Name: PHY625 (Fall 2012), 10/03/12

Homework Assignment #3

(*Due Date*: Fri, Oct. 12, 01:50pm, in class)

3.1 *Brueckner-Bethe-Goldstone (BBG) Theory of Nuclear Matter* *(3+3+2+1+1 pts.)*
Starting point is the A-nucleon Schrödinger equation,

$$\hat{H}\ \Psi_{\alpha_1\cdots\alpha_A} = E\ \Psi_{\alpha_1\cdots\alpha_A}\ , \quad \hat{H} = \sum_i^A \hat{T}_i + \sum_{i<j}^A \hat{V}_{ij} \tag{1}$$

with single-nucleon kinetic-energy operators $\hat{T}_i$ and two-nucleon potential operators $\hat{V}_{ij}$ (=$V(\vec{r}_i, \vec{r}_j)$ in coordinate space).

(a) Show that the plane wave $\phi_\alpha(\vec{r}) = \exp(i\vec{k}_\alpha \cdot \vec{r})/\sqrt{V}$ satisfies the in-medium 1-body Schrödinger equation in coordinate space,

$$T(r)\phi_\alpha(\vec{r}) + \int_{-\infty}^{+\infty} d^3r' U(\vec{r}-\vec{r}\,')\phi_\alpha(\vec{r}\,') = \epsilon_\alpha \phi_\alpha(\vec{r}) \tag{2}$$

with $\epsilon_\alpha = \vec{k}_\alpha^2/2m_N + U_\alpha$, where $U_\alpha = U(k_\alpha)$ denotes the Fourier transform of $U(\vec{r})$. Then use the non-interacting Fermi-gas approximation, $\Psi_{\alpha_1\cdots\alpha_A} = \Phi_{\alpha_1\cdots\alpha_A}$, to show that the nuclear ground-state energy is given by

$$E = \langle \alpha_1 \cdots \alpha_A | \hat{H} | \Phi_{\alpha_1\cdots\alpha_A} \rangle = \sum_{k_\alpha < k_F} \left(\frac{k_\alpha^2}{2m_N} + \frac{1}{2} U_\alpha \right)\ , \tag{3}$$

where $|\alpha_1 \cdots \alpha_A\rangle \propto \phi_{\alpha_1} \cdots \phi_{\alpha_A}$ with norm $\langle \alpha_2 \cdots \alpha_A | \Phi_{\alpha_1\cdots\alpha_A} \rangle = \phi_{\alpha_1}$. Start by multiplying Eq. (1) with $\langle \alpha_2 \cdots \alpha_A|$ to identify U_α in terms of V_{ij}. What is the main problem in calculating U_α with the non-interacting Fermi-gas wavefunction?

(b) Defining the 1- and *interacting* 2-nucleon part of the A-body wavefunction as $\Psi_{\alpha_1} = \langle \alpha_2 \cdots \alpha_A | \Psi_{\alpha_1\cdots\alpha_A} \rangle$ and $\Psi_{\alpha_1\alpha_2} = \langle \alpha_3 \cdots \alpha_N | \Psi_{\alpha_1\cdots\alpha_A} \rangle$, with norm $\langle \alpha_1 \cdots \alpha_N | \Psi_{\alpha_1\cdots\alpha_A} \rangle = 1$ as before, show that

$$E = \sum_{\alpha_i} \frac{k_{\alpha_i}^2}{2m_N} + \sum_{i<j} \langle \alpha_i \alpha_j | V_{ij} | \Psi_{\alpha_i\alpha_j} \rangle\ . \tag{4}$$

(c) Express the potential term in Eq. (4) via the Brueckner G-matrix and relate it to U_α. Write down the Bethe-Goldstone equation for $\Psi_{\alpha_i\alpha_j}$ and explain the emergence of a self-consistency problem when solving it.

(d) What is the main effect in the solution of the BBG 2-body wavefunction compared to the noninteracting Fermi gas result? How does this solve the problem (referred to in part (a)) of calculating the interaction-energy contribution to the nuclear ground-state energy, E, in the Fermi-gas approximation?

(e) Sketch the results of the BBG theory with realistic NN potentials for the saturation curve of nuclear matter, $\frac{E_B}{A}(k_F)$. Does BBG theory fully succeed in describing the empirical saturation regime?