Name: PHY625 (Fall 2012), 09/14/12

Homework Assignment #2

(*Due Date*: Wednesday, Sept. 26, 01:50pm, in class)

2.1 *Liquid Drop Model (LDM) of Nuclei* *(1+1+2+1+1 pts.)*

The empirical Weizsäcker/LDM formula for the binding energy of nuclei is given by

$$E_B = \sum_{i=1}^{5} E_i = -a_1 A + a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} + a_4 \frac{(A-2Z)^2}{A} + a_5 \frac{\lambda}{A^{3/4}} \tag{1}$$

with A: nuclear mass number, Z: nuclear charge (in units of e) and positive coefficients a_i (in units of MeV).

(a) Briefly discuss the physical motivation for each term.

(b) Derive the value Z^* for the charge which minimizes the binding energy for fixed A. Plot the resulting valley of stability, $Z^*(N)$.

(c) Use plotting software to graph $-E_B(A)$ using $Z^*(A)$ from part (b) by subsequently adding the terms of the LDM in numerical order as written above up to $i = 4$. For each curve read off the value of A, if any, for which $-E_B(A)$ is maximal. Here and below use $a_1 = 15.75$ MeV, $a_2 = 17.8$ MeV, $a_3 = 0.71$ MeV, $a_4 = 23.7$ MeV, $a_5 = 34$ MeV and λ=-1,0,1 for e-e,e-o,o-o nuclei, respectively.

(d) Use the LDM to calculate the the binding energies of ${}^{235}_{92}U$, ${}^{144}_{56}Ba$ and ${}^{89}_{36}Kr$, and compare to the experimental values that you find on the Internet.

(e) How much energy is set free when fissioning a ${}^{235}_{92}U$ nucleus into ${}^{144}_{56}Ba + {}^{89}_{36}Kr + 2n$ (once all are far separated)? Why does ${}^{235}_{92}U$ not fission spontaneously?
(*Hint: compute the Coulomb repulsion between* ${}^{144}_{56}Ba$ *and* ${}^{89}_{36}Kr$ *when their surfaces just touch, with* $V_C(d) = Z_1 Z_2 \alpha/d$ *and nuclear radii* $R_A = r_0 A^{1/3}$ *with* r_0*=1.1 fm*).

2.2 *One- and Two-Body Density Matrix in Fermi Gas Model (FGM)* *(2+1+1 pts.)*

The (reduced) 2-body density matrix in the FGM has been shown to take the form

$$\rho^{(2)}(\vec{r}_1, \vec{r}_2) = \frac{1}{A(A-1)V^2} \sum_{\vec{k}_1, \vec{k}_2} \left[1 - \cos(2\vec{k}_{ij} \cdot \vec{r})\right] \tag{2}$$

with $\vec{r} \equiv \vec{r}_2 - \vec{r}_1$ and $\vec{k}_{ij} = (\vec{k}_i - \vec{k}_j)/2$.

(a) Using the continuum limit for the momentum sums, $\sum_{\vec{k}} \to V \int_0^{k_F} \frac{d^3k}{(2\pi)^3}$ (also implying $A \to \infty$), show that $\rho^{(2)}(\vec{r}_1, \vec{r}_2) = g_-(x)/V^2$ with

$$g_-(x) = 1 - \left[\frac{3}{x^2}\left(\frac{\sin x}{x} - \cos x\right)\right]^2 . \tag{3}$$

(b) Quote the result for $\rho^{(2)}(\vec{r}_1, \vec{r}_2)$ for a symmetric spatial part of the A-body wave function. Use plotting software to graph both $g_\pm(x)$ over a relevant range in r for k_F=265 MeV. Interpret the behaviors for $x \to 0$.

(c) Discuss the relevance of both $g_\pm(x)$ in building the total A-nucleon wave functions, i.e., including the spin-isospin parts.