Name: PHY625 (Fall 2013), 10/18/13

Homework Assignment #5

(*Due Date*: Monday, Oct. 28, 01:50 pm, in class)

5.1 *Relativistic Mean-Field Theory of Nuclear Matter* *(1+3+2+1+3+1 pts.)*

The Lagrangian of the σ-ω (or Walecka) model is given by

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m_\sigma^2\phi^2 - \frac{1}{4}(V_{\mu\nu})^2 + \frac{1}{2}m_\omega^2 V_\mu^2 + \bar{\psi}\ [i\partial\!\!\!/ - M_N - g_\omega V\!\!\!/ + g_\sigma\phi]\ \psi \qquad (1)$$

with conventions as defined in class. The nucleon density operator is defined as $\hat{\rho}_N \equiv \psi^+\psi$. In numerical calculations, use M_N=939 MeV, m_σ=550 MeV m_ω=782.6 MeV, ρ_0=0.16 fm^{-3}, g_σ=10.555 and g_ω=13.05.

(a) Apply the mean-field approximation (MFA) to the scalar and vector fields at finite density and establish their relations to the scalar and number density of nucleons. Under what conditions is the MFA applicable?

(b) In MFA the Hamiltonian, $\mathcal{H} = \pi\dot{q} - \mathcal{L}$, has been shown to take the form

$$\mathcal{H}_{\rm MFA} = \frac{1}{2}m_\sigma^2\phi_0^2 - \frac{1}{2}m_\omega^2 V_0^2 + g_\omega V_0 \hat{\rho}_N + \frac{1}{V}\sum_\alpha E_k^*(b_\alpha^\dagger b_\alpha + d_\alpha^\dagger d_\alpha) + \delta\mathcal{H}\ . \qquad (2)$$

Use this expression to compute the energy density and pressure of nuclear matter as a function of density, ρ_N, and scalar mean field, ϕ_0.

(c) Derive the selfconsistency equation for the scalar mean field by minimizing the energy at fixed A, V, and rewrite it in terms of the effective nucleon mass as

$$M_N^* = M_N - \frac{g_\sigma^2}{m_\sigma^2}\rho_S \quad , \quad \rho_S = d_{\rm SI}\int_0^{k_F} \frac{d^3k}{(2\pi)^3}\frac{M_N^*}{E_k^*}\ . \qquad (3)$$

(d) Show that energy density and pressure can be written as

$$\varepsilon(\rho_N) = \frac{g_\sigma^2}{2m_\sigma^2}\rho_S^2 + \frac{g_\omega^2}{2m_\omega^2}\rho_N^2 + \langle E_k^*\rangle\rho_N\ , \quad P(\rho_N) = -\frac{g_\sigma^2}{2m_\sigma^2}\rho_S^2 + \frac{g_\omega^2}{2m_\omega^2}\rho_N^2 + \langle\frac{\vec{k}^2}{3E_k^*}\rangle\rho_N\ . \qquad (4)$$

(e) Compute numerically and plot the energy $E_B/A = \varepsilon/\rho_N - M_N$ (in [MeV]) and pressure for ρ_N=0-2ρ_0 in steps of 0.1. For each ρ_N, start by finding the selfconsistent solution for M_N^* (and ρ_S) by numerical iteration of eqs. (3). Interpret your results.

(f) Repeat part (e) but replace ρ_S by ρ_N. What does this tell you about relativistic effects for nuclear saturation?