *(3 pts.)*
Consider the properties of a wave function, $\psi(\vec{r})$, in 3-dimensional space under simple translations, $\mathcal{T}\vec{r} = \vec{r} + \vec{a}$.

(a) Explain why the set of all translations form a Lie group. What are its basis elements?

(b) If translations are a Symmetry Group (i.e., leave physical laws invariant), show that its representations, U, defined by $U(\mathcal{T})|\psi\rangle = |\psi'\rangle$, commute with the Hamiltonian.

(c) Construct the explicit form of U in coordinate space for an infinitesimal translation and generalize it to finite translations. Which conservation law emerges?

6.2 *Isospin Invariance of π-N Interactions* *(4 pts.)*
A simple π-N-N interaction Lagrangian may be written as

$$\mathcal{L}_{\pi NN} = g_{\pi NN}\ \bar{\psi}_N\, i\gamma_5\, \vec{\pi}\cdot\vec{\tau}\,\psi_N\ , \tag{1}$$

where the arrows indicate vectors in isospin space (ψ_N: isospin doublet; $\vec{\tau}$: Pauli matrices).

(a) Show that the above Lagrangian is invariant under infinitesimal rotations of the fields in isospin space, $\psi_N \to (1 - i\vec{\varepsilon}\cdot\vec{\tau}/2)\ \psi_N$ and $\vec{\pi} \to (1 + \vec{\varepsilon}\times)\,\vec{\pi}$.

(b) Using the decomposition of the physical pion fields in terms of their cartesian coordinates, $\pi^0 = \pi_3$, $\pi^\pm = \frac{1}{\sqrt{2}}(\pi_1 \pm i\pi_2)$, show that the above Lagrangian predicts relations between the physical coupling constants according to

$$g_{pp\pi^0} = -g_{nn\pi^0} = \frac{1}{\sqrt{2}} g_{pn\pi^+} = \frac{1}{\sqrt{2}} g_{pn\pi^-}\ . \tag{2}$$

6.3 *Baryons and Magnetic Moments in the Constituent-Quark Model* *(4 pts.)*
In the Constituent-Quark Model (CQM) baryon wave functions are fully symmetric in their quark flavor-spin parts (notation: $u^\uparrow$ ($s^\downarrow$) denotes an *up* (*strange*) quark with spin up (down), etc.; identify the position of writing a quark in the baryon wave function with the particle label, e.g., $|u^\uparrow d^\uparrow s^\downarrow\rangle$).

(a) The baryon decouplet is realized by fully symmetric flavor and spin wave functions. Start from the wave function of the $\Delta^-(S_z{=}+\frac{3}{2}){=}|d^\uparrow d^\uparrow d^\uparrow\rangle$, to construct the normalized wave functions of the remaining 3 Δ states, of the $\Sigma^{*,+}(S_z{=}+\frac{3}{2})$, $\Xi^{*,0}(S_z{=}+\frac{3}{2})$ and $\Omega^-(S_z{=}+\frac{3}{2})$, by using the flavor-step operators $\lambda^I_\pm$ and $\lambda^U_\pm$ for I- and U-spin (e.g., $\lambda^I_-|udu\rangle = |ddu\rangle + 0 + |udd\rangle$). Estimate the masses of these baryons using quark masses of $m_{u,d}$=0.4 GeV and m_s=0.55 GeV, and compare to experiment.

(d) In the CQM the magnetic moment operator for a hadron h is defined by

$$\mu_h = \sum_i \mu_i\ \sigma^i_z \quad , \quad \mu_i = \frac{e_i}{2m_i} \tag{3}$$

where the sum is over all constituent quarks (charge e_i and mass m_i) in the hadron. Calculate the magnetic moment of the Δ^0, Δ^{++}, $\Sigma^{*,+}$ and $\Omega^- = |s^\uparrow s^\uparrow s^\uparrow\rangle$ in units of [GeV^{-1}] (the quark charges are $e_u = \frac{2}{3}$, $e_{d,s}{=}-\frac{1}{3}$).