Name: PHY625 (Fall 2012), 11/26/12

Homework Assignment #7

(*Due Date*: Monday, Dec. 03, 01:50 pm, in class)

7.1 *Heavy-Quark Potential and Bottomonium Spectrum* *(2+2 pts.)*
The static QCD potential between a heavy quark and antiquark is well represented by

$$V_{Q\bar{Q}}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r \tag{1}$$

with a string tension of $\sigma = 1$ GeV/fm. The ground and first excited state of the Upsilon spectrum (bottom-antibottom bound states) have masses of $M(\Upsilon(1S)) = 9.46$ GeV and $M(\Upsilon(2S)) = 10.02$ GeV, respectively.

(a) Assuming α_s=0.5 and neglecting the linear confining term in the potential, use the hydrogen expression for the bound-state energy to estimate the bottomonium binding energies. Can you find a reasonable value for the bottom-quark mass, m_b, to establish approximate agreement with the above masses?

(b) Evaluate the correction to the 2 binding energies by evaluating the confining term assuming hydrogen-like radii of the two Υ states. Are the corrections in each case "large" or "small"?

7.2 *Bag Model of Hadron Structure* *(2+2+2 pts.)*
In the MIT bag model, the QCD vacuum is modeled by a background field generating pressure $P = -B$ and energy density $\epsilon = B$, where the bag constant B is defined as negative. Hadrons are constructed as "bags" of empty vacuum stabilized by the kinetic energy of the quarks inside. In the following assume a spherical bag.

(a) For simplicity, approximate the quarks as massless and spinless particles described by the free Klein-Gordon equation within the bag,

$$\Box\, \phi = 0\ . \tag{2}$$

Find the stationary ground-state solution for the radial wave function by requiring it to vanish at the boundary, to find the minimal momentum, $k_{\rm min}$, of each quark.

(b) Write down the total energy of a N_q-quark bag and find the radius, $R_{\rm min}(B, N_q)$, which minimizes this energy (N_q: no. of quarks in the bag); to improve your estimate, use $k_{\rm min} = 2.04/R$ from the Dirac equation.

(c) Equate the minimum energy to the proton mass to find the explicit values for bag radius (in fm) and bag constant, B (in GeV/fm^3). By how many percent does the calculated radius differ from the experimental result $\langle R_p^2\rangle^{1/2} \simeq 0.8$ fm? (Convert the latter into the proton radius assuming a 3-D spherical, homogeneous charge distribution.)