Name: PHY625 (Fall 2008), 11/11/08

Homework Assignment #7

(*Due Date*: Thursday, November 20, 05:30 pm, in class)

7.1 *Baryon Wave Functions and Magnetic Moments in the Constituent-Quark Model (7 pts.)*
As discussed in class, the physical realizations of the baryon wave functions in the Constituent-Quark Model (CQM) correspond to fully symmetric flavor-spin parts (notation: use $u^\uparrow$ ($s^\downarrow$) for an *up* (*strange*) quark with spin up (down), etc.; identify the position of writing a quark in the baryon wave function with the particle label, so that the particle exchange operator for, e.g., quark 1 and 3, acts as follows: $P_{13}|u^\uparrow d^\uparrow s^\downarrow\rangle = |s^\downarrow d^\uparrow u^\uparrow\rangle$).

(a) The baryon decuplet is realized by combining the fully symmetric flavor and spin wave functions in the $(10, 4)$ representation. Starting from the wave function of the $\Delta^-(S_z{=}+\frac{3}{2}){=}|d^\uparrow d^\uparrow d^\uparrow\rangle$, construct the (normalized) wave functions of the remaining 3 Δ states, as well as of the $\Sigma^{*,+}(S_z{=}+\frac{3}{2})$ and $\Xi^{*,0}(S_z{=}+\frac{3}{2})$, by using the flavor step operators, $\lambda^I_\pm = \frac{1}{2}(\lambda_1 \pm i\lambda_2)$ and $\lambda^U_\pm = \frac{1}{2}(\lambda_6 \pm i\lambda_7)$ for I- and U-spin, respectively.

(b) The baryon octet representation corresponds to $\frac{1}{\sqrt{2}}[(8_S, 2_S) + (8_A, 2_A)]$, where "$S$" ("$A$") denotes (anti-) symmetry with respect to the first 2 quarks. Construct the flavor-spin wave function of a $S_z{=}+\frac{1}{2}$ proton. Start from the 1↔2 antisymmetric flavor part, $p_A = \frac{1}{\sqrt{2}}(ud - du)u$, and construct p_S by requiring orthogonality to both p_A and the decuplet Δ^+; then combine p_A and p_S with the 1↔2 anti-/symmetric spin wave functions χ_A and χ_S, respectively.

(c) Explicitly verify symmetry of the flavor-spin proton wave function constructed in (b) under quark exchange P_{12}, P_{13} and P_{23}.

(d) In the CQM the magnetic moment operator for a hadron h is defined by

$$\mu_h = \sum_i \mu_i\, \sigma^i_z \quad , \quad \mu_i = \frac{e_i}{2m_i} \tag{1}$$

where the sum is over all constituent quarks (charge e_i and mass m_i) in the hadron. Compute the magnetic moment of the (spin-up) proton, μ_p, in terms of the u- and d-quark ones, $\mu_{u,d}$. Obtain the neutron magnetic moment by the interchange $u \leftrightarrow d$, and compute μ_n/μ_p assuming $m_u{=}m_d$. How does your result compare to the experimental value of -0.685?

7.2 *Electron Scattering and Parton Model* *(3 pts.)*
The elastic cross section for scattering an electron (with initial and final 4-momenta (E,$\vec{k}$) and (E',$\vec{k}'$), respectively) off a spin-$\frac{1}{2}$ point particle of mass M can be written as

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4(\theta/2)}\, \frac{E'}{E} \left(\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right) \tag{2}$$

where $\theta = \angle(\vec{k}, \vec{k}')$ is the scattering angle (neglect the electron mass).

(a) Show that, for given incident energy in the lab system, the scattering angle is the only independent variable, by expressing E' and the 4-momentum transfer q^2 in terms of θ.

(b) For inelastic scattering, the outgoing proton breaks up and another variable (usually taken as E' or $\nu = E - E'$) together with 2 structure functions, $W_{1,2}$, are needed. The differential cross section takes the form

$$\frac{d\sigma}{d\Omega\, dE'} = \frac{\alpha^2}{4E^2 \sin^4(\theta/2)} \left(W_2(\nu, q^2) \cos^2 \frac{\theta}{2} + 2W_1(\nu, q^2) \sin^2 \frac{\theta}{2} \right) . \qquad (3)$$

Show that for $W_2(\nu, q^2) = \delta(\nu + q^2/2M)$ and $2W_1(\nu, q^2) = -q^2/(2M^2)\, \delta(\nu + q^2/2M)$, the elastic "point-like" cross section, Eq. (2), is recovered.

(c) Alternatively, introduce "new" point-like constituents with mass $m = xM$ by setting $W_2(\nu, q^2) = \delta(\nu + q^2/2m)$ and $2W_1(\nu, q^2) = -q^2/(2m^2)\, \delta(\nu + q^2/2m)$ in Eq. (3). Show that the redefined structure functions $F_1 \equiv MW_1$ and $F_2 \equiv \nu W_2$ lead to a cross section which only depends on the "mass fraction" x of the "partons", and not separately on ν and q^2.