Name: PHY625 (Fall 2012), 10/26/12

Homework Assignment #4

(*Due Date*: Monday, Nov. 05, 01:50 pm, in class)

4.1 *σ-ω Model of Nuclear Matter* *(2+1+2+2+2+1 pts.)*

The Lagrangian of the σ-ω (or Walecka) model is given by

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m_\sigma^2\phi^2 - \frac{1}{4}(V_{\mu\nu})^2 + \frac{1}{2}m_\omega^2 V_\mu^2 + \bar{\psi}\,[i\partial\!\!\!/ - m_N - g_\omega V\!\!\!/ + g_\sigma \phi]\,\psi \qquad (1)$$

using the conventions defined in class; the nucleon number density operator is defined as $\hat{\rho}_B \equiv \psi^+\psi$.

(a) Use the Euler-Lagrange equations to derive the equations of motion for the σ and ω fields.

(b) Formulate and apply the mean-field approximation (MFA) to obtain the σ and ω mean fields in terms of nucleon scalar and number density. Write down the Lagrangian in MFA.

(c) The nuclear-matter Hamiltonian in MFA has been shown in class to take the form

$$\hat{\mathcal{H}} = \pi\dot{q} - \hat{\mathcal{L}} = \frac{1}{2}m_\sigma^2\phi_0^2 - \frac{1}{2}m_\omega^2 V_0^2 + g_\omega V_0 \hat{\rho}_N + \frac{1}{V}\sum_\alpha E_k^*(b_\alpha^+ b_\alpha + d_\alpha^+ d_\alpha)\ . \qquad (2)$$

Use it to obtain the energy density, ϵ, and pressure, $P = -dE/dV$, of nuclear matter.

(d) Derive the selfconsistency equation for the scalar mean field by minimizing the energy at fixed A, V, and rewrite it in terms of the effective nucleon mass as

$$m_N^* = m_N - \frac{g_\sigma^2}{m_\sigma^2}\rho_s \quad , \quad \rho_s = d_{\rm SI}\int_0^{k_F} \frac{d^3k}{(2\pi)^3}\frac{m_N^*}{E_k^*}\ . \qquad (3)$$

Express ϵ and P as functions of nucleon number density, ρ_N, and scalar density, ρ_s.

(e) Fix the 2 free parameters at $(g_\sigma/m_\sigma)^2 m_N^2$=325 and $(g_\omega/m_\omega)^2 m_N^2$=245 (or there about), to numerically verify a minimum of the binding energy per nucleon, $E_B/A = \epsilon/\rho_N - m_N$, as a function of k_F at the empirical values of -16 MeV and 1.35 fm^{-1}, respectively. Start by numerically solving for m_N^* at each k_F (e.g., by iteration). Also calculate and plot $P(k_F)$.

(f) What happens to the saturation mechanism if you approximate the scalar density non-relativistically, $\rho_s \simeq \rho_N$? Plot $E_B/A(k_F)$ and $P(k_F)$ to support your explanation.