Name: PHY625 (Fall 2008), 08/26/08

Homework Assignment #1

(*Due Date*: Tuesday, September 08, 05:30 pm, in class)

1.1 *Yukawa Potential* *(4 pts.)*

Show that the Fourier transform of the static scalar-isoscalar meson-exchange potential,

$$V_\sigma(q) = -g_\sigma^2 \, \frac{1}{\vec{q}^2 + m_\sigma^2} \ , \tag{1}$$

yields the standard Yukawa potential in coordinate space, $V(r) = -g_\sigma^2/(4\pi)\ \mathrm{e}^{-m_\sigma r}/r$.

1.2 *Electron Scattering off Nuclei* *(6 pts.)*

In Born approximation, the differential cross section for an ultrarelativistic electron of energy $E \gg m_e$ (m_e=0.511 MeV) elastically scattering off an electrostatic potential $A_0(r)$ is given by (neglecting any intrinsic spin dependencies)

$$\frac{d\sigma}{d\Omega} = \frac{E^2}{4\pi^2} \left[e \int d^3r A_0(r)\ \mathrm{e}^{i\vec{q}\cdot\vec{r}} \right]^2 \ , \tag{2}$$

where $\vec{q} \equiv \vec{p}_f - \vec{p}_i$: momentum transfer to the electron, and e: electron charge.
(Note: the units are such that $\hbar$=c=1, and a conversion factor from fm to MeV^{-1} (or MeV to fm^{-1}) of $\hbar c$=197.33 MeV fm)

(a) Show that the relation between the magnitude of the momentum transfer, $q = |\vec{q}|$, and the scattering angle, θ, is given by $q = 2E\sin(\theta/2)$.

(b) Using Possion's equation, $\vec{\nabla}^2 A_0 = -Ze\rho_{ch}$, as well as $\vec{\nabla}^2 \mathrm{e}^{i\vec{q}\cdot\vec{r}} = -\vec{q}^2 \mathrm{e}^{i\vec{q}\cdot\vec{r}}$ and a partial integration, show that the cross section takes the form

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma_R}{d\Omega} \right) |F(q^2)|^2 \ , \tag{3}$$

where $(d\sigma_R/d\Omega) = Z^2\alpha^2/(4E^2\sin^4(\theta/2))$ is the Rutherford cross section for a point charge Z ($\alpha = e^2/4\pi$) and

$$F(q) = \int d^3r \rho_{\mathrm{ch}}(r) \mathrm{e}^{i\vec{q}\cdot\vec{r}} \tag{4}$$

the formfactor of the charge distribution.

(c) Sketch the angular dependence of the cross section for a uniform charge distribution, $\rho_{ch}(r) = C\ \theta(R_0 - r)$, assuming a radius R_0=5 fm (C: constant) and an electron energy E=E_i=E_f=150 MeV, and compare it to the result for a point charge, $\rho_{ch} = Z\ \delta^{(3)}(\vec{r})$. Start by determining the constant C to normalize the extended distribution to one.