Name: PHY625 (Fall 2013), 10/02/13

Homework Assignment #4

(*Due Date*: Wednesday, Oct. 16, 01:50 pm, in class)

4.1 *Nuclear Shell Model* *(1+2+2 pts.)*

Starting point is the spherical harmonic oscillator potential,

$$U_{HO}(r) = -U_0 + \frac{1}{2} M_N \, \omega^2 \, r^2 \ , \tag{1}$$

for the collective nuclear mean field inside a nucleus of radius $R_A \simeq 1.2 A^{1/3}$, with $U_0 \simeq -50\,\mathrm{MeV}$ and $U(R_A) \simeq 0$.

(a) How large is the oscillator quantum $\hbar\omega$ for a Pb-208 nucleus?

(b) Given the energy levels of this potential, $E_N = (N + \frac{3}{2})\hbar\omega - U_0$ with $N = 2(n-1) + l$ and $n \geq 1$, $l \geq 0$, determine the nuclear shell filling for neutrons (or protons) and the pertinent totals for filled shells.

(c) Now include a spin-orbit coupling of type

$$U(r) = U_{HO}(r) - \frac{2}{\hbar^2} \, \alpha \, \vec{L} \cdot \vec{S} \tag{2}$$

where $\vec{S}$ is the nucleon's spin-$\frac{1}{2}$ operator and α carries the dimension of energy. Determine the eigenvalues of $\vec{L} \cdot \vec{S}$ in terms of j, l and s for $j = l \pm \frac{1}{2}$ (and s=$\frac{1}{2}$), and explain how this can give rise to the nuclear magic numbers 2, 8, 20, 28, 50 and 82. What is the approximate magnitude of α for this to work out?

4.2 *σ-ω Model Lagrangian* *(2+3 pts.)*

The Lagrangian of the σ-ω (or Walecka) model is given by

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m_\sigma^2 \phi^2 - \frac{1}{4}(V_{\mu\nu})^2 + \frac{1}{2} m_\omega^2 V_\mu^2 + \bar{\psi} \left[i\partial\!\!\!/ - M_N - g_\omega V\!\!\!/ + g_\sigma \phi \right] \psi \tag{3}$$

using the conventions defined in class.

(a) The free Dirac equation for the nucleon in momentum space is given by

$$(p\!\!\!/ - M_N)\psi = 0 \ . \tag{4}$$

Show that the ansatz $u_s = N[I_2, \vec{\sigma} \cdot \vec{p}/(\omega_p + M_N)]\chi_s$ for the 4-vector ψ is a solution, where s=1,2, χ_1=(1,0), χ_2=(0,1), $\omega_p = \sqrt{\vec{p}^{\,2} + M_N^2}$, I_2 is the 2×2 identitiy matrix and $\vec{\sigma}$ are the 3 standard Pauli matrices.
Requiring the normalization $u_r^\dagger u_s = \delta_{rs}$, determine the normalization factor N.

(b) Using the Euler-Lagrange equations, derive the equations of motion for the σ, ω and nucleon field.