Homework Assignment #1

(Due Date: Tuesday, September 08, 05:30 pm, in class)

1.1 Yukawa Potential

Show that the Fourier transform of the static scalar-isoscalar meson-exchange potential,

\[ V_\sigma(q) = -g_\sigma^2 \frac{1}{q^2 + m_\sigma^2}, \]  

yields the standard Yukawa potential in coordinate space, \( V(r) = -g_\sigma^2/(4\pi) e^{-m_\sigma r}/r \).

1.2 Electron Scattering off Nuclei

In Born approximation, the differential cross section for an ultrarelativistic electron of energy \( E \gg m_e \) (\( m_e = 0.511 \text{ MeV} \)) elastically scattering off an electrostatic potential \( A_0(r) \) is given by (neglecting any intrinsic spin dependencies)

\[ \frac{d\sigma}{d\Omega} = \frac{E^2}{4\pi^2} \left[ e^2 \int d^3 r A_0(r) e^{i\vec{q}\cdot\vec{r}} \right]^2, \]  

where \( \vec{q} \equiv \vec{p}_f - \vec{p}_i \): momentum transfer to the electron, and \( e \): electron charge. (Note: the units are such that \( \hbar = c = 1 \), and a conversion factor from fm to MeV\(^{-1} \) (or MeV to fm\(^{-1} \)) of \( \hbar c = 197.33 \text{ MeV fm} \)

(a) Show that the relation between the magnitude of the momentum transfer, \( q = |\vec{q}| \), and the scattering angle, \( \theta \), is given by \( q = 2E \sin(\theta/2) \).

(b) Using Possion’s equation, \( \nabla^2 A_0 = -Ze\rho_{ch} \), as well as \( \nabla^2 e^{i\vec{q}\cdot\vec{r}} = -q^2 e^{i\vec{q}\cdot\vec{r}} \) and a partial integration, show that the cross section takes the form

\[ \frac{d\sigma}{d\Omega} = \left( \frac{d\sigma_R}{d\Omega} \right) |F(q^2)|^2, \]  

where \( (d\sigma_R/d\Omega) = Z^2 \alpha^2/(4E^2 \sin^4(\theta/2)) \) is the Rutherford cross section for a point charge \( Z \) (\( \alpha = e^2/4\pi \)) and

\[ F(q) = \int d^3 r \rho_{ch}(r) e^{i\vec{q}\cdot\vec{r}} \]  

the formfactor of the charge distribution.

(c) Sketch the angular dependence of the cross section for a uniform charge distribution, \( \rho_{ch}(r) = C \theta(R_0 - r) \), assuming a radius \( R_0 = 5 \text{ fm} \) (\( C \): constant) and an electron energy \( E = E_i = E_f = 150 \text{ MeV} \), and compare it to the result for a point charge, \( \rho_{ch} = Z \delta^{(3)}(\vec{r}) \). Start by determining the constant \( C \) to normalize the extended distribution to one.