Name: PHY401 (Spring 2013), 03/05/13

# Homework Assignment #6

(*Due Date*: Thursday, March 21, 05:30 pm, in class)

6.1 *Ideal Wave Propagation, Stability and Spectral Decomposition* (cf. Ex. 6.1, 6.2, 6.4, 6.9) *(3+1+2+2 pts.)*

Write a FORTRAN program to numerically calculate the signal propagation on a uniform string (length $L = 1m$, tension $F_T = 850N$, total mass $25g$, both ends fixed) using the ideal wave equation,

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} , \tag{1}$$

for the displacement $y(x,t)$ at string position $x$ and time $t$. Set the numerical coefficient $r \equiv c/(\Delta x/\Delta t)$ equal to one unless otherwise noted.

(a) Impart an initial Gaussian pluck on the string, $y(x, t_{0,1}) = y_0 \exp[-(x-\bar{x})^2/2\sigma_x^2]$ with $y_0 = 0.25m$, $\bar{x} = 0.4m$ from the left end of the string and a full-width-at-half-maximum of $0.1m$ (what is the pertinent value of $\sigma_x$?). Study the signal propagation by plotting time snapshots of your solution, $y(x, t_s)$, as the signal moves across the string. How are the signals reflected at the ends? Also produce a 3-D "surface" plot for $y$ as a function of $x$ and $t$ (using, e.g., the splot command in gnuplot); the time axis should cover at least 1.2 times the period of the motion.

(b) Study and comment on the consequences of choosing values of $r = 0.8, 1.2$ in your numerical solution in (a).

(c) Perform a Fourier analysis of the string excitation in part (a), i.e., for a fixed position on the string (e.g. $x_s = 0.5m$), record $y(t)$ and evaluate the Fourier transform using a suitable time discretization. Plot the power spectrum vs. frequency and interpret the peaks in the spectrum in terms of the harmonics of the string. Repeat the Fourier analysis for a different position on the string (e.g. $x_s = 0.7m$) and comment on the difference to $x_s = 0.5m$.

(d) Initialize the string with a more realistic "triangular" pluck, with a maximum $y(0.2, t_{0,1}) = 0.25m$, which linearly connects to the ends. Produce a surface plot of $y(x,t)$ as in part (a).

6.2 *Stiffness Term in Wave Equation* (cf. Ex. 6.15) *(2 pts.)*

The wave equation with stiffness correction for a string under tension $F_T$ reads

$$\frac{\partial^2 y}{\partial t^2} = c^2 \left( \frac{\partial^2 y}{\partial x^2} - \frac{Y}{F_T} \frac{\partial^4 y}{\partial x^4} \right) , \tag{2}$$

with $Y$: Young's modulus (no numerical work required in this problem).

(a) Derive the symmetric finite difference expression of the 4. order derivative term suitable for numerical evaluation.

(b) Derive the finite difference expression for $y(i, n+1)$ for the string displacement $y$ at time $t_{n+1}$ and position $x_i$ in terms of $y(x_{i-2,i-1,i,i+1,i+2}, t_{n,n-1})$.

(c) Discuss qualitatively the effect of a finite stiffness.