Name: PHY625 (Fall 2008), 10/16/08

Homework Assignment #5

(*Due Date*: Tuesday, October 28, 05:30 pm, in class)

5.1 *Nuclear Matter Equation of State in the σ-ω Model* *(2+2+1+2 pts.)*
The energy density for cold (temperature T=0) nuclear matter in the σ-ω model reads

$$\epsilon(\rho_N;\phi_0) = \frac{1}{2}m_\sigma^2\phi_0^2 + \frac{1}{2}\frac{g_\omega^2}{m_\omega^2}\rho_N^2 + d_{\rm SI}\int_0^{k_F}\frac{d^3k}{(2\pi)^3}E_k^* \tag{1}$$

with $E_k^* = [(m_N^*)^2 + \vec{k}^2]^{1/2}$, $m_N^* = m_N - g_\sigma\phi_0$, $\rho_N = 2k_F^3/3\pi^2$, $d_{\rm SI}$=4.

(a) Using $dE = -PdV$, derive the expression for the pressure, $P(\rho_N;\phi_0)$.

(b) Derive the self-consistency equation for the effective mass,

$$m_N^* = m_N - \frac{g_\sigma^2}{m_\sigma^2}\rho_s \quad , \quad \rho_s = d_{\rm SI}\int_0^{k_F}\frac{d^3k}{(2\pi)^3}\frac{m_N^*}{E_k^*} \ , \tag{2}$$

by minimizing the energy at fixed nucleon number and volume.

(c) Explain why nuclear saturation in the σ-ω model is essentially a relativistic effect.

(d) Fix the 2 free parameters at $(g_\sigma/m_\sigma)^2m_N^2$=267.1 and $(g_\omega/m_\omega)^2m_N^2$=195.9, and numerically determine the binding energy per nucleon, $E_B/A = \epsilon/\rho_N - m_N$, as a function of k_F. Start by numerically solving for m_N^* at each k_F (e.g., by iteration). Perform a numerical estimate of the compression modulus, $K = k_F^2\,\partial^2(E_B/A)/\partial k_F^2$.

5.2 *σ-ω Model at Finite Temperature* *(3 pts.)*
Starting from the grandcanonical potential and partition function,

$$\Omega(\mu_N, T; V) = -T\ln\mathcal{Z}_G \quad , \quad \mathcal{Z}_G = {\rm tr}\exp[-(\hat{H} - \mu_N\hat{N})/T] \ , \tag{3}$$

derive the finite-T expression for the pressure in the σ-ω model,

$$P = \frac{1}{2}m_\omega^2V_0^2 - \frac{1}{2}m_\sigma^2\phi_0^2 + +d_{\rm SI}\int_0^{\infty}\frac{d^3k}{(2\pi)^3}\frac{\vec{k}^2}{3E_k^*}(f_k + \bar{f}_k) \tag{4}$$

with $f_k = [\exp((E_k^* - \mu_N^*)/T) + 1]^{-1}$, $\bar{f}_k = [\exp((E_k^* + \mu_N^*)/T) + 1]^{-1}$, $\mu_N^* = \mu_N - g_\omega V_0$, $\hat{H} = V\hat{\mathcal{H}}_{\rm MFT}$ and $\hat{\mathcal{H}}_{\rm MFT}$ as derived in class. Use the eigenstates of the nucleon-number operator to evaluate the trace in $\mathcal{Z}_G$.