Name: PHY625 (Fall 2012), 08/31/12

Homework Assignment #1

(*Due Date*: Friday, Sep. 07, 01:50pm, in class)

1.1 *Empirical Features of the N-N Force* *(4 pts.)*

(a) Nucleon-Nucleon scattering shows no significant evidence of a P-wave component up to center-of-mass (CM) energies of $E_{\rm CM} \simeq 10\,{\rm MeV}$. Use this information to estimate the range, R, of the nuclear force.

(c) Energy levels in atomic nuclei reveal the presence of a spin-orbit force associated with a potential operator $\hat{V}_{LS} = V_{LS}(r)\vec{L}\cdot\vec{S}$, where $\vec{L}$ and and $\vec{S}$ denote the angular momentum and spin of the $2N$ system, respectively.
(i) Express the eigenvalues of the operator $\vec{L}\cdot\vec{S}$ in terms of the ones of $|\vec{L}|^2$, $|\vec{S}|^2$ and $|\vec{J}|^2$, where $\vec{J} \equiv \vec{L} + \vec{S}$. (*hint: evaluate* $|\vec{J}|^2$)
(ii) Explain why the spin-orbit force is absence in spin-singlet and S-wave channels.

1.2 *Central Nuclear Force* *(6 pts.)*

In this problem we develop a schematic model for nuclear saturation.

(a) Show that the Fourier transform of a static scalar meson-exchange potential,

$$V_\alpha(q) = -g_\alpha^2 \, \frac{1}{\vec{q}^2 + m_\alpha^2} \, , \tag{1}$$

yields the standard Yukawa potential in coordinate space, $V(r) = -g_\alpha^2/(4\pi)\, {\rm e}^{-m_\alpha r}/r$.

(b) Graph the r-dependence (in units of [fm]) of the attractive scalar potential (in [MeV]) from σ exchange (α=σ, $m_\sigma = 500\,{\rm MeV}$, $g_\sigma = 10.0$) and of the repulsive scalar potential from ω exchange (α=ω, $m_\omega = 782\,{\rm MeV}$, $g_\omega = 20.5$), as well as their sum (use the conversion factor $\hbar c = 197.33\,{\rm MeV\,fm}$).

(c) Based on your result for the central potential in part (b) estimate the nuclear saturation (ground-state) density, as well as the binding energy per nucleon. For the latter, assume 4 nearest neighbors and include an average kinetic energy of $KE \simeq 30\,{\rm MeV}$ per nucleon.