Name: PHY625 (Fall 2012), 11/19/12

Homework Assignment #6

(*Due Date*: Monday, Nov. 26, 01:50 pm, in class)

6.1 *Electron Scattering and Parton Model* *(2+1+2+2 pts.)*

The elastic cross section for scattering an electron (with initial and final 4-momenta $(E,\vec{k})$ and $(E',\vec{k}')$, respectively) off a spin-$\frac{1}{2}$ point particle of mass M can be written as

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4(\theta/2)} \frac{E'}{E} \left(\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2} \sin^2\frac{\theta}{2} \right) \tag{1}$$

where $\theta = \angle(\vec{k}, \vec{k}')$ is the scattering angle (neglect the electron mass).

(a) Show that, for fixed incident lab energy, the scattering angle is the only independent variable, by expressing E' and the 4-momentum transfer q^2 in terms of θ.

(b) For inelastic scattering, the outgoing proton breaks up and another variable (usually taken as E' or $\nu = E - E'$) together with 2 structure functions, $W_{1,2}$, are needed. The differential cross section takes the form

$$\frac{d\sigma}{d\Omega\, dE'} = \frac{\alpha^2}{4E^2 \sin^4(\theta/2)} \left(W_2(\nu, q^2) \cos^2\frac{\theta}{2} + 2W_1(\nu, q^2) \sin^2\frac{\theta}{2} \right) . \tag{2}$$

Show that for $W_2(\nu, q^2) = \delta(\nu + q^2/2M)$ and $2W_1(\nu, q^2) = -q^2/(2M^2)\, \delta(\nu + q^2/2M)$, the elastic "point-like" cross section, Eq. (1), is recovered.

(c) Introduce "new" point-like constituents with mass $m = x'M$ by setting $W_2(\nu, q^2) = \delta(\nu + q^2/2m)$ and $2W_1(\nu, q^2) = -q^2/(2m^2)\, \delta(\nu + q^2/2m)$ in Eq. (2). Show that the redefined structure functions $F_1 \equiv MW_1$ and $F_2 \equiv \nu W_2$ "scale", i.e., only depend on the ratio $\omega = -2M\nu/q^2$ for given x' of a "parton", not separately on ν and q^2.

(d) Integrate over x' with charge (e_i^2)and probability ($f_i(x')$) weights to show

$$2xF_1(x) = F_2(x) = \sum_i e_i^2 x f_i(x) \tag{3}$$

where $x \equiv 1/\omega$ and the sum is over parton species in the proton (i=u,$\bar{u}$,d,$\bar{d}$,s,$\bar{s}$).

6.2 *Local Gauge Invariance of QCD Lagrangian* *(2+1 pts.)*

The QCD Lagrangian is defined by

$$\mathcal{L}_{\rm QCD} = \bar{q}\,(i\gamma^\mu D_\mu - \hat{m}_q)\, q - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a \tag{4}$$

with the covariant derivative $D_\mu \equiv \partial_\mu + ig_s T_a G^a_\mu$ where $T_a = \lambda_a/2$ are given by the Gellmann matrices $\lambda_{1...8}$ and satisfy $[T_a, T_b] = i f_{abc} T_c$ (double indices summed over).

(a) Show that the quark part of $\mathcal{L}_{\rm QCD}$ is invariant under the following *local* infinitesimal phase rotation:
$q(x) \to [1 + i\alpha_a(x)\, T_a] q(x)$, $G^a_\mu(x) \to G^a_\mu(x) - \frac{1}{g_s} \partial_\mu \alpha_a(x) - f_{abc}\, \alpha_b(x)\, G^c_\mu(x)$.

(b) What are key differences between the coupling "constants" in QCD, $\alpha_s(Q^2)$, and QED, $\alpha_{\rm em}(Q^2)$?