Name: PHY625 (Fall 2008), 10/02/08

Homework Assignment #4

(*Due Date*: Tuesday, October 14, 05:30 pm, in class)

2.1 *σ-ω Model Lagrangian* *(3+3 pts.)*

The Lagrangian of the σ-ω (or Walecka) model is given by

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m_\sigma^2\phi^2 - \frac{1}{4}(V_{\mu\nu})^2 - \frac{1}{2}m_\omega^2 V_\mu^2 + \bar{\psi}\,\left[i\partial\!\!\!/ - m_N + g_\omega V\!\!\!/ + g_\sigma\phi\right]\psi \qquad (1)$$

using the conventions defined in class.

(a) Using the Euler-Lagrange equations, derive the equations of motion for the σ, ω and nucleon field.

(b) For an infinitesimal field transformation, $\psi \to \psi + \Psi_a\,\delta\epsilon^a$ (Ψ_a: transformation matrix, a: summation index), Noether's theorem implies the existence of a conserved current,

$$\partial_\mu j^\mu_\nu = 0 \quad \text{with} \quad j^\mu_\nu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\Psi_\nu\ . \qquad (2)$$

Show that the invariance of the Lagrangian, eq. (1), under global phase rotations of the nucleon field, $\psi \to \psi e^{i\alpha}$ (with α constant), implies a conserved baryon current $B_\mu \equiv \bar{\psi}\gamma_\mu\psi$. (*Hint: start by identifying* Ψ_a *for this transformation*)

2.2 *Dirac Sea and Baryon Density* *(2+2 pts.)*

Following the previous problem, the baryon-density operator is defined as $\hat{\rho}_B \equiv \psi^+\psi$.

(a) Using the momentum-space expansion of the solutions for free Dirac field operators, ψ and ψ^+, as defined in class, in connection with the pertinent anti-commutation and orthogonality relations, show that

$$\hat{N}_B^{bare} \equiv \int d^3x\ \psi^+\psi = \sum_s \int d^3p\ \left[b_s^+(p)\ b_s(p) + d_s(p)\ d_s^+(p)\right]\ . \qquad (3)$$

(b) Explain why the expression, eq. (3), for $\hat{N}_B^{bare}$ implies any ground state (vacuum) of the theory to be unstable, and how this problem can be solved by subtracting an infinite constant leading to the "renormalized" baryon-density operator

$$\hat{\rho}_B = \sum_s \int d^3p\ \left[b_s^+(p)\ b_s(p) - d_s^+(p)\ d_s(p)\right]\ . \qquad (4)$$