Name: PHY625 (Fall 2013), 09/09/13

Homework Assignment #2

(*Due Date*: Wednesday, Sept. 18, 01:50pm, in class)

2.1 *Liquid Drop Model (LDM) of Nuclei* *(1+2+1+1+1 pts.)*

The empirical Weizsäcker formula for the binding energy of nuclei is given by

$$E_B = \sum_{i=1}^{5} E_i = -a_1 A + a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} + a_4 \frac{(A-2Z)^2}{A} + a_5 \frac{\lambda}{A^{3/4}} \tag{1}$$

with A: nuclear mass number, Z: nuclear charge (in units of e), a_1=15.75 MeV, a_2=17.8 MeV, a_3=0.71 MeV, a_4=23.7 MeV and a_5=34 MeV with λ=-1,0,1 for e-e,e-o,o-o nuclei.

(a) Briefly discuss the physical motivation (A and Z dependence) for each term.

(b) Estimate the value of a_3 theoretically by modeling a nucleus (A, Z) with radius $R_A = r_0\ A^{1/3}$ ($r_0 = 1.15$ fm) by a uniform spherical charge distribution, $\rho_c(r) = \rho_{c,0}\ \Theta(R_A - r)$. First use Poisson's law, $\Delta V(r) = -\rho(r)$, to find the static electric potential for $r < R_A$ (make sure it is continuous with the point-charge potential $V(r) = Ze/4\pi r$ for $r > R_A$). Then calculate the electric potential energy,

$$U(A,Z) = -\frac{1}{2}\int d^3r\ V_c(r)\ \rho_c(r)\ . \tag{2}$$

(c) Derive the value Z^* for the charge which minimizes the binding energy for fixed A. Plot the resulting valley of stability, $Z^*(N)$.

(d) Plot $|E_B(A)|$ using $Z^*(A)$ from part (c) by subsequently adding the terms of the LDM in numerical order as written above up to (including) $i = 4$. In each step determine the A, if any, for which $|E_B(A)|$ is maximal.

(e) How much energy is released when fissioning ${}^{235}U \to 2\ {}^{116}Pd + 3n$? How many kg of ${}^{235}U$ have to be fissioned in 30 days at an output of 1000 MW (neglect any losses)?

2.2 *2-Body Density Matrix in Fermi Gas Model* *(2+1+1 pts.)*

The (reduced) 2-body density matrix for a non-interacting Fermi gas of A nucleons is

$$\rho^{(2)}(\vec{r}_1,\vec{r}_2) = \frac{1}{A(A-1)V^2} \sum_{\vec{k}_i,\vec{k}_j} \left[1 - \cos(2\vec{k}_{ij}\cdot\vec{r})\right] \tag{3}$$

with V: volume, $\vec{r} \equiv \vec{r}_2 - \vec{r}_1$ and $\vec{k}_{ij} = (\vec{k}_i - \vec{k}_j)/2$.

(a) Use the continuum limit for the momentum sums, $\sum_{\vec{k}_i} \to V \int_0^{k_F} \frac{d^3k_i}{(2\pi)^3}$ (also implying $A \to \infty$) to show that $\rho^{(2)}(\vec{r}_1,\vec{r}_2) = g_-(x)/V^2$ with

$$g_-(x) = 1 - \left[\frac{3}{x^2}\left(\frac{\sin x}{x} - \cos x\right)\right]^2\ . \tag{4}$$

(b) Using Taylor expansions of the sine and cosine functions, show that $\lim_{x\to 0} g_-(x) = 0$, i.e., $\rho^{(2)}(\vec{r}_1,\vec{r}_2)$ exhibits "Pauli repulsion".

(c) Why does the Pauli repulsion in the spatial part of the 2-nucleon density matrix not suffice to stabilize atomic nuclei at a size that increases as $R_A \propto A^{1/3}$?