Name: PHY625 (Fall 2013), 08/28/13

Homework Assignment #1

(*Due Date*: Friday, Sep. 06, 01:50pm, in class)

1.1 *Empirical Features of the N-N Force* *(4 pts.)*

(a) In units where $\hbar = c = 1$, the Coulomb potential is given by $V_{\rm Coul} = \pm\alpha_{\rm em}/r$ with $\alpha_{\rm em} = 1/137$. The deuteron binding energy is $E_B^d = 2.2\,\rm MeV$ and its approximate size is $d_d \simeq 4\,\rm fm$. Can the Coulomb correction be the reason for not binding the *pp* system?

(b) Nucleon-nucleon scattering shows no significant evidence of a P-wave component up to center-of-mass (CM) energies of $E_{\rm CM} \simeq 10\,\rm MeV$. Use this information to estimate the range, R, of the nuclear force.

(c) When scattering a proton off a stationary hydrogen target, the elastic D-wave phase shifts are measured to turn from positive to negative at an incoming energy of $E_{\rm lab} \simeq 800\,\rm MeV$. What qualitative feature does this reveal about the NN force? Estimate an associated distance scale for this feature.

(d) Energy levels in atomic nuclei reveal the presence of a spin-orbit force associated with a potential operator $\hat{V}_{LS} = V_{LS}(r)\vec{L}\cdot\vec{S}$, where $\vec{L}$ and and $\vec{S}$ denote the angular momentum and spin of the NN system, respectively.
(i) Express the eigenvalues of the operator $\vec{L}\cdot\vec{S}$ in terms of the ones of $|\vec{L}|^2$, $|\vec{S}|^2$ and $|\vec{J}|^2$, where $\vec{J} \equiv \vec{L} + \vec{S}$. *(hint: evaluate* $|\vec{J}|^2$*)*
(ii) Explain why the spin-orbit force is absent in spin-singlet and S-wave channels.

1.2 *Central Nuclear Force* *(6 pts.)*
In this problem we develop a schematic model for nuclear saturation.

(a) Show that the Fourier transform of a static scalar meson-exchange potential,

$$V_\alpha(q) = -g_\alpha^2\,\frac{1}{\vec{q}^2 + m_\alpha^2}\ , \qquad (1)$$

yields the standard Yukawa potential in coordinate space, $V(r) = -g_\alpha^2/(4\pi)\,{\rm e}^{-m_\alpha r}/r$.

(b) Graph the r-dependence (in units of [fm]) of the attractive scalar potential (in [MeV]) from σ exchange (α=σ, $m_\sigma = 550\,\rm MeV$, $g_\sigma = 10$) and of the repulsive vector potential from ω exchange (α=ω, $m_\omega = 782\,\rm MeV$, $g_\omega = 17$), as well as their sum (use the conversion factor $\hbar c = 197.33\,\rm MeV\,fm$).

(c) From the minimum of the central potential in part (b) estimate the nuclear saturation (ground-state) density, as well as the binding energy per nucleon. For the latter, assume 6 nearest neighbors and an average kinetic energy of $KE \simeq 25\,\rm MeV$ per nucleon.