Name: PHY625 (Fall 2008), 09/11/08

Homework Assignment #2

(*Due Date*: Thursday, September 18, 05:30 pm, in class)

2.1 *2-Body Density Matrix in Fermi Gas Model* *(6+3+1 pts.)*

In class it has been shown that the (reduced) 2-body density matrix for a non-interacting Fermi gas of A nucleons within a volume V takes the form

$$\rho^{(2)}(\vec{r}_1, \vec{r}_2) = \frac{1}{A(A-1)V^2} \sum_{\vec{k}_1, \vec{k}_2} \left[1 - \cos(2\vec{k}_{ij} \cdot \vec{r})\right] \tag{1}$$

with $\vec{r} \equiv \vec{r}_2 - \vec{r}_1$ and $\vec{k}_{ij} = (\vec{k}_i - \vec{k}_j)/2$.

(a) Using the continuum limit for the momentum sums, $\sum_{\vec{k}} \to V \int_0^{k_F} \frac{d^3k}{(2\pi)^3}$ (also implying $A \to \infty$), show that the 2-body density matrix takes the form $\rho^{(2)}(\vec{r}_1, \vec{r}_2) = g_-(x)/V^2$ with

$$g_-(x) = 1 - \left[\frac{3}{x^2}\left(\frac{\sin x}{x} - \cos x\right)\right]^2 \ . \tag{2}$$

(b) Using Taylor expansions of the sine and cosine functions, show that $\lim_{x \to 0} g_-(x) = 0$, i.e., $\rho^{(2)}(\vec{r}_1, \vec{r}_2)$ exhibits "Pauli repulsion".

(c) Why does the Pauli repulsion in the spatial part of the 2-nucleon density matrix not suffice to stabilize atomic nuclei at a size that increases as $R_A \propto A^{1/3}$?