Homework Assignment #8

(Due Date: Tuesday, April 16, 05:30 pm, in class)

8.1 2-D Percolation and Critical Exponent (cf. Ex. 7.27+7.31 in the textbook) (3+2 pts.)
Write a FORTRAN program to simulate the percolation transition on a \(N \times N\) square lattice by subsequently filling lattice sites randomly, corresponding to an increase in occupation probability, \(p\).

(a) Determine the critical probability \(p_c\) by checking, after each new entry, for the appearance of a spanning cluster, and plot the latter when it first appears (once for each value of \(N\)). Perform this procedure for various lattice sizes \(N\) (e.g. \(N=5,10,15,20,30,50,80\)) using an average over ca. 50 simulations for each \(N\) and plot \(p_c(N^{-1})\) to extrapolate to the infinite-size limit, \(p_c(0)\).

(b) For fixed lattice size (as large as possible, e.g., \(N=100\)), compute the fraction

\[
F(p > p_c) = \frac{\text{no. of sites in spanning cluster}}{\text{no. of occupied sites}}
\]

as a function of \(p\) above the critical \(p_c\) (for chosen \(N\)). Average your results for each \(p\) over ca. 50 simulations. Fit your results to a power-law ansatz

\[
F = F_0(p - p_c)^\beta
\]

by plotting the logarithm of both sides and extracting the slope of a straight-line fit (note that the power-law only applies for \(p\) not “too far” above \(p_c\)).

9.2 2-D Ising Model (cf. Ex. 8.1,8.2+8.3 in the textbook) (2+3 pts.)
Consider the 2-D Ising Model on a square lattice with nearest-neighbor \((4)\) interaction strength \(J=2.5\) and at zero external magnetic field.

(a) Use the Newton-Raphson root finder algorithm to numerically solve the mean-field selfconsistency relation for the average magnetization,

\[
\langle s \rangle = \tanh \left( \frac{zJ}{k_B T} \langle s \rangle \right),
\]

in the temperature range from zero to just above \(T_c\). Compare your numerical results at temperature \(T\) near \(T_c\) and at small(!) \(T\) to analytic results obtained from a suitable Taylor expansion.

(b) Write a FORTRAN program to simulate the 2D Ising Model on a \(n \times n\) lattice with periodic boundary conditions (\(N=n^2\): total number of spins, \(M = \sum_i s_i = N\langle s \rangle\)). Calculate the (time-average) magnetization as a function of temperature (after sufficient Monte Carlo sweeps to reach equilibrium), and determine the critical temperature as well as the critical exponent \(\beta\) in \(\langle M \rangle \propto (T_c - T)^\beta\).

(Hints: to extract \(\beta\), focus on the temperature region \(T = (0.9 - 1) T_c\). For several “trial” values of \(T_c\), plot \(\log(M)\) vs. \(\log(T_c - T)\) and take as best estimate the case where a straight line fits best, and read off \(\beta\); perform this procedure for two lattice sizes, \(n=15\) and 25, to check for finite-size effects.)