Name: PHY625 (Fall 2008), 09/23/08

Homework Assignment #3

(*Due Date*: Thursday, October 02, 05:30 pm, in class)

3.1 *Free and In-Medium Nucleon-Nucleon Scattering: T- and G-matrix* *(3+3+3+2 pts.)*
In class the Lippmann-Schwinger equation (LSE) for the T-matrix for the scattering of 2 nucleons in free space has been introduced,

$$T(E;\vec{k}',\vec{k}) = V(\vec{k}',\vec{k}) + \int \frac{d^3p}{(2\pi)^3} V(\vec{k}',\vec{p})\ \frac{1}{E - p^2/m + i\eta}\ T(E;\vec{p},\vec{k})\ , \tag{1}$$

where $\pm\vec{k}$ and $\pm\vec{k}'$ denote the relative momentum of the 2 nucleons before and after the scattering, and E is the total kinetic energy, with $E = k^2/m = k'^2/m$, m=940 MeV the nucleon mass and η infinitesimal (neglect spin-isospin except for nuclear densities).

(a) Using the partial wave expansion for both potential V and T-matrix,

$$X(\vec{k}',\vec{k}) = 4\pi \sum_{l=0}^{\infty} (2l+1)\ P_l(u_{k'k})\ X_l(k',k)\ , \tag{2}$$

as well as azimuthal symmetry, show that the LSE can be reduced to

$$T_l(E;k',k) = V_l(k',k) + \frac{2}{\pi}\int p^2dp\ V_l(k',p)\ \frac{1}{E - p^2/m + i\eta}\ T_l(E;p,k)\ , \tag{3}$$

in each partial wave l. In eq. (2), $u_{k'k} = \cos(\alpha_{k'k})$ with $\alpha_{k'k} = \angle(\vec{k}',\vec{k})$; make use of the following identity for the Legendre polynomials

$$\int du_{pk}\ P_{l'}(u_{k'p})\ P_l(u_{pk}) = \frac{2\delta_{ll'}}{2l+1}\ P_l(u_{k'k})\ . \tag{4}$$

(b) Concentrate on the S-wave channel (l=0) and approximate the N-N interaction by the sum of 2 schematic potentials associated with σ- and ω-meson exchange,

$$V_0^\sigma(k',k) = -\frac{g_\sigma^2}{4\pi}\frac{F_\sigma(k)F_\sigma(k')}{m_\sigma^2}\quad ,\quad V_0^\omega(k',k) = +\frac{g_\omega^2}{4\pi}\frac{F_\omega(k)F_\omega(k')}{m_\omega^2}\ ; \tag{5}$$

hadronic formfactors, $F(k) = \Lambda^2/(\Lambda^2 + k^2)$, simulate the finite size of the hadrons and ensure the convergence of the 1-D LSE. Write down explicitly the first 3 terms of the Born series for the T-matrix; exploit the separability of the above potentials, $V(k',k) = v(k')\,v(k)$, to resum the T-matrix using a geometric series to obtain

$$T_0(E;k',k) = \frac{V_0(k',k)}{1-\Pi(E)}\quad ,\quad \Pi(E) = \frac{2}{\pi}\int p^2dp\ \frac{V_0(p)}{E - p^2/m + i\eta} \tag{6}$$

(c) For S-wave scattering, the total N-N cross section takes the form

$$\sigma_{\rm tot}^{l=0}(E) = 4\pi m^2 |T_0(E)|^2 \ . \tag{7}$$

To evaluate the "loop" function, $\Pi(E)$, make us of the following decomposition into real and imaginary parts:

$$\int dp \frac{f(p)}{p_0^2 - p^2 + i\eta} = PP \int_0^\infty dp \frac{f(p)}{p_0^2 - p^2} + \int dp \ f(p) \ (-i\pi)\delta(p_0^2 - p^2) \tag{8}$$

(here: $p_0^2 = mE$). The principle-value (PP) integral for the real part of $\Pi(E)$ requires a numerical integration. To avoid numerical instabilities when integrating over the pole, use the following "regularization" trick

$$PP \int_0^\infty dp \frac{f(p)}{p_0^2 - p^2} = PP \int_0^\infty dp \frac{f(p) - f(p_0)}{p_0^2 - p^2} \ , \quad \text{since} \quad PP \int_0^\infty \frac{dp}{p_0^2 - p^2} = 0 \tag{9}$$

Plot the cross section in [mb]=[0.1 fm^2] from threshold to E=150 MeV using the following values for the potential parameters: m_σ=550 MeV, g_σ=10.0, Λ_σ=1000 MeV, m_ω=782.6 MeV, g_ω=11.65, Λ_σ=1500 MeV; use $\hbar c$=197.33 MeVfm.

(d) Compute the in-medium N-N G-matrix by implementing a Pauli-Blocking factor, $[1 - f(\epsilon_p; \mu_N, T)]^2$, into the integral of the LSE (3), where

$$f(\epsilon_p; \mu_N, T) = \frac{1}{\exp[(\epsilon_p - \mu_N)/T] + 1} \tag{10}$$

with $\epsilon_p = p^2/2m$ and chemical potential $\mu_N = k_F^2/2m$ (k_F: Fermi momentum). Plot the in-medium N-N S-wave cross section for k_F=220 MeV with temperature T=1 MeV and T=5 MeV.