Name: PHY625 (Fall 2008), 10/30/08

Homework Assignment #6

(*Due Date*: Thursday, November 06, 05:30 pm, in class)

6.1 *Kinematics in Hadron Scattering* *(1+1+1 pts.)*

The 3 subproblems below are independent of each other.

(a) Fully ionized ^{12}C nuclei are directed toward a ^{208}Pb target. What is the minimal kinetic energy (in [MeV]) for a carbon nucleus to "touch" a lead nucleus in a central collision (use the nuclear radius formula $R_A = 1.2\,A^{1/3}$ fm in this estimate)?

(b) High-energy protons are directed toward a hydrogen target. What is the minimal (total) bombarding energy (in [GeV]) to being able to create antiprotons? (*hint: watch out for baryon number conservation*)

(c) High-energy protons traveling along the z direction hit a hydrogen target. 17 *cm* behind the collision point, a V-track emerges consisting of a negative pion with 3-momentum k=0.326 GeV at θ=17.2° above the z axis and a proton with 3-momentum p=2.189 GeV at θ=2.53° below the z-axis. Compute the mass of the parent particle and its lifetime in its rest system. (*hint: account for Lorentz time dilation in the decay*)

6.2 *Rotation Group and Algebra* *(2+2 pts.)*

Consider a wave function $\psi(\vec{r})$ in 3-dimensional space.

(a) Show that the 4 successive infinitesimal rotations of the coordinate system (ϵ about 1-axis, then η about 2-axis, then -ϵ about 1-axis and finally -η about 2-axis) are equivalent to the second-order rotation by $\epsilon\eta$ about the 3-axis.

(b) Apply the same 4 successive rotations as in (a) at the wave function level using the $SO(3)$ generators J_1 and J_2 and show that the equivalence to the result in (a) requires the relation $[J_1, J_2] = iJ_3$ to be satisfied.

6.3 *Isospin Invariance of* π-N *Interactions* *(2+1 pts.)*

A simple π-N-N interaction Lagrangian may be written as

$$\mathcal{L}_{\pi NN} = g_{\pi NN}\ \bar{\psi}_N\, i\gamma_5\, \vec{\pi}\cdot\vec{\tau}\,\psi_N\ , \qquad (1)$$

where the arrows indicate vectors in isospin space (recall that the nucleon spinors are doublets in this space).

(a) Show that the above Lagrangian is invariant under rotations in isospin space (to be applied to all field operators).

(b) Show that the above Lagrangian predicts relations between the physical coupling constants according to

$$g_{pp\pi^0} = -g_{nn\pi^0} = \frac{1}{\sqrt{2}} g_{pn\pi^+} = \frac{1}{\sqrt{2}} g_{pn\pi^-} \qquad (2)$$